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БИАНИЗОТРОПНЫЕ МАТЕРИАЛЫ С СИЛЬНЫМ ЭЛЕКТРОМАГНИТНЫМ ВЗАИМОДЕЙСТВИЕМ

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BIANISOTROPIC MATERIALS OPTIMIZED FOR STRONG INTERACTIONS WITH ELECTROMAGNETIC FIELDS

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В статье обсуждается новая концепция метаматериалов для оптимального взаимодействия с электромагнитными волнами. В этих материалах форма элементов специально выбрана таким образом, чтобы у частиц была максимально возможная энергия в поле заданной плоской электромагнитной волны в композитной среде, составленной из данных частиц. Оказывается, что оптимальные частицы – бианизотропны и метаматериал для взаимодействия с плоскими линейно поляризованными затухающими волнами – это материал на основе Ω-*элементов*, в то время как для распространяющихся волн это определенный невзаимный материал.

Ключевые слова: *метаматериалы, электромагнитные волны, композитные среды, взаимодействие,* Ω-*элементы.*

In this report we discuss the new concept of “optimal metamaterials” for interactions with electromagnetic waves. In these materials the inclusion shape is chosen so that the particles have the maximum possible energy in the field of a given plane electromagnetic wave propagating or decaying inside the composite medium composed of these particles. It appears that the optimal particles are bianisotropic, and the “optimal material” for interactions with plane linearly polarized evanescent waves is the omega material, while for propagating waves it is a certain nonreciprocal material.

Keywords: *metamaterial, electromagnetic waves, composite medium, interactions, omega material.*

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Introduction

Recently, it was noticed that the shape of chiral particles (for example, helices) can be optimized in such a way that these optimal helices radiate waves of only one circular polarization, whatever is the exciting field [1]. Later, it was shown that the optimal spiral shape corresponds to the maximum (or minimum) energy of the particle in a given plane-wave field [2] interacting with the wave in the most effective way. Furthermore, mixtures of optimal spirals realize effective media with the index of refraction $n = -1$ for one of the circular polarizations, while the same medium is transparent for the orthogonal circular polarization [3]. It has been therefore established that there exist metamaterials whose properties are optimal for interactions with circularly polarized waves.

The goal of the present work is to find the optimal inclusions for metamaterials which would interact most effectively with linearly polarized plane waves. The inclusion properties should be optimized so that the optimal material would store as much energy density as possible in the field of a given plane wave (for a given concentration of inclusions). This study is restricted to composites formed by electrically small inclusions, modeled as electric and magnetic dipoles, so that the effective medium description remains possible.

Omega particle as the “best particle” for interaction with linearly polarized evanescent waves

It is obvious that in order to enhance the field-material interaction we need to design a medium which would be polarizable by both electric and magnetic fields, and the particles should be bianisotropic, but not just a set of individual electrically and magnetically polarizable inclusions. Indeed, in the optimal spiral the electromotive forces induced by electric and magnetic fields sum up in phase enhancing the overall current induced in the spiral. With this in view, we assume that the optimal particle is a bianisotropic particle whose electric and magnetic moments are induced by both fields. Considering for simplicity of writing a linearly polarized plane wave whose electric field is directed along and magnetic field along, we write the generic polarizability relation as

$$p_z = \alpha_{ee} E_z + \alpha_{em} H_y,$$
$$m_y = \alpha_{mm} H_y + \alpha_{me} E_z.$$

Here **p** and **m** are the induced electric and magnetic dipole moments, respectively, and alphas are the polarizability coefficients. We define the optimal interaction with the fields as the extremum value of the electromagnetic energy of the particle in the given field, which is equal to

$$W = -\frac{1}{2}\mathrm{Re}\,(\mathbf{p}\cdot\mathbf{E}^* + \mathbf{m}\cdot\mathbf{H}^*) =$$

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$$=\frac{1}{2}\mathrm{Re}\left\{\alpha_{ee}|E|^2+\alpha_{em}HE^*+\alpha_{me}EH^*+\alpha_{mm}|H|^2\right\}.$$

Here and in the following we drop the indices y, z for shortness.

Considering reciprocal particles, we use the relation following from the reciprocity theorem (e.g., [4]): $\alpha_{em}=-\alpha_{me}$. Denoting the wave impedance of the exciting plane wave by Z, we can rewrite the particle energy as

$$W=-\frac{1}{2}|E|^2\,\mathrm{Re}\left\{\alpha_{ee}+\alpha_{em}\left(\frac{1}{Z}-\frac{1}{Z^*}\right)+\alpha_{mm}\frac{1}{|Z|^2}\right\}.$$

At this point we can observe that reciprocal particles in the field of linearly polarized plane waves can "benefit" from magnetoelectric coupling in the particles only if the exciting wave is evanescent.

Indeed, for travelling waves the wave impedance Z is real, and the term proportional to α_{me} cancels out. However, for evanescent waves, for which the wave impedance is purely imaginary, the situation is dramatically different. Considering waves propagating or decaying in the direction shown on figure 1, the wave impedance for evanescent waves reads

$$Z=\frac{j}{\beta}\sqrt{\frac{\mu_0}{\varepsilon_0}},$$

where $\beta=\sqrt{\frac{k_t^2}{k_0^2}-1}$ for TE polarization (with respect to the decay direction) and $\beta=\frac{1}{\sqrt{\frac{k_t^2}{k_0^2}-1}}$ for TM polarization. Now we can optimize the coupling coefficient to maximize the particle energy.

To do that, we note that there exists so called "hierarchy of polarizabilities" of small bianisotropic particles, where the electric polarizability is the strongest, then comes the cross-coupling coefficient (first-order spatial dispersion), and the weakest effect is the artificial magnetism (second–order effect). As a specific example, we consider an omega particle (figure 1) and make use of approximate expressions for its polarizabilities (neglecting the electric polarization of the loop) [4]

$$\alpha_{ee}=-j\frac{l^2}{\omega Z_{tot}},$$

$$\alpha_{me}=\pm j\alpha_{ee}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{k_0S}{l},$$

$$\alpha_{mm}=\alpha_{ee}\frac{\mu_0}{\varepsilon_0}\left(\frac{k_0S}{l}\right)^2.$$

Here Z_{tot} is the total impedance of the particle (the sum of the input impedances of the loop and the pair of strips). At the resonance of the particle $Z_{tot}=R_{tot}$ is a real number. One can note that the "scaling factor" $\frac{k_0S}{l}$ is small for electrically very small particles, but can be of the order of unity for inclusions used in the design of metamaterials.

Finally, we are ready to express the particle energy in the form suitable for optimization of its shape:

$$W=-\frac{1}{2}|E|^2\frac{l^2}{\omega X_{tot}}\left\{1\pm 2x+x^2\right\},$$

where $x=\frac{\beta k_0S}{l}$ (we consider the resonant frequency case). Obviously, the energy takes the extremal values at $x=\pm 1$. One of these solutions corresponds to zero value of W (the particle is "invisible"), and for the other solution W is two times larger than the sum of energies of the two dipoles in the absence of bianisotropic coupling.

The physical nature of this optimal behaviour is similar to the case of chiral particles in the field of circularly polarized waves: In one extreme case the electromotive forces induced by electric and magnetic fields cancel out, and the total induced current is zero, while in the other extreme case the two forces sum up in phase and interaction is optimally enhanced. Note that the extremum condition $(x=\pm 1)$ depends not only on the particle shape and the frequency but also on the transverse wave number (decay factor) of the exciting evanescent wave.

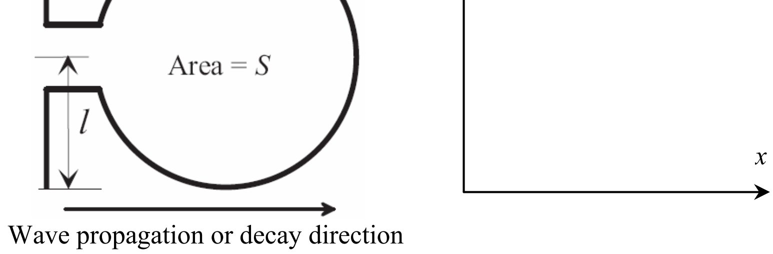

Figure 1 – An omega inclusion and the associated co-ordinate system

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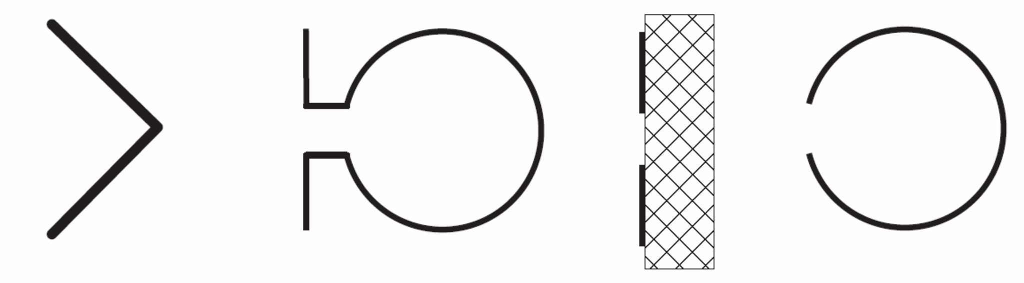

Figure 2 – Various geometries which exhibit omega coupling

The next step is to study eigenwaves in an effective medium formed by many optimal omega particles and check if the evanescent wave for which the particles have been tuned is actually an eigenwave in the medium. The answer to this question is positive. Furthermore, it is interesting to consider what inclusions are optimal for interactions with propagating linearly polarized waves. It can be shown that such optimal particles exist, but they are nonreciprocal inclusions, and the corresponding composite material belongs to the class of artificial moving media.

Conclusion

In summary, we have discussed the concept of optimal particles and composite materials for interactions with linearly polarized waves. The optimal particles extract maximum power from electromagnetic waves (for a given concentration and resonant frequency of inclusions). Apparently, these particles are also the optimal radiators of power. Many structures, widely used in various applications, have the symmetry which allows omega coupling (see some examples in figure 2).

The use of the optimal material is expected to allow optimization of electromagnetic performance of various devices, where the material interacts with linearly polarized plane waves (antennas, absorbers, sensors, lenses, and so on). This concept of optimal materials was first introduced in the conference presentation [5].

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