

**Министерство образования Республики Беларусь**

**Учреждение образования  
«Гомельский государственный университет  
имени Франциска Скорины»**

**Г.Н.ПЕТУХОВА**

**АНГЛИЙСКИЙ ЯЗЫК**

**ПРАКТИЧЕСКОЕ РУКОВОДСТВО**

*для студентов заочного факультета*

*специальности 1-31 03 01-02 «Математика (научно-педагогическая деятельность)»*

**Гомель 2011**

УДК 811.111(075.8)

ББК 81.432.1-923

П 77

Рецензенты:

Пузенко И.Н. – заведующий кафедрой белорусского и иностранных языков учреждения образования «Гомельский государственный технический университет имени П.О.Сухого», кандидат филологических наук, доцент;

кафедра английского языка учреждения образования «Гомельский государственный университет имени Франциска Скорины»

Рекомендовано к изданию на заседании научно-методическим советом учреждения образования «Гомельский государственный университет имени Франциска Скорины».

**Петухова, Г.Н.**

П 77 Английский язык: практическое руководство для студентов заочного факультета специальности 1-31 03 01-02 «Математика (научно-педагогическая деятельность)» / Г.Н.Петухова; М-во образов РБ, Гомельский гос. ун-т им. Ф.Скорины. – Гомель: ГГУ им.Ф.Скорины, 2011 – 38 с.

ISBN 978-985-439-327-8

Целью практического руководства является оказание помощи студентам факультета заочного обучения в накоплении и систематизации словарного запаса профессиональной лексики по предлагаемым темам.

Практическое руководство адресовано студентам 1 и 2 курса специальности «Математика (научно-педагогическая деятельность)» 1-31 03 01-02 и состоит из 19 тематических текстов и заданий к ним.

УДК 811.111(075.8)

ББК 81.432.1-923

ISBN 978-985-439-327-8

© Г.Н.Петухова, 2011

© УО «ГГУ им.Ф.Скорины», 2011

## СОДЕРЖАНИЕ

ВВЕДЕНИЕ .....	4
ТЕХТ 1 Arithmetic .....	5
ТЕХТ 2 Decimal arithmetic .....	7
ТЕХТ 3 Arithmetic operations .....	8
ТЕХТ 4 History of elementary algebra.....	11
ТЕХТ 5 Babylonian algebra .....	13
ТЕХТ 6 Egyptian algebra .....	14
ТЕХТ 7 Greek geometric algebra.....	15
ТЕХТ 8 Diophantine algebra.....	17
ТЕХТ 9 Indian algebra .....	18
ТЕХТ 10 Islamic algebra.....	20
ТЕХТ 11 European algebra .....	22
ТЕХТ 12 Modern algebra.....	23
ТЕХТ 13 History of geometry .....	26
ТЕХТ 14 Babylonian geometry.....	27
ТЕХТ 15 Classical Greek geometry .....	28
ТЕХТ 16 Hellenistic geometry .....	30
ТЕХТ 17 After Archimedes .....	31
ТЕХТ 18 Modern geometry .....	34
ТЕХТ 19 The 18th and 19th centuries.....	35
ЛИТЕРАТУРА .....	38

## **ВВЕДЕНИЕ**

В практическое руководство включены тематические тексты и упражнения, направленные на понимание содержания.

Практическое руководство состоит из 19 тематических текстов разного уровня сложности, что дает возможность преподавателю использовать их по своему усмотрению. Тексты снабжены упражнениями, направленными на контроль понимания прочитанного. В практическом руководстве содержатся также аутентичные тексты для самостоятельного чтения.

Целью практического руководства является оказание помощи студентам 1 и 2 курса факультета заочного обучения в накоплении и систематизации необходимого словарного запаса.

## TEXT 1 Arithmetic

*Read and translate the following text. Answer the questions.*

*Arithmetic* is the oldest and most elementary branch of mathematics, used by almost everyone, for tasks ranging from simple day-to-day counting to advanced science and business calculations. It involves the study of quantity, especially as the result of combining numbers. In common usage, it refers to the simpler properties when using the traditional operations of addition, subtraction, multiplication and division with smaller values of numbers. Professional mathematicians sometimes use the term (*higher*) *arithmetic* when referring to more advanced results related to number theory, but this should not be confused with elementary arithmetic.

### **History**

The prehistory of arithmetic is limited to a very small number of small artifacts indicating a clear conception of addition and subtraction, the best-known being the Ishango bone from central Africa, dating from somewhere between 20,000 and 18,000 BC.

The earliest written records indicate the Egyptians and Babylonians used all the elementary arithmetic operations as early as 2000 BC. These artifacts do not always reveal the specific process used for solving problems, but the characteristics of the particular numeral system strongly influence the complexity of the methods. The hieroglyphic system for Egyptian numerals, like the later Roman numerals, descended from tally marks used for counting. In both cases, this origin resulted in values which used a decimal base but did not include positional notation. Although addition was generally straightforward, multiplication in Roman arithmetic required the assistance of a counting board to obtain the results.

The early number systems which included positional notation were not decimal, including the sexagesimal system for Babylonian numerals and the vigesimal system which defined Maya numerals. Because of this place-value concept, the ability to reuse the same digits for different values contributed to simpler and more efficient methods of calculation.

The continuous historical development of modern arithmetic starts with the Hellenistic civilization of ancient Greece, although it originated much later than the Babylonian and Egyptian examples. Prior to the works of Euclid around 300 BC, Greek studies in mathematics overlapped with

philosophical and mystical beliefs. For example, Nicomachus summarized the viewpoint of the earlier Pythagorean approach to numbers, and their relationships to each other, in his *Introduction to Arithmetic*.

Greek numerals, derived from the hieratic Egyptian system, also lacked positional notation, and therefore imposed the same complexity on the basic operations of arithmetic.

The gradual development of Arabic numerals independently devised the place-value concept and positional notation which combined the simpler methods for computations with a decimal base and the use of a digit representing zero, allowing the system to consistently represent both large and small integers. This approach eventually replaced or superseded all other systems. In the early 6th century AD, the Indian mathematician Aryabhata incorporated an existing version of this system in his work, and experimented with different notations. In the 7th century, Brahmagupta established the use of zero as a separate number and determined the results for multiplication, division, addition and subtraction of zero and all other numbers, except for the result of division by zero.

In the Middle Ages, arithmetic was one of the seven liberal arts taught in universities.

The flourishing of algebra in the medieval Islamic world and in Renaissance Europe was an outgrowth of the enormous simplification of computation through decimal notation.

Various types of tools exist to assist in numeric calculations. Examples include slide rules (for multiplication, division, and trigonometry) and nomograph in addition to the electrical calculator.

**Notes:**

to refer to – ссылаться на

BC = Before Christ – до нашей эры

to reveal – обнаруживать, показывать

to descend – происходить, переходить от

decimal – десятичный

place-value concept – позиционная система разрядов

sexagesimal – шестидесятиричный

vigesimal – основанный на двадцати (о счислении)

integer – целое число

digit – цифра; однозначное число

nomograph – номограмма

***Answer the following questions:***

1. What is Arithmetic?
2. What is the prehistory of arithmetic limited to?
3. What does the continuous historical development of modern arithmetic start with?
4. What did gradual development of Arabic numerals devise?
5. What did arithmetic of the Middle Ages look like?
6. What tools were used in numeric calculations?

## **TEXT 2 Decimal arithmetic**

***Read and translate the following text. Answer the questions.***

Although *decimal notation* may conceptually describe any numerals from a system with a decimal base, it is commonly used exclusively for the written forms of numbers with Arabic numerals as the basic digits, especially when the numeral includes a decimal separator preceding a sequence of these digits to represent a fractional part of the number. In this common usage, the written form of the number implies the existence of positional notation. For example, 507.36 denotes 5 hundreds ( $10^2$ ), plus 0 tens ( $10^1$ ), plus 7 units ( $10^0$ ), plus 3 tenths ( $10^{-1}$ ) plus 6 hundredths ( $10^{-2}$ ). The conception of zero as a number comparable to the other basic digits, and the corresponding definition of multiplication and addition with zero, is an essential part of this notation.

Algorithm comprises all of the rules for performing arithmetic computations using this type of written numeral. For example, addition produces the sum of two arbitrary numbers. The result is calculated by the repeated addition of single digits from each number which occupy the same position, proceeding from right to left.

The process for multiplying two arbitrary numbers is similar to the process for addition. A multiplication table with ten rows and ten columns will list the results for each pair of digits.

Similar techniques exist for subtraction and division.

The creation of a correct process for multiplication relies on the relationship between values of adjacent digits. The value for any single digit in a numeral depends on its position. Also, each position to the left represents a value which is ten times larger than the position to the right. In mathematical terms, the exponent for the base of ten increases by one (to the left) or decreases by one (to the right). Therefore, the value for any

arbitrary digit is multiplied by a value of the form  $10^n$  with integer  $n$ . The list of values corresponding to all possible positions for a single digit is written as  $\{\dots, 10^2, 10, 1, 10^{-1}, 10^{-2}, \dots\}$ .

Repeated multiplication of any value in this list by ten will produce another value in the list. In mathematical terminology, this characteristic is defined as closure, and the previous list is described as *closed under multiplication*.

**Notes:**

exclusively – единственно, исключительно, только

preceding – предшествующий; упомянутый, данный выше

sequence – последовательность;

fractional – дробный;

corresponding – соответствующий;

to comprise – составлять;

arbitrary – произвольный;

addition – сложение;

multiplication – умножение;

subtraction and division – вычитание и деление

closure – замкнутость.

***Answer the following questions:***

1. What is decimal notation used for?
2. What does Algorithm comprise?
3. What does the creation of correct process for multiplication rely on?
4. How do you understand the term “closure”?

### **TEXT 3 Arithmetic operations**

***Read and translate the following text. Answer the questions.***

The basic arithmetic operations are addition, subtraction, multiplication and division, although this subject also includes more advanced operations, such as manipulations of percentages, square roots, exponentiation, and logarithmic functions. Arithmetic is performed according to an order of operations. Any set of objects upon which all four operations of arithmetic can be performed (except division by zero), and wherein these four operations obey the usual laws, is called a field.



**Notes:**

to include – включать в себя;  
according to – согласно чему-либо;  
to perform – производить.

**Addition (+)**

Addition is the basic operation of arithmetic. In its simplest form, addition combines two numbers, the *addends* or *terms*, into a single number, the *sum* of the numbers.

Adding more than two numbers can be viewed as repeated addition; this procedure is known as summation and includes ways to add infinitely many numbers in an infinite series; repeated addition of the number one is the most basic form of counting.

Addition is commutative and associative so the order in which the terms are added does not matter. The identity element of addition (the additive identity) is 0, that is, adding zero to any number will yield that same number. Also, the inverse element of addition (the additive inverse) is the opposite of any number, that is, adding the opposite of any number to the number itself will yield the additive identity, 0. For example, the opposite of 7 is -7, so  $7 + (-7) = 0$ .

**Notes:**

repeated addition – повторяющееся сложение;  
does not matter – не имеет значения;  
identity element – тождественный элемент;  
to yield – производить, приносить, давать (плоды, доход).

**Subtraction (-)**

Subtraction is the opposite of addition. Subtraction finds the *difference* between two numbers, the *minuend* minus the *subtrahend*. If the minuend is larger than the subtrahend, the difference will be positive; if the minuend is smaller than the subtrahend, the difference will be negative; and if they are equal, the difference will be zero.

Subtraction is neither commutative nor associative. For that reason, it is often helpful to look at subtraction as addition of the minuend and the opposite of the subtrahend, that is  $a - b = a + (-b)$ . When written as a sum, all the properties of addition hold.

**Notes:**

difference – разность;  
minuend – уменьшаемое;

subtrahend – вычитаемое;  
properties – свойства.

### **Multiplication ( $\times$ , $\cdot$ , or $*$ )**

Multiplication is the second basic operation of arithmetic. Multiplication also combines two numbers into a single number, the *product*. The two original numbers are called the *multiplier* and the *multiplicand*, sometimes both simply called *factors*.

Multiplication is best viewed as a scaling operation. If the real numbers are imagined as lying in a line, multiplication by a number, say  $x$ , greater than 1 is the same as stretching everything away from zero uniformly, in such a way that the number 1 itself is stretched to where  $x$  was. Similarly, multiplying by a number less than 1 can be imagined as squeezing towards zero. (Again, in such a way that 1 goes to the multiplicand.)

Multiplication is commutative and associative; further it is distributive over addition and subtraction. The multiplicative identity is 1, that is, multiplying any number by 1 will yield that same number. Also, the multiplicative inverse is the reciprocal of any number (except zero; zero is the only number without a multiplicative inverse), that is, multiplying the reciprocal of any number by the number itself will yield the multiplicative identity.

#### **Notes:**

product – произведение;  
multiplicand – множимое;  
multiplier – множитель;  
factors – сомножители;  
similarly – подобным образом;  
inverse – обратный;  
reciprocal – обратный, взаимный.

### **Division ( $\div$ or $/$ )**

Division is essentially the opposite of multiplication. Division finds the *quotient* of two numbers, the *dividend* divided by the *divisor*. Any dividend divided by zero is undefined. For positive numbers, if the dividend is larger than the divisor, the quotient will be greater than one, otherwise it will be less than one (a similar rule applies for negative numbers). The quotient multiplied by the divisor always yields the dividend.

Division is neither commutative nor associative. As it is helpful to look at subtraction as addition, it is helpful to look at division as multiplication of the dividend times the reciprocal of the divisor, that is  $a \div b = a \times \frac{1}{b}$ . When written as a product, it will obey all the properties of multiplication.

**Notes:**

quotient – частное;  
dividend – делимое;  
divisor – делитель.

**Number theory**

The term *arithmetic* is also used to refer to number theory. This includes the properties of integers related to primality, divisibility, and the solution of equations in integers, as well as modern research which is an outgrowth of this study. It is in this context that one runs across the fundamental theorem of arithmetic and arithmetic functions. *A Course in Arithmetic* by Jean-Pierre Serre reflects this usage, as do such phrases as *first order arithmetic* or *arithmetical algebraic geometry*. Number theory is also referred to as *the higher arithmetic*, as in the title of Harold Davenport's book on the subject.

**Notes:**

primality – простота  
equation – уравнение.

**Answer the following questions:**

- 1 How many arithmetic operations do you know?
- 2 What is the essence of addition?
- 3 What is the inverse operation of addition?
- 4 What is multiplication characterised by?
- 5 What is the inverse operation of multiplication?

**TEXT 4 History of elementary algebra**

**Read and translate the following text. Answer the questions.**

*Algebra* is a branch of mathematics concerning the study of structure, relation, and quantity. Elementary algebra is the branch that deals with solving for the operands of arithmetic equations. Modern or abstract

algebra has its origins as an abstraction of elementary algebra. Some historians believe that the earliest mathematical research was done by the priest classes of ancient civilizations, such as the Babylonians, to go along with religious rituals. The origins of algebra can thus be traced back to ancient Babylonian mathematicians roughly four thousand years ago.

The word *Algebra* is derived from the Arabic word *Al-Jabr*, and this comes from the treatise written in 820 by the Persian mathematician, Muhammad ibn Mūsā al-Khwārizmī, which can be translated as *The Compendious Book on Calculation by Completion and Balancing*. The treatise provided for the systematic solution of linear and quadratic equations. Although the exact meaning of the word *al-jabr* is still unknown, most historians agree that the word meant something like "restoration", "completion" and "reuniter of broken bones" or "bonesetter." The term is used by al-Khwarizmi to describe the operations that he introduced, "reduction" and "balancing", referring to the transposition of subtracted terms to the other side of an equation, that is, the cancellation of like terms on opposite sides of the equation.

Algebra did not always make use of the symbolism that is now ubiquitous in mathematics; rather, it went through three distinct stages. The stages in the development of symbolic algebra are roughly as follows:

- *Rhetorical algebra*, where equations are written in full sentences. For example, the rhetorical form of  $x + 1 = 2$  is "The thing plus one equals two" or possibly "The thing plus 1 equals 2". Rhetorical algebra was first developed by the ancient Babylonians and remained dominant up to the 16<sup>th</sup> century.

- *Syncopated algebra*, where some symbolism is used but which does not contain the entire characteristic of symbolic algebra. For instance, there may be a restriction that subtraction may be used only once within one side of an equation, which is not the case with symbolic algebra. Syncopated algebraic expression first appeared in *Diophantus' Arithmetica*, followed by Brahmagupta's *Brahma Sphuta Siddhanta*.

- *Symbolic algebra*, where full symbolism is used. Early steps toward this can be seen in the work of several Islamic mathematicians such as Ibn al-Banna and al-Qalasadi, though fully symbolic algebra sees its culmination in the work of René Descartes.

As important as the symbolism, or lack thereof, that was used in algebra was the degree of the equations that were used. Quadratic equations played an important role in early algebra; and throughout most

of history, until the early modern period, all quadratic equations were classified as belonging to one of three categories:

- $x^2 + px = q$
- $x^2 = px + q$
- $x^2 + q = px$

where  $p$  and  $q$  are positive. This trichotomy comes about because quadratic equations of the form  $x^2 + px + q = 0$ , with  $p$  and  $q$  positive, have no positive roots.

In between the rhetorical and syncopated stages of symbolic algebra, a *geometric constructive algebra* (algebraic equations were solved through geometry) was developed by classical Greek and Vedic Indian mathematicians in which. For instance, an equation of the form  $x^2 = A$  was solved by finding the side of a square of area  $A$ .

**Notes:**

trichotomy – трихотомия (деление на три части, на три элемента);

syncopated algebra – алгебра с использованием элементов символизма.

***Answer the following questions:***

1. What is Algebra?
2. What is the word Algebra devised from?
3. What are the main stages of Algebra?

## **TEXT 5 Babylonian algebra**

***Read and translate the following text. Answer the questions.***

The origins of algebra can be traced to the ancient Babylonians, who developed a positional number system which greatly aided them in solving their rhetorical algebraic equations. The Babylonians were not interested in exact solutions but approximations, and so they would commonly use linear interpolation to approximate intermediate values. One of the most famous tablets is the Plimpton 322 tablet, created around 1900 - 1600 BCE, which gives a table of Pythagorean triples and represents some of the most advanced mathematics prior to Greek mathematics.

Babylonian algebra was much more advanced than the Egyptian algebra of the time; whereas the Egyptians were mainly concerned with linear equations the Babylonians were more concerned with quadratic and cubic equations. The Babylonians had developed flexible algebraic

operations with which they were able to add equals to equals and multiply both sides of an equation by like quantities so as to eliminate fractions and factors. They were familiar with many simple forms of factoring, three-term quadratic equations with positive roots, and many cubic equations although it is not known if they were able to reduce the general cubic equation.

**Notes:**

solutions – решения;

approximations – приближения;

interpolation – интерполяция;

intermediate values – промежуточные значения;

tablets – дощечки, таблички (с надписью);

triple – тройной, утроенный.

***Answer the following questions:***

1. What can the origins of algebra be traced to?
2. Was the Babylonian algebra much more advanced than the Egyptian algebra?
3. What operations were the Babylonians familiar with?

## **TEXT 6 Egyptian algebra**

***Read and translate the following text. Answer the questions.***

Ancient Egyptian algebra dealt mainly with linear equations while the Babylonians found these equations too elementary and developed mathematics to a higher level than the Egyptians.

The Rhind Papyrus, also known as the Ahmes Papyrus, is an ancient Egyptian papyrus written circa 1650 BCE by Ahmes, who transcribed it from an earlier work that he dated to between 2000 and 1800 BCE. It is the most extensive ancient Egyptian mathematical document known to historians. The Rhind Papyrus contains problems where linear equations of the form  $x + ax = b$  and  $x + ax + bx = c$  are solved, where  $a$ ,  $b$ , and  $c$  are known and  $x$ , which is referred to as "aha" or heap, is the unknown. The solutions were possibly, but not likely, arrived at by using the "method of false position".

**Notes:**

higher level – более высокий уровень;

to arrive at – приходить к (заключению);

false position – метод ложного положения.

**Answer the following questions:**

1. What does Ancient Egyptian algebra deal with?
2. What is the method of “false position” characterised by?
3. What problems does the Rhind Papyrus contain?

## **TEXT 7 Greek geometric algebra**

**Read and translate the following text. Answer the questions.**

It is sometimes alleged that the Greeks had no algebra, but this is inaccurate. By the time of Plato, Greek mathematics had undergone a drastic change. The Greeks created a geometric algebra where terms were represented by sides of geometric objects, usually lines, that had letters associated with them and with this new form of algebra they were able to find solutions to equations by using a process that they invented which is known as "the application of areas". "The application of areas" is only a part of geometric algebra and it is thoroughly covered in Euclid's *Elements*.

An example of geometric algebra would be solving the linear equation  $ax = bc$ . The ancient Greeks would solve this equation by looking at it as an equality of areas rather than as an equality between the ratios  $a:b$  and  $c:x$ . The Greeks would construct a rectangle with sides of length  $b$  and  $c$ , then extend a side of the rectangle to length  $a$ , and finally they would complete the extended rectangle so as to find the side of the rectangle that is the solution.

**Notes:**

ratio – отношение;

rectangle – прямоугольник.

### **Euclid of Alexandria**

Euclid was a Greek mathematician who flourished in Alexandria, Egypt, almost certainly during the reign of Ptolemy I (323–283 BC). Neither the year nor place of his birth have been established, nor the circumstances of his death.

Euclid is regarded as the "father of geometry". His *Elements* is the most successful textbook in the history of mathematics. Although he is one of the most famous mathematicians in history there are no new discoveries attributed to him, rather he is remembered for his great explanatory skills. The *Elements* is not, as is sometimes thought, a collection of all Greek

mathematical knowledge to its date, rather, it is an elementary introduction to it.

**Notes:**

to flourish – процветать;

circumstance – обстоятельства;

to attribute to – приписывать (чему-либо, кому-либо).

**Elements**

The geometric work of the Greeks, typified in Euclid's *Elements*, provided the framework for generalizing formulae beyond the solution of particular problems into more general systems of stating and solving equations.

Book II of the *Elements* contains fourteen propositions, which in Euclid's time were extremely significant for doing geometric algebra. These propositions and their results are the geometric equivalents of our modern symbolic algebra and trigonometry. Today, using modern symbolic algebra, we let symbols represent known and unknown magnitudes (i.e. numbers) and then apply algebraic operations on them. While in Euclid's time magnitudes were viewed as line segments and then results were deduced using the axioms or theorems of geometry.

Many basic laws of addition and multiplication are included or proved geometrically in the *Elements*.

**Notes:**

proposition – предположение;

significant – значительный;

magnitude – величина.

**Data**

*Data* is a work written by Euclid for use at the University of Alexandria and it was meant to be used as a companion volume to the first six books of the *Elements*. The book contains some fifteen definitions and ninety-five statements, of which there are about two dozen statements that serve as algebraic rules or formulas. Some of these statements are geometric equivalents to solutions of quadratic equations. For instance, *Data* contains the solutions to the equations  $dx^2 - adx + b^2c = 0$  and the familiar Babylonian equation  $xy = a^2$ ,  $x \pm y = b$ .

**Notes:**

definition – определение;

to contain – содержать.



**Answer the following questions:**

1. What kind of algebra did the Greeks create?
2. Who is regarded as “the father of geometry”?
3. What book did Euclid write?
4. What propositions do the Elements contain?
5. What laws are included in the Elements?
6. What solutions does Data contain?

## **TEXT 8 Diophantine algebra**

**Read and translate the following text. Answer the questions.**

Cover of the 1621 edition of Diophantus' *Arithmetica*, translated into Latin by Claude Gaspard Bachet de Méziriac.

Diophantus was a mathematician who lived circa 250 AD, but the uncertainty of this date is so great that it may be off by more than a century. He is known for having written *Arithmetica*, a treatise that was originally thirteen books but of which only the first six have survived. *Arithmetica* has very little in common with traditional Greek mathematics since it is divorced from geometric methods, and it is different from Babylonian mathematics in that Diophantus is concerned primarily with exact solutions, both determinate and indeterminate, instead of simple approximations.

In *Arithmetica*, Diophantus is the first to use symbols for unknown numbers as well as abbreviations for powers of numbers, relationships, and operations; thus he used what is now known as *syncopated* algebra.

*Arithmetica* is a collection of some 150 solved problems with specific numbers and there is no postulation development nor is a general method explicitly explained, although generality of method may have been intended and there is no attempt to find all of the solutions to the equations. *Arithmetica* does contain solved problems involving several unknown quantities, which are solved, if possible, by expressing the unknown quantities in terms of only one of them. *Arithmetica* also makes use of the identities:

$$\begin{aligned} (a^2 + b^2)(c^2 + d^2) &+ (-ad)^2 = (ac + db)^2 + (bc - ad)^2 \\ &= (ad + bc)^2 + (ac - bd)^2 \end{aligned}$$

$$-bd)^2$$

**Notes:**

determinate – детерминированный, определенный, установленный; заданный, фиксированный

**Answer the following questions:**

1. When did Diophantus live?
2. What did Diophantus use for unknown numbers?
3. Is Arithmetica a collection of problems?

**TEXT 9 Indian algebra****Read and translate the following text. Answer the questions.**

The method known as "Modus Indorum" or the method of the Indians has become our algebra today. This algebra came along with the Hindu Number system to Arabia and then migrated to Europe. The earliest known Indian mathematical documents are dated to around the middle of the first millennium B.C.E (around the sixth century B.C.E.).

The recurring themes in Indian mathematics are, among others, determinate and indeterminate linear and quadratic equations, simple mensuration, and Pythagorean triples.

**Aryabhata**

Aryabhata (476–550 A.D.) was an Indian mathematician who authored *Aryabhataiya*. In it he gave the rules,

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

and

$$1^3 + 2^3 + \dots + n^3 = (1 + 2 + \dots + n)^2$$

**Notes:**

mensuration – измерение, замер

**Answer the following questions:**

1. What was Aryabhata?
2. What rules did Aryabhata give?

### **Brahma Sphuta Siddhanta**

Brahmagupta (fl. 628) was an Indian mathematician who authored *Brahma Sphuta Siddhanta*. In his work Brahmagupta solves the general quadratic equation for both positive and negative roots. In indeterminate

analysis Brahmagupta gives the Pythagorean triads  $m, \frac{1}{2}\left(\frac{m^2}{n} - n\right),$

$\frac{1}{2}\left(\frac{m^2}{n} + n\right),$  but this is a modified form of an old Babylonian rule that

Brahmagupta may have been familiar with. He was the first to give a general solution to the linear Diophantine equation  $ax + by = c,$  where  $a, b,$  and  $c$  are integers.

Like the algebra of Diophantus, the algebra of Brahmagupta was syncopated. Addition was indicated by placing the numbers side by side, subtraction by placing a dot over the subtrahend, and division by placing the divisor below the dividend, similar to our notation but without the bar. Multiplication, evolution, and unknown quantities were represented by abbreviations of appropriate terms. The extent of Greek influence on this syncopation, if any, is not known and it is possible that both Greek and Indian syncopation may be derived from a common Babylonian source.

#### **Notes:**

positive and negative roots – положительные и отрицательные корни;

to be derived from – БЫТЬ ЗАИМСТВОВАННЫМ ИЗ ...

#### **Answer the following questions:**

1. What equations does Brahmagupta solve in his work?
2. Was Brahmagupta the first to give a general solution to linear Diophantine equations?
3. Who influenced on his work?

### **Bhāskara II**

Bhāskara II (1114-ca. 1185) was the leading mathematician of the twelfth century. In Algebra, he gave the general solution of the Pell equation. He is the author of *Lilavati* and *Vija-Ganita*, which contain problems dealing with determinate and indeterminate linear and quadratic

equations, and Pythagorean triples and he fails to distinguish between exact and approximate statements. Many of the problems in *Lilavati* and *Vija-Ganita* are derived from other Hindu sources, and so Bhaskara is at his best in dealing with indeterminate analysis.

Bhaskara uses the initial symbols of the names for colors as the symbols of unknown variables.

***Answer the following questions:***

1. When did Bhaskara live?
2. What problems do his books contain?
3. What does he fail to distinguish?

## **TEXT 10 Islamic algebra**

***Read and translate the following text. Answer the questions.***

The first century of the Islamic Arab Empire saw almost no scientific or mathematical achievements since the Arabs, with their newly conquered empire, had not yet gained any intellectual drive and research in other parts of the world had faded. In the second half of the eighth century Islam had a cultural awakening, and research in mathematics and the sciences increased.

There are three theories about the origins of Arabic Algebra. The first emphasizes Hindu influence, the second emphasizes Mesopotamian or Persian-Syriac influence and the third emphasizes Greek influence. Many scholars believe that it is the result of a combination of all three sources.

Throughout their time in power, before the fall of Islamic civilization, the Arabs used a fully rhetorical algebra, where often even the numbers were spelled out in words. The Arabs would eventually replace spelled out numbers (e.g. twenty-two) with Arabic numerals (e.g. 22), but the Arabs never adopted or developed a syncopated or symbolic algebra, until the work of Ibn al-Banna in the 13th century and Abū al-Hasan ibn Alī al-Qalasādī in the 15th century.

### **Al-jabr wa'l muqabalah**

The Muslim Persian mathematician Muhammad ibn Mūsā al-Khwārizmī was a faculty member of the "House of Wisdom" (*Bait al-Hikma*) in Baghdad, which was established by Al-Mamun. Al-Khwarizmi, who died around 850 CE, wrote more than half a dozen mathematical and

astronomical works; some of which were based on the Indian *Sindhind*. One of al-Khwarizmi's most famous books is entitled *Al-jabr wa'l muqabalah* or *The Compendious Book on Calculation by Completion and Balancing*, and it gives an exhaustive account of solving polynomials up to the second degree. The book also introduced the fundamental concept of "reduction" and "balancing", referring to the transposition of subtracted terms to the other side of an equation, the second chapter deals with squares equal to number ( $ax^2 = c$ ), the third chapter deals with roots equal to a number ( $bx = c$ ), the fourth chapter deals with squares and roots equal a number ( $ax^2 + bx = c$ ), the fifth chapter deals with squares and number equal roots ( $ax^2 + c = bx$ ), and the sixth and final chapter deals with roots and number equal to squares ( $bx + c = ax^2$ ).

In *Al-Jabr*, al-Khwarizmi uses geometric proofs, he does not recognize the root  $x = 0$ , and he only deals with positive roots. He also recognizes that the discriminant must be positive and described the method of completing the square, though he does not justify the procedure.

**Notes:**

CE = AN (Anno Domini) – наша эра;

to refer to – ссылаться на ...;

to justify – подтверждать, доказывать.

**Omar Khayyám, Sharaf al-Dīn, and al-Kashi**

Omar Khayyám (ca. 1050 - 1123) wrote a book on Algebra that went beyond *Al-Jabr* to include equations of the third degree. Omar Khayyám provided both arithmetic and geometric solutions for quadratic equations, but he only gave geometric solutions for general cubic equations since he mistakenly believed that arithmetic solutions were impossible. Omar Khayyám generalized the method to cover all cubic equations with positive roots. He only considered positive roots and he did not go past the third degree. He also saw a strong relationship between Geometry and Algebra.

**Notes:**

ca = circa – приблизительно, примерно;

mistakenly – ошибочно.

**Al-Hassār, Ibn al-Banna, and al-Qalasadi**

Al-Hassār, a mathematician from the Maghreb (North Africa) specializing in Islamic inheritance jurisprudence during the 12th century, developed the modern symbolic mathematical notation for fractions, where the numerator and denominator are separated by a horizontal bar. This

same fractional notation appeared soon after in the work of Fibonacci in the 13th century.

Abū al-Hasan ibn Alī al-Qalasādī (1412–1482) was the last major medieval Arab algebraist, who made the first attempt at creating an algebraic notation since Ibn al-Banna two centuries earlier, who was himself the first to make such an attempt since Diophantus and Brahmagupta in ancient times. The syncopated notations of his predecessors, however, lacked symbols for mathematical operations. Al-Qalasadi "took the first steps toward the introduction of algebraic symbolism by using letters in place of numbers" and by "using short Arabic words, or just their initial letters, as mathematical symbols."

**Notes:**

inheritance – наследие;

notation – условные знаки, применяемые для выражения каких-либо понятий.

***Answer the following questions:***

1. How many theories are there about the origins of Arabic algebra?
2. What kind of algebra did the Arabs use?
3. What are the names of the Arabian mathematicians of the 13-th and 15-th centuries?
4. Who gave an exhaustive account of solving polynomials up to the second degree?
5. What fundamental concepts did the book introduce?
6. What kinds of roots did al Kwarizmi deal with?
7. Who wrote about the equations of the third degree?
8. Did Omar Khayyam see a strong relationship between Geometry and Algebra?
9. Who was the last major medieval Arabian algebraist?
10. What is Al-Hassar famous for?

## **TEXT 11 European algebra**

***Read and translate the following text. Answer the questions.***

### **Dark Ages**

The year 529 is now taken to be the beginning of the medieval period. Scholars fled the West towards the more hospitable East,

particularly towards Persia, where they found haven under King Chosroes and established what might be termed an "Athenian Academy in Exile". Under a treaty with Justinian, Chosroes would eventually return the scholars to the Eastern Empire. During the Dark Ages, European mathematics was at its nadir with mathematical research consisting mainly of commentaries on ancient treatises; and most of this research was centered in the Byzantine Empire. The end of the medieval period is set as the fall of Constantinople to the Turks in 1453.

### **Late Middle Ages**

The twelfth century saw a flood of translations from Arabic into Latin and by the thirteenth century, European mathematics was beginning to rival the mathematics of other lands. In the thirteenth century, the solution of a cubic equation by Fibonacci is representative of the beginning of a revival in European algebra.

As the Islamic world was declining after the fifteenth century, the European world was ascending. And it is here that Algebra was further developed.

#### **Notes:**

hospitable – восприимчивый, открытый;

treaty – договор;

nadir – противоположный;

medieval – средневековый.

#### ***Answer the following questions:***

1. What is the year 529 famous for?
2. What is the 12-th century characterised by?

## **TEXT 12 Modern algebra**

### ***Read and translate the following text. Answer the questions.***

Another key event in the further development of algebra was the general algebraic solution of the cubic and quartic equations, developed in the mid-16th century. The idea of a determinant was developed by Japanese mathematician Kowa Seki in the 17th century, followed by Gottfried Leibniz ten years later, for the purpose of solving systems of simultaneous linear equations using matrices. Gabriel Cramer also did some work on matrices and determinants in the 18th century.

The symbol  $x$  commonly denotes an unknown variable. Even though any letter can be used,  $x$  is the most common choice. This usage can be traced back to the Arabic word “thing,” used in Arabic algebra texts such as the *Al-Jabr*, and was taken into Old Spanish with the pronunciation “šei,” which was written *xei*, and was soon habitually abbreviated to  $x$ . (The Spanish pronunciation of “ $x$ ” has changed since). Some sources say that this  $x$  is an abbreviation of Latin *causa*. This started the habit of using letters to represent quantities in algebra. In mathematics, an “italicized  $x$ ” ( $x$ ) is often used to avoid potential confusion with the multiplication symbol.

**Notes:**

simultaneous – одновременно;

to avoid – избегать чего-либо;

confusion – путаница;

quartic – уравнение четвертой степени, поверхность четвертого порядка.

**Gottfried Leibniz**

Although the mathematical notion of function was implicit in trigonometric and logarithmic tables, which existed in his day, Gottfried Leibniz was the first, in 1692 and 1694, to employ it explicitly, to denote any of several geometric concepts derived from a curve, such as abscissa, ordinate, tangent, chord, and the perpendicular. In the 18th century, “function” lost these geometrical associations.

Leibniz realized that the coefficients of a system of linear equations could be arranged into an array, now called a matrix, which can be manipulated to find the solution of the system, if any. This method was later called Gaussian elimination. Leibniz also discovered Boolean algebra and symbolic logic, also relevant to algebra.

**Notes:**

curve – кривая;

relevant – релевантный, значимый, существенный, важный.

***Answer the following questions:***

1. Who used matrices for solving the systems of simultaneous linear equations?
2. What started the habit of using letters to represent quantities in potential confusion with the multiplication symbol?
3. What was Leibniz famous for?



## **Abstract algebra**

Abstract algebra was developed in the 19th century, initially focusing on what is now called Galois Theory, and on constructability issues.

## **The father of algebra**

The Hellenistic mathematician Diophantus has traditionally been known as "the father of algebra" but debate now exists as to whether or not Al-Khwarizmi deserves this title instead. Those who support Diophantus point to the fact that the algebra found in *Al-Jabr* is more elementary than the algebra found in *Arithmetica* and that *Arithmetica* is syncopated while *Al-Jabr* is fully rhetorical.

Those who support Al-Khwarizmi point to the fact that he gave an exhaustive explanation for the algebraic solution of quadratic equations with positive roots, and was the first to teach algebra in an elementary form and for its own sake, whereas Diophantus was primarily concerned with the theory of numbers. Al-Khwarizmi also introduced the fundamental concept of "reduction" and "balancing" (which he originally used the term *al-jabr* to refer to), referring to the transposition of subtracted terms to the other side of an equation, that is, the cancellation of like terms on opposite sides of the equation. Other supporters of Al-Khwarizmi point to his algebra no longer being concerned "with a series of problems to be resolved." They also point to his treatment of an equation for its own sake and "in a generic manner, insofar as it does not simply emerge in the course of solving a problem, but is specifically called on to define an infinite class of problems." Al-Khwarizmi's work established algebra as a mathematical discipline that is independent of geometry and arithmetic.

### **Notes:**

exhaustive – исчерпывающий;

cancellation – отмена;

insofar – до такой степени.

### ***Answer the following questions:***

1. Who has been known as the "father of algebra"?
2. What debate does exist?

## TEXT 13 History of geometry

*Read and translate the following text. Answer the questions.*

*Geometry* (Greek geo = earth, metria = measure) arose as the field of knowledge dealing with spatial relationships. Geometry was one of the two fields of pre-modern mathematics, the other being the study of numbers.

Classic geometry was focused in compass and straightedge constructions. Geometry was revolutionized by Euclid, who introduced mathematical rigor and the axiomatic method still in use today. His book, *The Elements* is widely considered the most influential textbook of all time, and was known to all educated people in the West until the middle of the 20th century.

In modern times, geometric concepts have been generalized to a high level of abstraction and complexity, and have been subjected to the methods of calculus and abstract algebra, so that many modern branches of the field are barely recognizable as the descendants of early geometry. (See areas of mathematics and algebraic geometry.)

### **Early geometry**

The earliest recorded beginnings of geometry can be traced to cavemen, who discovered obtuse triangles in the ancient Indus Valley, and ancient Babylonia from around 3000 BC. Early geometry was a collection of empirically discovered principles concerning lengths, angles, areas, and volumes, which were developed to meet some practical need in surveying, construction, astronomy, and various crafts. Among these were some surprisingly sophisticated principles, and a modern mathematician might be hard put to derive some of them without the use of calculus. For example, both the Egyptians and the Babylonians were aware of versions of the Pythagorean Theorem about 1500 years before Pythagoras; the Egyptians had a correct formula for the volume of a frustum of a square pyramid; the Babylonians had a trigonometry table.

#### **Notes:**

straightedge – устройство для проведения прямых линий;

rigor – жесткий;

cavemen – пещерные люди

sophisticated – сложный, замысловатый.

***Answer the following questions:***

1. How did Geometry arise as the field of knowledge?
2. What was classic geometry focused in?
3. When have geometric concepts been generalized to a high level of abstraction?
4. Who discovered obtuse triangles in the ancient Indus Valley?
5. What was early geometry?

## **TEXT 14 Babylonian geometry**

***Read and translate the following text. Answer the questions.***

The Babylonians may have known the general rules for measuring areas and volumes. They measured the circumference of a circle as three times the diameter and the area as one-twelfth the square of the circumference, which would be correct if  $\pi$  is estimated as 3. The volume of a cylinder was taken as the product of the base and the height, however, the volume of the frustum of a cone or a square pyramid was incorrectly taken as the product of the height and half the sum of the bases. The Pythagorean Theorem was also known to the Babylonians. Also, there was a recent discovery in which a tablet used  $\pi$  as 3 and  $1/8$ . The Babylonians are also known for the Babylonian mile, which was a measure of distance equal to about seven miles today. This measurement for distances eventually was converted to a time-mile used for measuring the travel of the Sun, therefore, representing time.

**Notes:**

volume – объем;

circumference – окружность;

frustum – усеченная пирамида, усеченный конус

***Answer the following questions:***

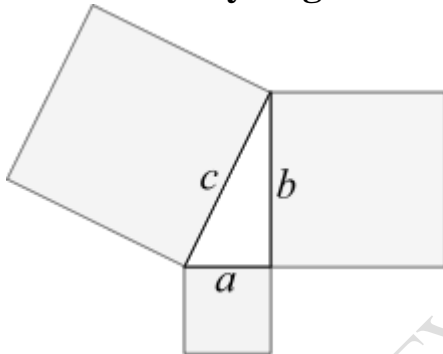
- 1 What may the Babylonians have known?
- 2 How did the Babylonians measure the circumference of a circle and the volume of a cylinder?
- 3 Was the Pythagorean Theorem known to the Babylonians?
- 4 What is the Babylonian mile?

## TEXT 15 Classical Greek geometry

*Read and translate the following text. Answer the questions.*

For the ancient Greek mathematicians, geometry was the crown jewel of their sciences, reaching a completeness and perfection of methodology that no other branch of their knowledge had attained. They expanded the range of geometry to many new kinds of figures, curves, surfaces, and solids; they changed its methodology from trial-and-error to logical deduction; they recognized that geometry studies "eternal forms", or abstractions, of which physical objects are only approximations; and they developed the idea of the "axiomatic method", still in use today.

### Thales and Pythagoras



Pythagorean Theorem:  $a^2 + b^2 = c^2$

Thales (635-543 BC) of Miletus (now in southwestern Turkey), was the first to whom deduction in mathematics is attributed. There are five geometric propositions for which he wrote deductive proofs, though his proofs have not survived. Pythagoras (582-496 BC) of Ionia, and later, Italy, then colonized by Greeks, may have been a student of Thales, and traveled to Babylon and Egypt. The theorem that bears his name may not have been his discovery, but he was probably one of the first to give a deductive proof of it. He gathered a group of students around him to study mathematics, music, and philosophy, and together they discovered most of what high school students learn today in their geometry courses. In addition, they made the profound discovery of incommensurable lengths and irrational numbers. (There is no evidence that Thales provided any deductive proofs, and in fact, deductive mathematical proofs did not appear until after Parmenides. At best, all that we can say about Thales is that he introduced various geometric theorems to the Greeks. The idea that mathematics was from its inception deductive is false. At the time of

Thales, mathematics was inductive. This means that Thales would have "provided" empirical and direct proofs, but not deductive proofs.)

**Notes:**

surface – поверхность;

solid – тело.

***Answer the following questions:***

1. What was the crown jewel of sciences for the ancient Greeks?
2. What is the essence of the Pythagorean Theorem?
3. Whom is deduction in mathematics attributed to?
4. Did Thales introduce various geometric theorems to the Greeks?

**Plato**

Plato (427-347 BC), the philosopher most esteemed by the Greeks, had inscribed above the entrance to his famous school, "Let none ignorant of geometry enter here." Though he was not a mathematician himself, his views on mathematics had great influence. Mathematicians thus accepted his belief that geometry should use no tools but compass and straightedge – never measuring instruments such as a marked ruler or a protractor, because these were a workman's tools, not worthy of a scholar. This dictum led to a deep study of possible compass and straightedge constructions, and three classic construction problems: how to use these tools to trisect an angle, to construct a cube twice the volume of a given cube, and to construct a square equal in area to a given circle. The proofs of the impossibility of these constructions, finally achieved in the 19th century, led to important principles regarding the deep structure of the real number system. Aristotle (384-322 BC), Plato's greatest pupil, wrote a treatise on methods of reasoning used in deductive proofs, which were not substantially improved upon until the 19th century.

**Notes:**

influence – влияние;

to accept – принимать;

reasoning – доказательство.

***Answer the following questions:***

1. What had Plato inscribed above the entrance to his famous school?
2. What did Aristotle write?

## TEXT 16 Hellenistic geometry

*Read and translate the following text. Answer the questions.*

Euclid (c. 325-265 BC), of Alexandria, probably a student of one of Plato's students, wrote a treatise in 13 books (chapters), titled *The Elements of Geometry*, in which he presented geometry in an ideal axiomatic form, which came to be known as Euclidean geometry.

*The Elements* began with definitions of terms, fundamental geometric principles (called *axioms* or *postulates*), and general quantitative principles (called *common notions*) from which all the rest of geometry could be logically deduced. Following are his five axioms, somewhat paraphrased to make the English easier to read.

1. Any two points can be joined by a straight line.
2. Any finite straight line can be extended in a straight line.
3. A circle can be drawn with any center and any radius.
4. All right angles are equal to each other.
5. If two straight lines in a plane are crossed by another straight line (called the transversal), and the interior angles between the two lines and the transversal lying on one side of the transversal add up to less than two right angles, then on that side of the transversal, the two lines extended will intersect (also called the parallel postulate).

**Notes:**

treatise – научный труд;

interior – внутренний;

transversal – пересекающая, секущая линия

*Answer the following questions:*

1. How many books did Euclid write?
2. What are fundamental principles presented in the Elements?

### **Archimedes**

Archimedes (287-212 BC), of Syracuse, Sicily, when it was a Greek city-state, is often considered to be the greatest of the Greek mathematicians, and occasionally even named as one of the three greatest of all time (along with Isaac Newton and Carl Friedrich Gauss). Had he not been a mathematician, he would still be remembered as a great physicist, engineer, and inventor. In his mathematics, he developed methods very similar to the coordinate systems of analytic geometry, and the limiting

process of integral calculus. The only element lacking for the creation of these fields was an efficient algebraic notation in which to express his concepts.

*Answer the following questions:*

1. Who is considered to be the greatest of the Greek mathematicians?
2. What methods did he develop?

### **TEXT 17 After Archimedes**

*Read and translate the following text. Answer the questions.*

Geometry was connected to the divine for most medieval scholars. The compass in this 13th Century manuscript is a symbol of God's act of Creation.

After Archimedes, Hellenistic mathematics began to decline. There were a few minor stars yet to come, but the golden age of geometry was over. Proclus (410-485), author of *Commentary on the First Book of Euclid*, was one of the last important players in Hellenistic geometry. He was a competent geometer, but more importantly, he was a superb commentator on the works that preceded him. Much of that work did not survive to modern times, and is known to us only through his commentary. The Roman Republic and Empire that succeeded and absorbed the Greek city-states produced excellent engineers, but no mathematicians of note.

The great Library of Alexandria was later burned. There is a growing consensus among historians that the Library of Alexandria likely suffered from several destructive events, but that the destruction of Alexandria's pagan temples in the late 4th century was probably the most severe and final one. The evidence for that destruction is the most definitive and secure. Caesar's invasion may well have led to the loss of some 40,000-70,000 scrolls in a warehouse adjacent to the port (as Luciano Canfora argues, they were likely copies produced by the Library intended for export), but it is unlikely to have affected the Library or Museum, given that there is ample evidence that both existed later.

Civil wars, decreasing investments in maintenance and acquisition of new scrolls and generally declining interest in non-religious pursuits likely contributed to a reduction in the body of material available in the Library, especially in the fourth century. The Serapeum was certainly destroyed by

Theophilus in 391, and the Museum and Library may have fallen victim to the same campaign.

**Notes:**

superb – благородный, величественный;

acquisition – приобретение;

scroll – свиток (с текстом);

pursuit – преследование, гонение.

**Islamic geometry**

The Islamic Caliphate established across the Middle East, North Africa, Spain, Portugal, Persia and parts of Persia, began around 640 CE. Islamic mathematics during this period was primarily algebraic rather than geometric, though there were important works on geometry. Scholarship in Europe declined and eventually the Hellenistic works of antiquity were lost to them, and survived only in the Islamic centers of learning.

Although the Muslim mathematicians are most famed for their work on algebra, number theory and number systems, they also made considerable contributions to geometry, trigonometry and mathematical astronomy, and were responsible for the development of algebraic geometry. Geometrical magnitudes were treated as "algebraic objects" by most Muslim mathematicians however.

The successors of Muhammad ibn Mūsā al-Kwārizmī who was Persian Scholar, mathematician and Astronomer who invented the Algorithm in Mathematics which is the base for Computer Science (born 780) undertook a systematic application of arithmetic to algebra, algebra to arithmetic, both to trigonometry, algebra to the Euclidean theory of numbers, algebra to geometry, and geometry to algebra. This was how the creation of polynomial algebra, combinatorial analysis and numerical analysis, the numerical solution of equations, the new elementary theory of numbers, and the geometric construction of equations arose.

**Omar Khayyám**

Omar Khayyám (born 1048) was a Persian mathematician, astronomer, philosopher and poet who described his philosophy through poems known as quatrains in the *Rubaiyat of Omar Khayyam*. Along with his fame as a poet, he was also famous during his lifetime as a mathematician, well known for inventing the general method of solving cubic equations by intersecting a parabola with a circle. In addition he discovered the binomial expansion, and authored criticisms of Euclid's



theories of parallels which made their way to England, where they contributed to the eventual development of non-Euclidean geometry. Omar Khayyam also combined the use of trigonometry and approximation theory to provide methods of solving algebraic equations by geometrical means. He was mostly responsible for the development of algebraic geometry.

His *Treatise on Demonstration of Problems of Algebra* contained a complete classification of cubic equations with geometric solutions found by means of intersecting conic sections. In fact Khayyam gives an interesting historical account in which he claims that the Greeks had left nothing on the theory of cubic equations. However, Khayyam himself seems to have been the first to conceive a general theory of cubic equations.

In *Commentaries on the difficult postulates of Euclid's book* Khayyam made a contribution to non-Euclidean geometry, although this was not his intention. In trying to prove the parallel postulate he accidentally proved properties of figures in non-Euclidean geometries. Khayyam also gave important results on ratios in this book, extending Euclid's work to include the multiplication of ratios. The importance of Khayyam's contribution is that he examined both Euclid's definition of equality of ratios (which was that first proposed by Eudoxus) and the definition of equality of ratios as proposed by earlier Islamic mathematicians such as al-Mahani which was based on continued fractions. Khayyam proved that the two definitions are equivalent. He also posed the question of whether a ratio can be regarded as a number but leaves the question unanswered.

The Khayyam-Saccheri quadrilateral was first considered by Omar Khayyam in the late 11th century in Book I of *Explanations of the Difficulties in the Postulates of Euclid*. Unlike many commentators on Euclid before and after him (including of course Saccheri), Khayyam was not trying to prove the parallel postulate as such but to derive it from an equivalent postulate he formulated from "the principles of the Philosopher" (Aristotle):

Two convergent straight lines intersect and it is impossible for two convergent straight lines to diverge in the direction in which they converge.

Khayyam then considered the three cases right, obtuse, and acute that the summit angles of a Saccheri quadrilateral can take and after proving a number of theorems about them, he (correctly) refuted the obtuse and acute cases based on his postulate and hence derived the classic postulate of

Euclid. It wasn't until 600 years later that Giordano Vitale made an advance on the understanding of this quadrilateral in his book *Euclide restituo* (1680, 1686), when he used it to prove that if three points are equidistant on the base AB and the summit CD, then AB and CD are everywhere equidistant. Saccheri himself based the whole of his long, heroic and ultimately flawed proof of the parallel postulate around the quadrilateral and its three cases, proving many theorems about its properties along the way.

Persian mathematician Sharafeddin Tusi (born 1135) did not follow the general development that came through al-Karaji's school of algebra but rather followed Khayyam's application of algebra to geometry. He wrote a treatise on cubic equations, which represents an essential contribution to another algebra which aimed to study curves by means of equations, thus inaugurating the study of algebraic geometry.

**Notes:**

expansion – расширение;

contribution – вклад;

quadrilateral – четырехугольник.

***Answer the following questions:***

1. What was geometry after Archimedes connected to?
2. Who contributed to the development of non-Euclidean geometry?

## **TEXT 18 Modern geometry**

***Read and translate the following text. Answer the questions.***

### **The 17th century**

When Europe began to emerge from its Dark Ages, the Hellenistic and Islamic texts on geometry found in Islamic libraries were translated from Arabic into Latin. The rigorous deductive methods of geometry found in Euclid's *Elements of Geometry* were relearned, and further development of geometry in the styles of both Euclid (Euclidean geometry) and Khayyam (algebraic geometry) continued, resulting in an abundance of new theorems and concepts, many of them very profound and elegant.

### **Discourse on Method by René Descartes**

In the early 17th century, there were two important developments in geometry. The first and most important was the creation of analytic geometry, or geometry with coordinates and equations, by René Descartes

(1596-1650) and Pierre de Fermat (1601-1665). This was a necessary precursor to the development of calculus and a precise quantitative science of physics. The second geometric development of this period was the systematic study of projective geometry by Girard Desargues (1591-1661). Projective geometry is the study of geometry without measurement, just the study of how points align with each other. There had been some early work in this area by Hellenistic geometers, notably Pappus (c. 340). The greatest flowering of the field occurred with Jean-Victor Poncelet (1788-1867).

In the late 17th century, calculus was developed independently and almost simultaneously by Isaac Newton (1642-1727) and Gottfried Wilhelm von Leibniz (1646-1716). This was the beginning of a new field of mathematics now called analysis. Though not itself a branch of geometry, it is applicable to geometry, and it solved two families of problems that had long been almost intractable: finding tangent lines to odd curves, and finding areas enclosed by those curves. The methods of calculus reduced these problems mostly to straightforward matters of computation.

**Notes:**

to emerge – возникать (о вопросе);  
abundance – множество, совокупность;  
profound – основательный;  
intractable – неподатливый.

***Answer the following questions:***

1. What were there two important developments in geometry in the early 17-th century?
2. Who developed calculus in the late 17-th century?
3. What was the beginning of analysis?

**TEXT 19 The 18th and 19th centuries**

***Read and translate the following text. Answer the questions.***

**Non-Euclidean geometry**

The old problem of proving Euclid's Fifth Postulate, the "Parallel Postulate", from his first four postulates had never been forgotten. Beginning not long after Euclid, many attempted demonstrations were given, but all were later found to be faulty, through allowing into the

reasoning some principle which itself had not been proved from the first four postulates. Though Omar Khayyám was also unsuccessful in proving the parallel postulate, his criticisms of Euclid's theories of parallels and his proof of properties of figures in non-Euclidean geometries contributed to the eventual development of non-Euclidean geometry. By 1700 a great deal had been discovered about what can be proved from the first four, and what the pitfalls were in attempting to prove the fifth. Saccheri, Lambert, and Legendre each did excellent work on the problem in the 18th century, but still fell short of success. In the early 19th century, Gauss, Johann Bolyai, and Lobachevski, each independently, took a different approach. Beginning to suspect that it was impossible to prove the Parallel Postulate, they set out to develop a self-consistent geometry in which that postulate was false. In this they were successful, thus creating the first non-Euclidean geometry. By 1854, Bernhard Riemann, a student of Gauss, had applied methods of calculus in a ground-breaking study of the intrinsic (self-contained) geometry of all smooth surfaces, and thereby found a different non-Euclidean geometry. This work of Riemann later became fundamental for Einstein's theory of relativity.

William Blake's "Newton" is a demonstration of his opposition to the 'single-vision' of scientific materialism; here, Isaac Newton is shown as 'divine geometer' (1795)

It remained to be proved mathematically that the non-Euclidean geometry was just as self-consistent as Euclidean geometry, and this was first accomplished by Beltrami in 1868. With this, non-Euclidean geometry was established on an equal mathematical footing with Euclidean geometry.

While it was now known that different geometric theories were mathematically possible, the question remained, "Which one of these theories is correct for our physical space?" The mathematical work revealed that this question must be answered by physical experimentation, not mathematical reasoning, and uncovered the reason why the experimentation must involve immense (interstellar, not earth-bound) distances. With the development of relativity theory in physics, this question became vastly more complicated.

**Notes:**

pitfall – ошибка, просчет; заблуждение;  
to suspect – предполагать.

*Answer the following questions:*

1. What had been discovered by 1700?
2. What had Bernhard Riemann applied by 1854?
3. Was the non-Euclidean geometry as self-consistent as Euclidean geometry?

### **The 20th century**

Developments in algebraic geometry included the study of curves and surfaces over finite fields as demonstrated by the works of among others André Weil, Alexander Grothendieck, and Jean-Pierre Serre as well as over the real or complex numbers. Finite geometry itself, the study of spaces with only finitely many points, found applications in coding theory and cryptography. With the advent of the computer, new disciplines such as computational geometry or digital geometry deal with geometric algorithms, discrete representations of geometric data, and so forth.

РЕПОЗИТОРИЙ ГГУ ИМЕНИ Ф.СКОРНЬЯ

## ЛИТЕРАТУРА

1 Wikipedia, the free encyclopedia – Режим доступа:  
[http://en.wikipedia.org/wiki/Main\\_Page](http://en.wikipedia.org/wiki/Main_Page)

2 Дорожкина В. П., Английский язык для математиков (книга вторая):– М.: Изд-во МГУ, 1974 – 402 с.

3 Англо-русский словарь математических терминов. /Под ред. П.С.Александрова. – 2-е, исправл. и дополн. изд. – М.: Мир, 1994 – 416 с.

РЕПОЗИТОРИЙ ГГУ ИМЕНИ Ф. СКОРИНА

**Учебное издание**

**Петухова Галина Николаевна**

**АНГЛИЙСКИЙ ЯЗЫК**

**ПРАКТИЧЕСКОЕ РУКОВОДСТВО**

*для студентов заочного факультета  
специальности 1-31 03 01-02 «Математика (научно-педагогическая деятельность)»*

**В авторской редакции**

Лицензия №02330/0133208 от 30.04.04.

Подписано в печать . . . Формат 60x84 1/16.

Бумага писчая №1. Гарнитура «Таймс». Усл.п.л. , .

Уч.-изд.л. , . Тираж экз. Заказ № .

Отпечатано с оригинала-макета на ризографе  
учреждения образования

«Гомельский государственный университет  
имени Франциска Скорины».

Лиц №02330/0056611 от 16.04.04.

246019, г. Гомель, ул. Советская, 104.