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Design and numerical modeling of an optical birefringent quarter waveplate

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В работе представлен способ проектирования оптической четвертьволновой пластинки для инфракрасной области спектра. Проведено компьютерное моделирование основанное на использовании метода конечных элементов. Осуществлено измерение интенсивности прошедшего светового потока, а также его поляризационные характеристики. Использую данную модель возможно проектировать четвертьволновую пластинку (а также другие волновые пластинки) в широком частотном диапазоне, не ограниченном только оптическими частотами. Предложенный в работе пример хорошо согласуется с теоретическими расчетами. Результаты данного моделирования могут быть использованы в качестве базиса при проектировании некоторых других комплексных оптических устройств, метаматериалов и фотонных кристаллов.
Ключевые слова: волновая пластинка, двупреломление, моделирование, эллиптичность, интенсивность.

In this article, a design of an optical quarter wave plate operating in an infrared part of the spectrum is proposed. Computer modeling utilizing a finite-element electromagnetic algorithm is developed. It provides measurements of an intensity of the transmitted flux as well as its polarization characteristics. Using this model, it is possible to design a quarter wave plates (as well as other wave plates) in a wide frequency range not only limited by optical frequencies. The proposed example is in good agreement with theoretical calculations. Results of this modeling can be used as a basis for designing some other complex optical devices, metamaterials, and photonic crystals.
Keywords: waveplate, birefringence, modeling, ellipticity, intensity.

Introduction. A waveplate or retarder is an optical device that alters the polarization state of a light wave travelling through it. Two common types of waveplates are the half-wave plate, which shifts the polarization direction of linearly polarized light, and the quarter-wave plate (which converts linearly polarized light into circularly polarized light and vice versa). A typical illustration of a quarter waveplate is shown in Figure 1.

Waveplates are constructed out of a birefringent material (such as quartz or mica), for which the index of refraction is different for different orientations of light passing through it. The behavior of a waveplate (that is, whether it is a half-wave plate, a quarter-wave plate, etc.) depends on the thickness of the crystal d, the wavelength of light λ, and the variation of the index of refraction n between ordinary 'o' and extraordinary 'e' components of light:

$$(n_e - n_o)d = \lambda\Delta\varphi / 2\pi \qquad (1)$$

For a light wave normally incident upon the plate, polarization component along the ordinary axis travels through the crystal with a speed $v_0 = c / n_0$, while the polarization component along the extraordinary axis travels with a speed $v_e = c / n_e$. This leads to the phase difference $\Delta\phi$ between the two above mentioned components as they exit the crystal. By appropriate choice of the relationship between these parameters, it is possible to introduce a controlled phase shift $\Delta\phi$ between the two polarization components of an electromagnetic wave, thereby altering its polarization.

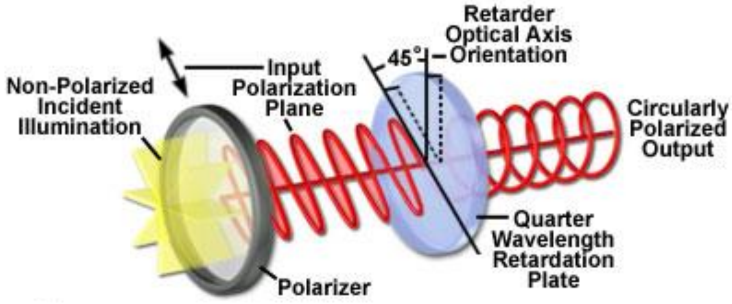


Figure 1 – Quarter wavelength retardation plate (property of Olympus Inc.).

Although the birefringence Δn may vary slightly due to dispersion, this is negligible compared to the variation in phase difference initiated by the wave propagation path. Waveplates are thus manufactured to work for a particular narrow range of wavelengths. The phase variation can be minimized by stacking two waveplates that differ by a tiny amount in thickness back-to-back, with the slow axis of one along the fast axis of the other [1]. With this configuration, the relative phase imparted can be, for the case of a quarter-wave plate, one-fourth a wavelength rather than three-fourths or one-fourth plus an integer. This is called a zero-order waveplate.

A broadband quarter waveplate provided by nature was discovered recently in Ref. [2]. The eyes of stomatopod crustaceans operate as an achromatic quarter wave retarders in the visible wavelength range due to their sophisticated internal composition. A wide band of operation can also be achieved by using an array of micro helices arranged properly as it is shown in Ref. [3]. Meander-line broadband quarter waveplates are well known too [4].

Polarization characteristics. An ellipticity, which is of the main interest in this paper, characterizes the polarization type of the wave and it is introduced through the reciprocal axial ratio as defined in general case:

$$\gamma = A/B \tag{2}$$

where

$$A = \left[\frac{1}{2}\left(E_x^2 + E_y^2 - \left[E_x^4 + E_y^4 + 2E_x^2E_y^2\cos(2\Delta\varphi)\right]^{1/2}\right)\right]^{1/2} \tag{3}$$

and

$$B = \left[\frac{1}{2}\left(E_x^2 + E_y^2 + \left[E_x^4 + E_y^4 + 2E_x^2E_y^2\cos(2\Delta\varphi)\right]^{1/2}\right)\right]^{1/2} \tag{4}$$

are the minor and the major semiaxes of the polarization ellipse, respectively [5].

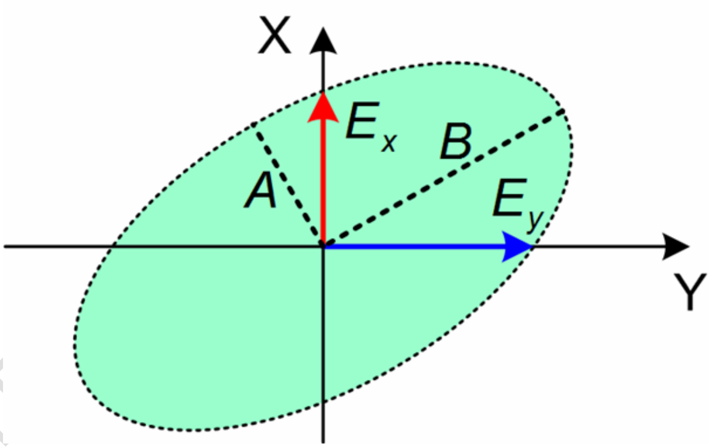


Figure 2 – An ellipse of polarization.

Here $\Delta\varphi$ is the time-phase difference between the two components E_x and E_y of the field. One can find that $0 \le \varepsilon \le 1$, where the zero value stands for a pure linear polarization, the unit value denotes a pure circular polarization. In fact, the ellipticity higher than 0.8 is often considered as circular. Sometimes it is merely preferable to use an alternative parameter for characterizing the polarization state of wave which is polarization angle τ, introduced as $\tan\tau = A/B$. It is clear that $\tau = 45°$ corresponds for a circular polarization.

Quarter waveplate design results. Figure 3 illustrates a basic 3D model of a birefringent quarter waveplate to be designed. The waveplate is composed of a solid homogeneous anisotropic piece of transparent for infrared light material. Relative permittivities of the material are the following: $\varepsilon_x = 1.8$, $\varepsilon_y = 1.5$, $\varepsilon_z = 1$. The Z-thickness of the anisotropic material [d in Eq. (1)] is adjusted to be $4.3\,\mu\text{m}$. Therefore, the theoretically calculated [using Eq. (1)] operation frequency of the device is 150 THz. The transmitted wave is supposed to be circularly polarized at 150 THz.

The wave vector $\vec{k}$ is directed towards the *z*-axis while the vector $\vec{E}_0$ is directed at 45° above the *x*-axis in the *xy*-plane. The transmitted flux is measured on the *z*-axis in a point which is located at a distance from the nearest slab's surface greater than the wavelength. The surfaces of the wave-plate which are perpendicular to the *x*- and *y*-axes are covered with periodic boundary conditions. The other two surfaces are covered with so-called radiation boundary conditions which assume strong absorption of the radiation.

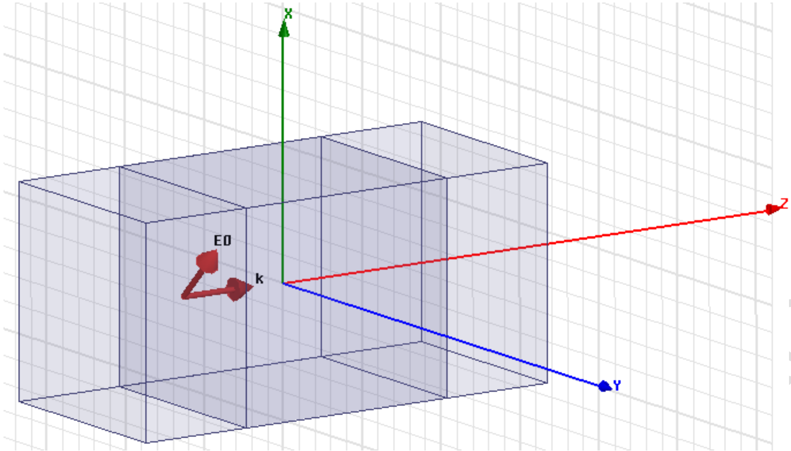
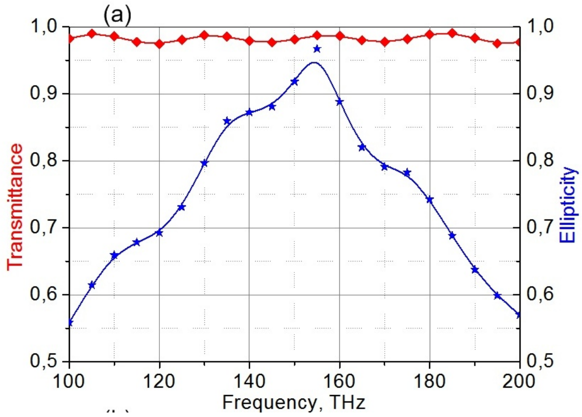


Figure 3 – Model of a THz-range quarter wave plate. An anisotropic slab which represents birefringent material is shown in the center with darker color. The slab is covered with boundary periodic and absorption conditions. An incident infrared wave is linearly polarized.

The results of modeling are presented in Figure 4. The most important characteristics of the designing device are the net transmittance and polarization type of the transmitted flux [ellipticity (2)]. The resultant transmitted flux is shown in Figure 4 (a) by the red-color graph marked with diamonds. One can find that the transmittance is very high providing the device almost an ideal performance. This property was achieved due to the low-thickness low-loss material used. In practice, the experimental samples are much thicker and, therefore, they are not so perfect from the point of view of transmittance. The blue graph in Figure 4 (a) corresponds to the right-hand scale and represents the ellipticity which is calculated using formulas (2-4). Actually the left-hand and right-hand scales are identical in this case. The peak of the ellipticity which is near unity at around 150 THz is the frequency at which the device provides its ultimate functionality. The frequency corresponds to an infrared light having wavelength of $2\,\mu\text{m}$. In Eqs. (3) and (4) there are electric field components E_x and E_y. Their magnitudes are responsible for the ellipticity, therefore Figure 4 (b) illustrates them in arbitrary units. The magnitudes themselves cannot say definitely the resultant ellipticity because there is the phase difference $\Delta\varphi$ in (3) and (4). Thus, it is necessary to provide information about the phase difference [Figure 4 (c)]. It is noticeable, that using graphs of Figure 4 (b) and Figure (c) it is not obvious that the resultant polarization is circular near 150 THz, but mathematical calculations and computer analysis confirm the results of the modeling.



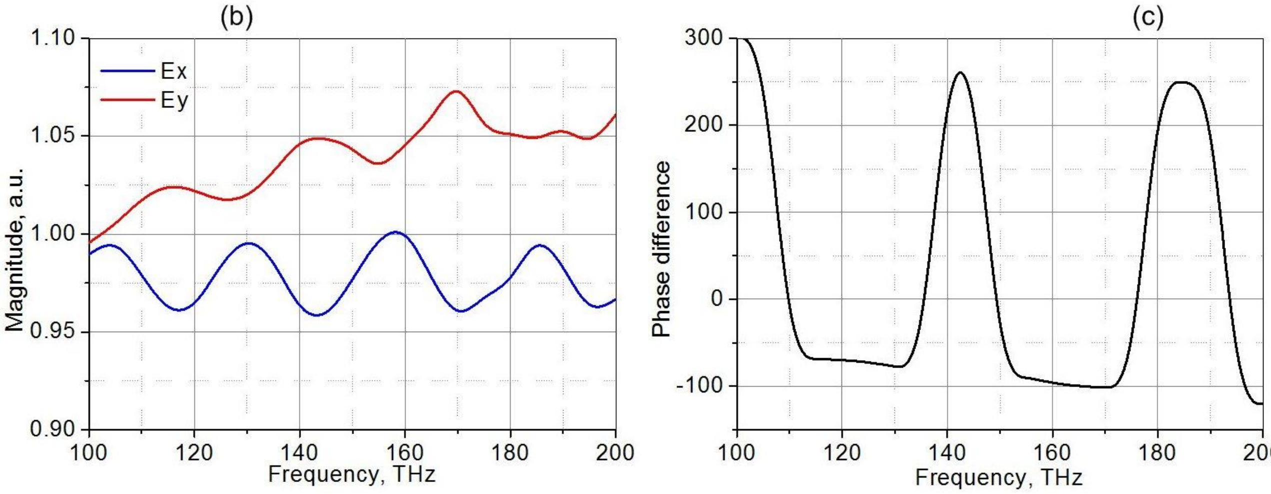


Figure 4 – (a) Transmittance and ellipticity of the radiation transmitted through the optical quarter waveplate (note that the vertical scales are identical); (b) Magnitudes E_x and E_y (in arbitrary units) of the transmitted wave; (c) Phase difference $\Delta\varphi$ between E_x and E_y components of the transmitted field.

Conclusion. In this paper, an example of modeling of a quarter waveplate is provided. The model can be extended towards similar optical devices as well as non-uniform materials such as, for example, spiral metal arrays [3], photonic crystals [6], and metamaterials. In turn, complex inhomogeneous arrays provide wider operation frequency range due to multiple resonances between an individual layers inside the whole array. Therefore, from this point of view, this example of modeling is considered as a basic foundation for start-up of more sophisticated designs.

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