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Meta-algebras of formal languages and their applications[1]

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1. Introduction

The formation of algebraic fundamentals of algorithmics, an applied theory of algorithms, which have important applications to the solution of diverse problems in modern programming theory and practice, is one of the promising directions of the Ukrainian algebraic-cybernetical school [1, 2]. The algebraic technique is known to be based on a description of objects with the help of formulae composed of operations, i.e. the constructions that satisfy fundamental laws presented, for example, in the form of equalities (identities, relationships, etc.). On the basis of the laws mentioned the formulae are transformed for optimizing in accordance with certain criteria, reducing to standard presentations (polynomials, normal forms, etc.), solving the problem of object equivalency etc. The algebraic formulae are superpositions, i.e. substitutions of some operations (functions) instead of arguments into other ones. The operation of superposition, one of the most fundamental algebraic constructions, is the base for the concept of meta-algebra (MA) oriented toward the construction of concrete algebras for the objects of our interest.

Two-based algorithmical MA associated with the known techniques for constructing algorithms and programs (structural, non-structural, object-oriented) have been studied. The general scheme of constructing a concrete algebra of algorithms (AA) is derived from the investigation of an MA associated with certain program design technique. First, for a chosen MA, a criterion of functional completeness is established basing on the proof of the corresponding theorem, and then a fixed system of operator and logical constructions (operations) is analyzed for its completeness and considered as a system of operations for a concrete AA.

The most interesting subalgebras of an algorithmical MA associated with known techniques for constructing algorithms and programs were studied in [3-6].

A classical example of an MA is algebra of logic and the Post theory being the base to establish the criteria of functional completeness for constructing the systems of operations in various algebras of the Boolean functions (BF) [7, 8].

For the first time the problem of constructing an algebra of algorithms and the corresponding MA was formulated by L.A. Kaluzhnin [9] as applied to flowgraphs of algorithms which were called the Kaluzhnin flowgraphs [10].

The solution of the problem of constructing the algebra of algorithms belongs to V.M. Glushkov [11], who not only constructed the first algebra of algorithms but also proved the fundamental theorem on presentability of an arbitrary algorithm in the algebra constructed, that was later called a system of algorithmical algebras, SAA of Glushkov [1]. Those investigations were several years ahead of the concept for an algebra of programs proposed by Dijkstra as well as of foreign publications on programming methodology and technology [12]. It should be noted that control structures being the base of the structural programming concept can be also used to formalize data structures since programs = algorithms + data structures [14].

The problem of constructing MA of the Kaluzhnin flowgraphs was solved in [10] that became an impact on research in the theory of MA and its applications.

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[1]key words: Kleene's algebra, regular event (RE), meta-algebra, functional completeness, multeity of Post, formalized program specifications.

2. Algebra of Logic Multeities and Isomorphisms

This chapter deals with research in the functional completeness problem for multeities of a two-valued algebra of logic and isomorphic to it set-theoretic MAs.

As a meta-algebra MA, we will generally imply a multi-based algebraic system $MAS = < O; SUPER >$, where $O = \{O_i | i \in I\}$ are bases (different for function sets depending on variables, which take on the values from the corresponding sets), SUPER is the MA signature consisting of operations being a superposition of functions as well as an identification and relettering of variables nested [15, 8].

An important analogy with the MA concept for a single-based case is the idea of clone [16]. A single-based MA is, in particular, the Post algebra (PA). It is a double-valued algebra of logic, $PA =< L(2); SUPER >$, where $L(2)$ is a set of all BFs.

Among the fundamental problems of algebra of logic we would mention the construction of its subalgebras' lattice and studying its basic properties. This problem was solved by Post for a double-valued case [7, 8] (see also [17]), who obtained the following main results:

- countability of the set of all subalgebras (closed classes) of PA has been established;
- finite-generation of such subalgebras has been proved;
- an inclusion diagram of PA subalgebras has been built.

Thus, the solution of the functional completeness problem for PA has been obtained in general by means of constructing a family of its maximal subalgebras (precomplete closed classes), and for each subalgebra separately, by means of describing the subalgebras, maximal against a given subalgebra.

Theorem 1. *Functional completeness criterion for PA [7, 8, 16].*

A system $SYST \subset L(2)$ is complete in PA if and only if it is not included in any of the following PA maximal subalgebras: T_0, T_1, S, M, L, where T_0, T_1 are sets of all functions preserving constant 0,1, respectively, S, M, and L are sets of self-dual, monotonic, and linear BFs, respectively.

Constructing and studying various BF algebras are among important problems of PA: the Boolean algebra, Zhegalkin's algebra, and other algebras whose signatures of operations form functionally complete systems of algebra of logic. The BF algebras' diversity is determined by:

- multeity of this theory important applications for synthesizing combined schemes of different sets of functional elements;
- difference in techniques and language tools used to design combined schemes;
- orientation toward modern and promising technological media etc.

A fundamental significance of the Post theorem consists in providing the functional completeness criterion to construct required complete systems with their subsequent inclusion into the signatures of operations for constructing the corresponding BF algebras. These algebras are subject to further studies because of necessity for them to solve the problem of axiomatization, build a theory of normal forms, minimize them, etc.

It is to be noted, in particular, that the apparatus of algebra of logic possesses important applications associated with the description of logical conditions while designing algorithms and programs as well as the objects processed by them within the framework of modern programming languages and computation media [17].

Let us consider a universe E, a finite set of objects, where the Boolean $B(E)$ and the associated with it set of characteristic predicates $P/B(E)$ are determined so that to every element M from $B(E)$ corresponds its characteristic predicate $P(M)$ from $P/B(E)$. On the set $P/B(E)$ consider as the base a set $L(2)$ of all BFs, each of which being an operation on $P/B(E)$.

In such a way a double-valued algebra of logic has been built:

$MA \setminus P/B(E) =< L(2); SUPER >$, associated with the Boolean $B(E)$. On the Boolean $B(E)$ we determine the operations $M \cup M'$ (integration), $M \cap M'$ (intersection), and $C(M)/E$ (complementation). By closing the operations introduced on superposition, we obtain $MA/B(E) =< B(2); SUPER >$, which is isomorphic to $M \setminus A \setminus P/B(E)$, where $B(2)$ is the base consisting of set-theoretic operations, isomorphic to BFs from $L(2)$.

Theorem 2. *For the constructed MAs the following is true: I. Meta-algebras $MA \setminus P/B(E)$ and $MA/B(E)$ are isomorphic; II. $MA/B(E)$ satisfies an analog of the Post theorem (Theorem 1), which is the base of the functional completeness criterion for a given algebra.*

Based on Theorem 2, the following statement is true: **Corollary.** *A meta-algebra*

$MA/B(2)$ is isomorphic to MA generated by a system of generatrices (SG), which is a signature of an arbitrary Boolean algebra(see, for example, the Stone theorem [18], page 216).

For applications, together with Boolean algebras, of interest are also monotonic algebras of the form:

$$ALMO ::=< B(E/k); SIGMO >,$$

where $B(E/k)$ is the Boolean over universe (E/k) consisting of a finite number of objects; SIGMO is a signature, that includes monotonic operations only (e.g., integration, intersection, taking the Boolean over a subset, etc.).

The following system belongs, specifically, to SG $ALMO : W/0 = \{q|q \in E/k\}$ is a set of all single-element objects belonging to the universe E/k.

Let us consider the Boolean $M/q = B(E/k - q)$ over a subset E/kq, which forms a subalgebra ALMO. The following statement is true:

Theorem 3. *An arbitrarily chosen system $Z \subset B(E/K)$ is SG ALMO if and only if for any object q and an adjointed subalgebra M/q in the system Z can be found at least one subset, which does not belong to M/q.*

Corollary. *The family $\{M/q \mid for\ anyq \in E/k\}$ constitutes a set of maximal subalgebras ALMO.*

Based on Theorem 3, the completeness criterion may be useful, in particular, in designing modern tools oriented toward constructing various multi-media applications (computer games, encyclopedias, etc.) [19].

Reference [20] is devoted to one of the important multi-media applications — game training program on mastering the algebra of logic, which is a classical MA, a component and archetype of the present work.

The algebra of logic and the theory of Post are used as a pattern and a component of algorithmical MAs oriented toward constructing varieties of algebras of algorithms associated with modern techniques of the program design [3–6]. In this case, along with two-based MAs, also multi-based, in particular three-based [21], MAs are considered, that include the bases associated with the data structures and objects of various origins.

3. Meta-algebras of Formal Languages

This chapter presents formal languages operations, which form SG of the corresponding meta-algebras of languages (MAL), and main results on functional completeness relating to the MAs constructed.

Let A be a finite alphabet consisted of terminal letter symbols and $F(A)$ - a free semigroup (with the unity e) of words of finite length in this alphabet, where e is an empty word.

As formal language L, we will imply some set of words (i.e., successions of letters from the alphabet A having finite length).

Let $M/A = \{L | L \subset F(A)\}$ be a set of languages in the alphabet A.

A language operation will be called the function $f(x_1, x_2, ..., x_n)$ defined on the set M/A and taking on the values in this set.

As usual, we suggest that the operation $L_1 * L_2$ of a semigroup multiplication of languages L_1 and L_2 generates such a language $L_3 = L_1 * L_2$ that: $L_3 ::= \{ww' | w \in L_1, w' \in L2\}$. In other words, the language L_3 is obtained as a result of diverse concatenations (gluing together) of ww' words, the first of which w belongs to the language L_1, and the second w' - to L_2.

The operation $x_1 * x_2$ of language multiplication is associative and satisfies the left and right distributivity laws on integration.

At the same time, according to the operation $x_1 * x_2$, the language $E = \{e\}$ (where e is an empty word) plays the role of constant 1, while the language $\emptyset$ (empty set) - constant 0.

A subcase of the language multiplication operation is their raising to power $L^n = L * L * \ldots * L$ (n times). here $L^0 = E$.

Infinite languages are generated by the following iteration species:

- $IT(*) = \{L\}^* = \cup L^k | k = 0, 1, 2, \ldots$;
- $IT(+) = \{L\}^+ = \cup L^k | k = 1, 2, \ldots$

In other words, the iterations $IT(*)$ and $IT(+)$ differ only in the inclusion of an empty word. The basic iteration properties are reflected in the equalities:

$$\{L\}^* = E \cup \{L\}+, \quad (1)$$

$$\{L\}^* = E \cup L * \{L\}^+, \quad (2)$$

$$\{L\}^+ = L * \{L\}^*, \quad (3)$$

$$\{\{L\}^@\}^@ = \{L\}^@, \quad (4)$$

$$\{\emptyset\}^+ = \emptyset, \{\emptyset\}^* = E, \quad (5)$$

$$\{E\}^@ = E, \quad (6)$$

where $@ \in \{*, +\}$.

Eqs. (1–6) reflect an interconnection of the iterations considered and their differences; in particular, the application of *Eq.* (2) in the direction from left to right we will call a development of a given iteration, while its application in the opposite direction — its compression; Eqs. (4–6) promote the reduction of the iteration nesting. Noteworthy are investigations on the algebra of regular events axiomatization using the technique for solving equations [22, 23].

Let a finite recognizer $R =< A, C, f >$ be given [24] with an adjustment (c_0, F), where A is the input alphabet, C — the alphabet of internal states, $f : C \times A \to C$ is a function of transitions, c_0 is an initial state, $F subseteq C$ is a set of final states of the recognizer R (here x is the Cartesian multiplication).

RE $L(R)$ is an input language of the recognizer R, i.e. a set of all words in the input alphabet A, which transfer R from the initial state c_0 into one of the final states belonging to F.

Let us consider the Kleene algebra [25] $KA =< RE'; SIGN >$, where the base RE' is a set of all REs; $SIGN$ is a signature, consisting of operations:

$x_1 * x_2$ is a semigroup multiplication of languages;

$x_1 \cup x_2$ is a set-theoretic integration of languages;

$\{x\}^{@}$ are various iteration species.

As MAL we will imply a single-based meta-algebra of languages:

$MAL ::=< OB(M/A); SUPER >$, where $OB(M/A)$ is the base formed by a set of various operations defined on M/A;

$SUPER$ is a signature, that includes only the superposition of functions with a possible identification and relettering of their variables.

The closure $[Z]$, where Z is from $OB(M/A)$, forms a subalgebra MAL, while the system Z forms the SG of a given subalgebra.

Specifically, the SG of Z may consist of operations nested in the signature of KA, then the subalgebra $[Z]$ will be called the Kleene meta-algebra (KMA), that can be also denoted as $KMA/0$.

Let $x_1 * x_2$ be an operation of semigroup multiplication of languages, which is associative and noncommutative; the closure $[x_1 * x_2]$ forms a semigroup SEM/L and also is a MAL subalgebra.

Let SEM be an arbitrary subalgebra of some MA generated by a semigroup noncommutative operation $v\&v'$ (herefrom the sign & will often be omitted for the sake of briefness). We build a set $V/P = \{v^p | p \in P\}$ from SEM, where $v^p = vv...v$ (p times); P is a set of all primes. By closing V/P over superposition, we obtain a subalgebra $SEM/P = [V/P]$ from the semigroup SEM of the chosen MA. The following statement is true.

Theorem 4. *Let $v\&v'$ (Z and $[Z]$ be a subalgebra of a given MA, then:*

I. V/P will be a countable base of the subalgebra SEM/P;

II. SEM/P is isomorphically nestable in any subalgebra $[Z]$ of a given MA;

III. Subalgebras SEM/P, $[Z]$ are algebras of a continual type (possess a continuum of subalgebras [26,1]).

Corollary. *A subalgebra SEM/P is isomorphically nestable in any subalgebra $[Z']$ MAL, such that $x_1 * x_2 \in Z'$; KMA is an algebra of a continual type.*

Thereby, MAL and its subalgebras including an isomorphic image of SEM/P are algebras of a continual type.

Let KMA/s be a Kleene meta-algebra whose SG contains an empty language F as a constant as well as a language consisted of an empty word e. The presence of constants provides the possibility to shorten the words in the course of language generation that, in turn, promotes increasing the given process generation efficiency. The functional completeness problem for KMA/s has been studied.

Let KMA/n be generated by an SG, which contains constants e and F. The superpositions with the constants are oriented toward shortening words that substantially increases the SG generating efficiency.

Let $A =< A, SUPER >$ be some algebra of functions and $h = f(f_1, f_2, \dots, f_m)$ be a superposition of arbitrarily chosen functions from the base A. Let further B (A be some subalgebra of the algebra A and Bm be an arbitrarily chosen subalgebra, maximal for B.

The subalgebra B will be called q-isolated if $S(Bm) = D \cup Bm$ is a subalgebra of the algebra A for any maximal subalgebra Bm, where $D = A - B$.

Theorem 5.

I. A family of subalgebras, maximal for A, consists of all subalgebras of a type $S(Bm)$ *and extensions of* $Sm(B)$*, such that the nesting is fulfilled:* $B =\subset Sm(B)$.

II. A family of maximal subalgebras for KMA/s *has been built that solves the functional completeness problem for a given* MA.

Corollary. *An arbitrarily chosen system* Z *is complete in* A *if and only if* Z *is not nested in any of subalgebras being maximal for* A*. In particular, the subsystem* $Z' \subset Z$ *is such that* $Z' \subset B$ *is complete in* B *if and only if* Z' *is not nested in any of subalgebras* Bm*, maximal for* B.

Theorem 5 presents the solution of the functional completeness problem for KMA/n with a non- shortening SG as a subalgebra of B and KMA/s corresponding to the algebra A. The concept of q-isolation is a natural generalization of the concept of isolated subalgebra [26, 1].

A monotonically increasing succession of generalized KMA/n has been built. Its set-theoretic limit forms an MAL subalgebra, that does not have a finite basis. The construction obtained is analogous to generalized DMA, whose SGs include generalized cycles with n outputs [5].

Thus, MAL, similar to algorithmical MAs, is adequate in subalgebra lattice properties to k-valued logics ($k > 3$) [27].

4. Integration Processes in Discrete Mathematics and Programming Education

In this Chapter on the base of the results on the theory of MA, an approach is proposed to integrate programming-theoretic sections of DM as well as the training process at the faculties with programming specialization. A natural combination of differential and integration trends is typical for modern state of scientific and technological development. Specifically, such a combination is also characteristic for DM being one of cornerstones of computer science. We will trace an interconnection of the trends mentioned first and foremost on some sections of DM [28], which belong to the theory of programming by definition: the Boole functions algebra, theory of automata and formal languages, theory of discrete transformers, and algorithmical algebras.

The DM sections listed and some others present independent directions in mathematical logic and programming theory and have their own objects and techniques of studies as well as a number of fundamental results obtained in accordance with those techniques. It should also be noted that special monographs, manuals, and educational guidances are devoted to the lines mentioned [1,2,7,8,17,24,27–31]. The common features of the lines mentioned are associated with the discreteness of the corresponding objects and processes, and that is just because they belong to DM.

The development of theoretical, applied, and systems programming is based on known paradigms: An organic imperative, functional, algebraic, and logical. combination of basic paradigms finds its embodiment in modern techniques and technologies of programming including corresponding instrumental packets and media.

The differentiation of modern directions in the development of theoretical, applied, and systems programming is accompanied by setting the problem of their integration. One of the promising approaches to the solution of this problem is the apparatus of algebra of algorithmics [2] and the development of the theory of meta-algebras, that belongs to the upper level of this algebraic system, whose one direction is just presented in this report.

The integration processes in the methodology and technology of programming also find their embodiment in setting an educational process at the faculties of computer sciences with programming specialization. In particular, we would note an integrated training program based on the concept of algebra of algorithmics and it is realized by the graduation chair "Software of Automated Systems" of the faculty of computer sciences of the ISU [32]. The core of this curriculum is a cycle of interconnected courses on algorithmics and fundamentals of discrete mathematics, which comprise a four-year bachelorship and subsequent annual specialization. The algorithmical core is supported by a fundamental classical education in mathematics and a series of traditional programming courses of applied and system orientation. A further integration of the algorithmical core mentioned with tightly adjacent courses in programming is based on the development of common curricula, methodical guidances, laboratory work, assays, graduation projects, in which the proposed approach to a common use of modern methodology and technology in program design finds its embodiment.

5. Conclusion

The results obtained can be classified in the following three groups:

- isomorphizm of algebra of logic and set-theoretic MA, which is comprised by the Post theory of multeities, isomorphizm of the Boolean algebras (the Stone theorem as the result of constructions performed), functional completeness for the case of constructing monotonic objects in connection with multimedia applications;

- the construction of MAL having subalgebras with an infinite basis and infinitely-generated subalgebras without any basis, the continuality criterion for MAL subalgebras has been established; thereby the MAL and its subalgebras are adequate in their properties with k-valued logics for $k > 2$;

- the solution of the functional completeness problem for KMA and its modifications, description of maximal subalgebra's families and relevant completeness criteria for the modifications studied.

Further investigations are associated with the solution of the functional completeness problem for KMA with generalized iterations (see Chap. 3) as well as with constructing multeities for MAL of context-free languages and connected with classes belonging to the Khomskij classification [1].

Thus, the basic aim of the present investigation avenue is: constructing a new, more detailed, classification of formal languages and grammars in context with the results obtained for algorithmical MA and its utilization in various subject domains, integration of programming sections of discrete mathematics, in computer education , etc.

The author hopes to devote his future publications to such investigations.

Abstract. The report is a review of research in algebraic fundamentals of algorithmics. Results on the solution of the functional completeness problem aimed at generalizing Kleene's algebra are presented. These results are a further development of the investigations on algorithmical meta-algebras associated with known techniques used to design algorithms and programs (structural, non-structural, object-oriented). In particular, interconnected aspects

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of programming-theoretic sections of discrete mathematics and development of curricula for computer education at the faculties with the programming specialization are considered.

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