G. Yu. Tyumenkov
Francisk Skorina Gomel State University, Gomel, Belarus

QUASI-FREE DOUBLE-TIME GREEN'S FUNCTION FOR Θ^{+}-PENTAQUARK

The lightest pentaquark Θ^{+} is a relativistic bound system of antiquark $\bar{s}$ and four quarks u, u, d and d. Its experimental observation [1] is actively discussed, which confirms the importance of such systems for the physics of elementary particles [2]. There is no doubt the importance of their theoretical description in the framework of quantum field representations. Since the spin of Θ^{+} is equal to ½, it can be modeled as a bound state of one spinor antiquark and two scalar diquarks (uu) and (dd). And such an approximation greatly simplifies the description problem.

In the theory of relativistic bound systems, the generally accepted research method is the covariant single-time approach in quantum field theory [3]. Its most consistent version is based on the application of covariant double-time Green's functions (GF) $\tilde{G}$ [4]. The inverse free double-times GF $\{\tilde{G}_{(0)}\}^{-1}$ plays an important role in the construction of integral equations for relativistic wave functions. When studying systems in an external electromagnetic field A_μ, such a role is played by the inverse quasi-free double-time GF $\{\tilde{G}_{(0)}^{qf}\}^{-1}$ [5]. The inversion procedure assumes knowledge of the type of non-inversed GF, so let's start finding them. We note here that for a system with a spin structure (0; 0; 1/2), which includes two scalar diquarks and one spinor antiquark, the inversion procedure does not lead to a singularity and is possible without projecting the GF on Dirac's bispinors with preservation their matrix structure.

Let's concider such a three-particle system. Let the first and second particles in it be scalar diquarks with zero spins, masses m_1 and m_2, and electric charges $Q_1=4/3$ and $Q_2=-2/3$. Their initial 4-momenta will be denoted by $p_1=(p_{10}, \vec{p}_1)$ and $p_2=(p_{20}, \vec{p}_2)$. The third particle is an antiquark with characteristics $S_3=1/2$, m_3, $p_3=(p_{30}, \vec{p}_3)$, $Q_3=1/3$.

Placing the system in an external electromagnetic field leads to a three-component $\tilde{G}_{(0)}^{qf}$ of the form:

$$\tilde{G}_{(0)}^{qf} = \tilde{G}_{(0)}^{[1]} + \tilde{G}_{(0)}^{[2]} + \tilde{G}_{(0)}^{[3]}, \qquad (1)$$

where $\tilde{G}_{(0)}^{[j]}$ ($j = 1,2,3$) – quasi-free double-time GF, taking into account the fact of interaction of the field A_μ with the *j-th* particle in the so-called momentum approximation [6]. The terms of formula (1) are obtained from four-time free GF $G_{(0)}^{[j]}$, defined as the vacuum mathematical expectations of the chronological product of the Heisenberg fields of the particles of the system, and the field A_μ in momentum space by integral equating the times in the initial and final states [4].

In this case, the parametrization of the GF by the total energy of the system P_0, which is characteristic of this version of the approach, arises. For the first and second diquarks, the structure of $G_{(0)}^{[j]}$ is similar and they have the form

$$G_{(0)}^{[1]} = Q_1 \cdot \frac{m_3 + \hat{p}_3}{p_3^2 - m_3^2 + i0} \cdot \frac{\Gamma_{1\mu} A^\mu(\vec{q}_1)}{k_1^2 - m_1^2 + i0} \cdot \frac{1}{p_1^2 - m_1^2 + i0} \cdot \frac{1}{p_2^2 - m_2^2 + i0}, \qquad (2)$$

$$G_{(0)}^{[2]} = Q_2 \cdot \frac{m_3 + \hat{p}_3}{p_3^2 - m_3^2 + i0} \cdot \frac{\Gamma_{2\mu} A^\mu(\vec{q}_2)}{k_2^2 - m_2^2 + i0} \cdot \frac{1}{p_1^2 - m_1^2 + i0} \cdot \frac{1}{p_2^2 - m_2^2 + i0}, \qquad (3)$$

where k_j – final 4-momenta of diquarks, $\vec{q}_j = \left(\vec{k}_j - \vec{p}_j\right)$ – 3-momenta of photons, $\Gamma_{j\mu}=(k_j+p_j)_\mu$ – vertex functions. For antiquark in similar symbols

$$G_{(0)}{}^{[3]} = Q_3 \cdot \frac{m_3 + \hat{k}_3}{k_3^2 - m_3^2 + i0} \cdot \hat{A}(\vec{q}_3) \cdot \frac{m_3 + \hat{p}_3}{p_3^2 - m_3^2 + i0} \cdot \frac{1}{p_1^2 - m_1^2 + i0} \cdot \frac{1}{p_2^2 - m_2^2 + i0}. \quad (4)$$

The formulas (2) – (4) lead to the following form of GF appearing in (1)

$$\tilde{G}_{(0)}{}^{[1]} = \frac{1}{12\, \omega_{1p}\, \omega_{2p} \omega_{3p}\, \omega_{1k}} A^{\mu}(\vec{q}_1)\, [(m_3 + \hat{\tilde{p}}_3) \times$$

$$\times \left(\frac{\Gamma_{\mu}^{(-)} R_p}{P_0 - \omega_{1k} - \omega_{2p} - \omega_{3p} + i0} - \frac{\left[\Gamma_{(p)}^{(+)}\right]_{\mu} R_p}{\omega_{1p} + \omega_{1k}} - \frac{\left[\Gamma_{(k)}^{(+)}\right]_{\mu}}{(P_0 - \omega_{1k} - \omega_{2p} - \omega_{3p} + i0)(\omega_{1p} + \omega_{1k})} \right) +$$

$$+ \left(\frac{\Gamma_{\mu}^{(+)} A_p}{P_0 + \omega_{1k} + \omega_{2p} + \omega_{3p} - i0} + \frac{\left[\Gamma_{(p)}^{(-)}\right]_{\mu} A_p}{\omega_{1p} + \omega_{1k}} + \frac{\left[\Gamma_{(k)}^{(-)}\right]_{\mu}}{(P_0 + \omega_{1k} + \omega_{2p} + \omega_{3p} - i0)(\omega_{1p} + \omega_{1k})} \right) \times$$

$$\times (m_3 - \hat{\tilde{p}}_3')], \quad (5)$$

$$\tilde{G}_{(0)}{}^{[2]} = -\frac{1}{24\, \omega_{1p}\, \omega_{2p} \omega_{3p}\, \omega_{2k}} A^{\mu}(\vec{q}_2)\, [(m_3 + \hat{\tilde{p}}_3) \times$$

$$\times \left(\frac{\Pi_{\mu}^{(-)} R_p}{P_0 - \omega_{1p} - \omega_{2k} - \omega_{3p} + i0} - \frac{\left[\Pi_{(p)}^{(+)}\right]_{\mu} R_p}{\omega_{2p} + \omega_{2k}} - \frac{\left[\Pi_{(k)}^{(+)}\right]_{\mu}}{(P_0 - \omega_{1p} - \omega_{2k} - \omega_{3p} + i0)(\omega_{2p} + \omega_{2k})} \right) +$$

$$+ \left(\frac{\Pi_{\mu}^{(+)} A_p}{P_0 + \omega_{1p} + \omega_{2k} + \omega_{3p} - i0} + \frac{\left[\Pi_{(p)}^{(-)}\right]_{\mu} A_p}{\omega_{2p} + \omega_{2k}} + \frac{\left[\Pi_{(k)}^{(-)}\right]_{\mu}}{(P_0 + \omega_{1p} + \omega_{2k} + \omega_{3p} - i0)(\omega_{2p} + \omega_{2k})} \right) \times$$

$$\times (m_3 - \hat{\tilde{p}}_3')], \quad (6)$$

$$\tilde{G}_{(0)}{}^{[3]} = \frac{1}{48\, \omega_{1p}\, \omega_{2p} \omega_{3p}\, \omega_{3k}} \times$$

$$\times \left[\frac{\left(m_3 + \hat{\tilde{k}}_3\right) \hat{A}(\vec{q}_3)\left(m_3 + \hat{\tilde{p}}_3\right) R_p}{P_0 - \omega_{1p} - \omega_{2p} - \omega_{3k} + i0} + \frac{\left(m_3 - \hat{\tilde{k}}_3'\right) \hat{A}(\vec{q}_3)\left(m_3 - \hat{\tilde{p}}_3'\right) A_p}{P_0 + \omega_{1p} + \omega_{2p} + \omega_{3k} - i0} + \right.$$

$$+\frac{\left(m_3-\hat{\tilde{k}}_3'\right)\hat{A}(\vec{q}_3)(m_3+\hat{\tilde{p}}_3)}{(\omega_{3p}+\omega_{3k})}\left(\frac{1}{P_0+\omega_{1p}+\omega_{2p}+\omega_{3k}-i0}-R_p\right)+$$

$$+\frac{\left(m_3+\hat{\tilde{k}}_3\right)\hat{A}(\vec{q}_3)(m_3-\hat{\tilde{p}}_3')}{(\omega_{3p}+\omega_{3k})}\left(A_p-\frac{1}{P_0-\omega_{1p}-\omega_{2p}-\omega_{3k}+i0}\right)\Bigg] \quad (7)$$

Here additional symbols are used:

$$\tilde{p}_j = (\omega_{jp}, \vec{p}_j),\ \tilde{p}_j' = (\omega_{jp}, -\vec{p}_j),$$

$$\omega_{jp} = \sqrt{m_j^2 + \vec{p}_j^2} \quad (j = 1,2,3)$$

and similar parameters by replacing (p ↔ k), as well as

$$R_p = (P_0 - \omega_{1p} - \omega_{2p} - \omega_{3p} + i0)^{-1},$$

$$A_p = (P_0 + \omega_{1p} + \omega_{2p} + \omega_{3p} - i0)^{-1};$$

$$\Gamma_\mu^{(\pm)} = \{2[P_0 \pm (\omega_{2p} + \omega_{3p})],\ \vec{p}_1 + \vec{k}_1\},$$

$$\left[\Gamma_{(p)}^{(\pm)}\right]_\mu = \left(\pm 2\omega_{1p}, \vec{p}_1 + \vec{k}_1\right), \qquad \left[\Gamma_{(k)}^{(\pm)}\right]_\mu = \left(\pm 2\omega_{1k}, \vec{p}_1 + \vec{k}_1\right);$$

$$\Pi_\mu^{(\pm)} = \{2[P_0 \pm (\omega_{1p} + \omega_{3p})],\ \vec{p}_2 + \vec{k}_2\},$$

$$\left[\Pi_{(p)}^{(\pm)}\right]_\mu = \left(\pm 2\omega_{2p}, \vec{p}_2 + \vec{k}_2\right), \qquad \left[\Pi_{(k)}^{(\pm)}\right]_\mu = \left(\pm 2\omega_{2k}, \vec{p}_2 + \vec{k}_2\right).$$

The problem can be simplified by replacement of two scalar diquarks with one scalar tetraquark. In this case, formula (1) loses one term and takes the form

$$\tilde{G}_{(0)}{}^{qf} = \tilde{G}_{(0)}{}^{[1]} + \tilde{G}_{(0)}{}^{[3]}. \quad (8)$$

GF (3) and (6) vanish, and in formulas (2), (4), (5) and (7) all parameters with index $j = 2$ are deleted.

Then, for convenience, the index 3 is replaced by 2, that is, the spinor quark becomes the second particle.

The bulkiness of quasi-free double-time GF (1), (8) does not interfere with the procedure for their nonsingular inversion using well-known software packages for analytical calculations.

The Green's function $\tilde{G}_{(0)}{}^{qf}$ can include a term $\tilde{G}_{(0)}$ without interaction with the external field. For a three-particle system, $\tilde{G}_{(0)}$ has the form

$$\tilde{G}_{(0)} = \frac{1}{8\,\omega_{1p}\,\omega_{2p}\omega_{3p}}\left[\frac{(m_3+\hat{\tilde{p}}_3)}{P_0-\omega_{1p}-\omega_{2p}-\omega_{3p}+i0} - \frac{(m_3-\hat{\tilde{p}}_3')}{P_0+\omega_{1p}+\omega_{2p}+\omega_{3p}-i0}\right] \quad (9)$$

and reverse form

$$\{\tilde{G}_{(0)}\}^{-1} = 4\,\omega_{1p}\,\omega_{2p}\left[P_0\gamma^0 - \frac{\omega_{1p}+\omega_{2k}+\omega_{3p}}{\omega_{3p}}\,(\vec{p}_3\vec{\gamma}+m_3)\right], \quad (10)$$

what is shown in [4].

Thus, in this work, we have obtained an explicit form for the quasi-free double-time Green's functions for the Θ^+ pentaquark, considered as a 3-particle quark–diquark system. The obtained FGs admit nonsingular inversion and further use in constructing integral equations for finding wave functions in the presence of an external electromagnetic field.

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