Известия Гомельского государственного университета
имени Ф. Скорины, №6(39), **1**, 2006

UDC 535.42:534.8

Diffraction of light by ultrasound under the conditions of fresnel reflection

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The diffraction of light by ultrasound is usually investigated using the theory of coupled waves, which ignores the Fresnel reflection of light from the boundaries of the region of acousto-optical (AO) interaction [1, 2]. However, as was shown in [3, 4], the consideration of the Fresnel reflection changes significantly the dependence of the diffraction efficiency on the index of sine modulation of the light wave. In this case, the modulation depth of the refractive index should be rather large as compared to the difference between the refractive indices of the adjacent media. It should be noted that some unique crystals widely used in acousto-optics, in particular, tellurium (Te), paratellurite (TeO_2) quartz (α-SiO_2), iodic acid (α-HIO_3), and cubic crystals such as sillenite ($Bi_{12}GeO_{20}$, $Bi_{12}SiO_{20}$, $Bi_{12}TiO_{20}$), possess the property of gyrotropy. Some of these crystals have been used for the creation of waveguide structures, promising for acousto-optical applications [5, 6]. If, in cubic crystals, gyrotropy manifests itself for any direction of light propagation, then, in crystals from the medium- and low-symmetry crystal systems, it should be taken into account only for directions close to optic axes [7, 8]. Our further consideration is restricted to the case of the AO diffraction in gyrotropic cubic crystals. In [2], it was noted that the electro-optical modulation of light under the conditions of Fresnel reflection from the boundaries of a modulated layer is promising due to a rather steep dependence of the transmission coefficient on the strength of the applied electric field.

In this paper, we examine the Bragg diffraction of light by ultrasound under the conditions of Fresnel reflection of the zeroth- and first-order diffracted waves from the boundaries of a modulated gyrotropic layer. A plane-parallel layer with the thickness h and the mean permittivity ε_2 is located between homogeneous transparent media with the permittivities ε_l and ε_3.

The origin of the system of coordinates *XYZ* lies on the top of the layer, and the *OY* axis is directed normally to the plane of incidence. The modulated layer occupies the space between the planes $x=0$ and $x=h$. A ultrasonic (US) wave with the frequency Ω and the wave vector $K \parallel OZ$ propagates parallel to the layer surfaces and creates a periodic lattice of permittivity in space and time,

$$\hat{\varepsilon}(x,t) = \varepsilon_2 + \Delta\hat{\varepsilon}_2 \cos(Kx - \Omega t), \quad (1)$$

where $K = 2\pi/\Lambda$, Λ is the US wavelength; $\Delta\hat{\varepsilon}_2 = -\varepsilon_2^2 \hat{p}\hat{U}$; and $\hat{U}$ is the modulation depth, determined by the acoustic power and the effective photoelastic constant ($\hat{p}$ is the tensor of photoelastic constants, and $\hat{U}$ is the strain tensor).

Assume that a plane light wave with the frequency $\omega >> \Omega$, and the wave vector $k_1 = e_x k_{1x} + e_z k_{1z}$ ($e_x \parallel OX$, are the unit vectors and $k_{1x} = kn_1 \cos\varphi_1$, $k_{1z} = kn_1 \sin\varphi_1$, $k = \omega/c$ and $n_1 = \sqrt{\varepsilon_1}$) has an arbitrary linear polarization with respect to the plane of incidence *XZ* and is incident on the face at an angle φ_1 to its normal. The refraction angle ($\varphi_2 = \arcsin\left(\sqrt{\varepsilon_1/\varepsilon_2}\sin\varphi_1\right)$ is close to the Bragg angle $\varphi_2 \approx \varphi_B \approx K/2k_2$, where $k_2 = kn_2$, $(n_2 = \sqrt{\varepsilon_2})$. The wave equation describing the behavior of a light wave in a gyrotropic medium can be found in [9, 10].

The solution of the wave equation for the field of a diffracted electromagnetic wave in the layer can be written as [3, 10]

$$E = \sum_{m=-\infty}^{+\infty} \left[e_m A_m(x) + e_y B_m(x)\right] \times \exp\left[i\left(k_{mz} - \omega_m t + \pi m/2\right)\right], \quad (2)$$

where $k_{mz} = k_{0z} + mk$, $\omega_m = \omega + m\Omega$, e_y is defined above, and $e_m = \left[e_y k_m\right] / \left|\left[e_y k_m\right]\right|$. At $k_{0z} \approx K/2$, the two most significant diffracted waves are separated from set (2), which correspond to the Bragg diffraction regime with the diffraction orders $m = 0$ and $m = -1$ [3,4].

The waves refracted by the upper boundary $E^{+}_{0s,p}(\omega)$ diffract into the waves $E^{+}_{1s,p}(\omega - \Omega)$. In turn, the waves reflected from the lower boundary $E^{+}_{0s,p}(\omega)$ diffract into the waves $E^{+}_{1s,p}(\omega - \Omega)$. The diffracted waves reflected from the lower boundary $E^{+}_{1s,p}(\omega - \Omega)$ also contribute to the diffracted waves propagating in the negative direction of the *OX* axis. Thus, as a result of multiple reflections, the light field in the layer is a superposition of eight waves, including four coupled waves (refracted $E^{+}_{0s,p}(\omega)$ and diffracted $E^{+}_{1s,p}(\omega - \Omega)$) propagating along the *OX* axis and four coupled waves $E^{+}_{0s,p}(\omega)$ and $E^{+}_{1s,p}(\omega - \Omega)$ propagating in the negative direction of the *OX* axis.

The constants of propagation of the diffracted waves in the layer can be found from the solution of the characteristic (dispersion) equation of coupled wave system. In this case, system of coupled wave equations is reduced to the equivalent system of first-order equations with constant coefficients [3, 11]. The characteristic equation for the parameter *q* has the form

$$\left(k_{0x}^2 + q^2\right)^2 \left(k_{-1x}^2 + q^2\right)^2 - k_2^4 \left(\eta_{||}^2 + \eta_{\perp}^2\right)\left(k_{0x}^2 + q^2\right)^2 \left(k_{-1x}^2 + q^2\right)^2 +$$
$$+ k_2^8 \eta_{||}^2 \eta_{\perp}^2 + 4\rho^2 \left[\left(k_{0x}^2 + q^2\right)^2 + \left(k_{-1x}^2 + q^2\right)^2 + k_2^4 \eta_{||}^2 \eta_{\perp}^2\right] q^2 + 16\rho^4 q^4 = 0, \tag{3}$$

where $k_{0x} = \sqrt{k_2^2 - k_{0z}^2}$, $k_{-1x} = \sqrt{k_2^2 - k_{1z}^2}$, $k_{0z} = k_2 \sin\varphi_B$, $\eta_{||} = n_2^2 p_{eff}^{||} U/2$, and $\eta_{\perp} = n_2^2 p_{eff}^{\perp} U/2$. Here $p_{eff}^{||}$ and $p_{eff}^{\perp}$ are the effective photoelastic constants responsible for the scattering of the s- and p- components of the diffracted waves and ρ is the parameter of the specific rotation of the gyrotropic layer.

In the general case, dispersion equation (3) can be solved only numerically. In the presence of gyrotropy, four roots of the dispersion equation are physically meaningful rather than two as in [3, 4]. With the use of the approximation $\rho / k << 1$, the solution of Eq. (3) can be represented in the form

$$q_{1,2} = \pm i k_{xs}^{a}, \quad q_{3,4} = \pm i k_{xs}^{b}, \quad q_{5,6} = \pm i k_{xp}^{a}, \quad q_{7,8} = \pm i k_{xp}^{b},$$

where

$$k_{xs}^{a,b} = k_2\left(1 - \rho^2 \eta_{||}\eta_{\perp} / 16 k_2^2\right)\sqrt{\left(1 \mp \eta_{\perp}\right) - K^2 / 4k_2^2},$$
$$k_{xs}^{a,b} = k_2\left(1 - \rho^2 \eta_{||}\eta_{\perp} / 16 k_2^2\right)\sqrt{\left(1 \mp \eta_{||}\right) - K^2 / 4k_2^2},$$

joining the solutions for the strengths of the electric and magnetic fields in the layer [3, 4], as well as in the regions $x < 0$ and $x > h$, we can find the reflection and transmission coefficients (the relative intensities) for the diffracted waves at the layer boundaries. The solution of the system of 16 algebraic equations can be found in the closed form. For the incident light wave with the azimuth of polarization ψ with respect to the plane of incidence XZ, the reflection ($R_{0,1}$) and transmission ($T_{0,1}$) coefficients for the diffracted waves can be represented as

$$R_{0,1} = \left|\Delta^{r}_{0,1} / \Delta\right| \sin^2\psi + \left|\tilde{\Delta}^{r}_{0,1} / \tilde{\Delta}\right| \cos^2\psi =$$
$$= (n_3/n_1)\left(\left|2\Delta^{t}_{0,1} / n_3\Delta\right| \sin^2\psi + \left|2\tilde{\Delta}^{t}_{0,1} / n_3\tilde{\Delta}\right| \cos^2\psi\right), \tag{4}$$

where

$$\begin{aligned}
\Delta &= \left(-\alpha^{-}_{1+}\alpha^{+}_{3+}e_1^{-*} + \alpha^{-}_{1-}\alpha^{+}_{3-}e_1^{-}\right)\left(\alpha^{+}_{1+}\alpha^{+}_{3+}e_1^{+*} - \alpha^{+}_{1-}\alpha^{+}_{3-}e_1^{-}\right) + \\
&+ \left(\alpha^{+}_{1+}\alpha^{+}_{3-}e_1^{+} - \alpha^{+}_{1+}\alpha^{+}_{3+}e_1^{+*}\right)\left(\alpha^{+}_{1+}\alpha^{+}_{3+}e_1^{-*} - \alpha^{+}_{1+}\alpha^{+}_{3+}e_1^{-}\right), \\
\Delta^{r}_{0} &= \left(-\alpha^{-}_{1+}\alpha^{-}_{3+}e_1^{-*} + \alpha^{-}_{1-}\alpha^{-}_{3+}e_1^{-}\right)\left(\alpha^{+}_{1-}\alpha^{+}_{3+}e_1^{+*} - \alpha^{+}_{1+}\alpha^{+}_{3-}e_1^{+}\right) + \\
&+ \left(-\alpha^{+}_{1+}\alpha^{+}_{3+}e_1^{+*} + \alpha^{+}_{1-}\alpha^{+}_{3+}e_1^{+}\right)\left(\alpha^{-}_{1-}\alpha^{-}_{3+}e_1^{-*} - \alpha^{-}_{1+}\alpha^{-}_{3-}e_1^{-}\right), \\
\Delta^{r}_{1} &= \left(b_1^{-} + b_1^{+}\right)\left(\alpha^{+}_{3-}\alpha^{+}_{3+}e_1^{+}e_1^{-*} - \alpha^{-}_{3+}\alpha^{+}_{3-}e_1^{-}e_1^{+*}\right) + \\
&+ \left(b_1^{-} + b_1^{+}\right)\left((\alpha^{+}_{3-})^2 e_1^{+}e_1^{-} - (\alpha^{+}_{3+})^2 e_1^{-*}e_1^{+*}\right), \\
\Delta^{r}_{0,1} &= \mp\alpha^{+}_{1+}e_1^{+*}\left[\alpha^{+}_{3+}b_1^{-} + n_2\alpha^{-}_{3+} + \alpha^{-}_{3-}\left(b_1^{-} - b_1^{+}\right)/2\right] \\
&\pm \alpha^{+}_{1-}e_1^{+}\left[\alpha^{+}_{3-}b_1^{-} + n_2\alpha^{-}_{3-} + \alpha^{-}_{3+}\left(b_1^{-} - b_1^{+}\right)/2\right] - \\
&- \alpha^{-}_{1+}e_1^{-*}\left[\alpha^{-}_{3+}b_1^{+} + n_2\alpha^{+}_{3+} + \alpha^{+}_{3+}\left(b_1^{+} - b_1^{-}\right)/2\right] \\
&+ \alpha^{-}_{1}e_1^{-}\left[\alpha^{-}_{3-}b_1^{+} + n_2\alpha^{+}_{3-} + \alpha^{+}_{3+}\left(b_1^{+} - b_1^{-}\right)/2\right]
\end{aligned} \tag{5}$$

$\alpha^{\pm}_{1,3+} = \left(1 + n^{-1}_{1,3}b_1^{\pm}\right)$, $\alpha^{\pm}_{1,3-} = \left(1 - n^{-1}_{1,3}b_1^{\pm}\right)$ and $b_1^{\pm} = k^{b,a}_{xs}/k$, $e_1^{\pm} = \exp\left(ihk^{a,b}_{xs}\right)$; the asterisk denotes complex conjugation, and the tilde in expressions (4) denotes the substitution $k^{d,a}_{xs} \to k^{b,a}_{xp}$ in expressions (5). A feature of the AO diffraction considered is the presence of a weak interaction between the $s-$ and p - polarized components of the diffracted waves. A strong interaction can be observed, for example, on taking into account the gyrotropic properties of the modulated layer from a plasma waveguide in a magnetic field [12]. For transparent media in the absence of Fresnel reflection and gyrotropy ($\rho = 0$), the transmission coefficients for the diffracted waves (4) are described by the relationships $T_0 = \cos^2\left(\pi n_2^3 p^{\perp,||}_{eff} Uh / 2\lambda_0\right)$ and $T_1 = \sin^2\left(\pi n_2^3 p^{\perp,||}_{eff} Uh / 2\lambda_0\right)$ [1, 2].

Numerical calculations were performed for the following optical systems: air-bismuth germanate ($Bi_{12}GeO_{20}$)-bismuth silicate ($Bi_{12}SiO_{20}$) and air-$Bi_{12}GeO_{20}$-air [5, 13]. It was assumed that the longitudinal US wave concentrates in a thin layer of the $Bi_{12}GeO_{20}$ crystal and propagates along the crystallo-graphic direction [111]. The wavelength of light in a vacuum is $\lambda_0 = 0{,}6328\,\mu m$; the amplitude of the strain tensor is $U = \sqrt{2P_a / hW\sigma\upsilon^3}$, where P_a is the power of the US wave, σ is the phase velocity of the US wave, a is the crystal density, and W is the bandwidth of the US piezoelectric transducer.

It shows that the peaks of the reflection coefficients R_1 of the diffracted waves for the system air-$Bi_{12}GeO_{20}$-air are much higher than the reflection coefficients of the system air-$Bi_{12}GeO_{20}$-$Bi_{12}SiO_{20}$. This is explained by the insignificant Fresnel reflection of light by the interface $Bi_{12}GeO_{20}$-$Bi_{12}SiO_{20}$ ($n_2 = 2.55, n_3 = 2.54$) as compared to the reflection by the interface air-$Bi_{12}GeO_{20}$ ($n_2 = 2.55, n_3 = 1$). For example, for the perturbed layer adjacent to air, the maximal value $R_1 = 0.13$ is attained at $U \approx 0.0038$, while the reflection coefficient for the $Bi_{12}GeO_{20}$ substrate is $R_1 = 0.004$. The reflection coefficient for the zeroth-order diffraction R_0 is virtually independent of the acoustic power P_a, while the reflection coefficient for the first-order diffracted wave R_1 is negligibly small. The reflectivity of the zeroth-order diffracted wave is largely determined by the Fresnel reflection from the layer [14]. For the symmetric system with the layer adjacent to air, in a wide range of variation of the acoustic power ($0.013 < U < 0.004$), the transmission coefficient T_1 is also independent of U and determined by the Fresnel reflection of light from the layer boundary. In contrast to the structure air-$Bi_{12}GeO_{20}$-$Bi_{12}SiO_{20}$, efficient AO modulation in a system with sharp boundaries (air-$Bi_{12}GeO_{20}$-air) takes place in the reflected light as well. This feature of diffraction is explained by the efficient AO interaction of the light waves in the layer due to the multiple reflection of the light from the layer boundaries.

The dependences depicted are indicative of the possibility of using multilayer structures on the basis of AO crystals as efficient light modulators in the regime of transmission and reflection of diffracted waves. The relative intensities of the diffracted light under the conditions of Fresnel reflection depend significantly on the relation between the refractive indices of the coating, the modulated layer, and the substrate. In this case, however, there is no need to take into account the particular features of the excitation of optical waveguide modes used in integral acousto-optics [14].

Abstract. The features of the Bragg diffraction of light by ultrasound under the conditions of Fresnel reflection from the boundaries of a perturbed layer have been investigated. It has been shown that layers based on acoustooptical gyrotropic crystals perturbed by ultrasound can be used as efficient light modulators. The relative intensities of diffracted waves have been found to be determined by the relation of the refractive indices of adjacent media and by the ultrasound intensity. With an increase in the thickness of the modulated layer, the reflection and transmission coefficients of the first-order diffracted waves achieve their maxima at lower ultrasound intensity.

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