Известия Гомельского государственного университета
имени Ф.Скорины, №6(39), 1, 2006

УДК 539.172

Exclusive radiative tail contribution to the semi-inclusive deep inelastic scattering

I. Akushevich, A. Ilyichev, M. Osipenko

1. Introduction

The Semi-Inclusive Deep Inelastic Scattering (SIDIS) of a lepton on the nucleon represents an important tool of studying QCD dynamics. The knowledge of longitudinal parton momentum distributions obtained in inclusive DIS allows one to extract from SIDIS information about parton orbital momentum. Moreover, it can be also used to study the hadronization process in kinematic regime complementary to e^+e^- colliders. Unfortunately, the kinematics of some SIDIS events is altered by the real photon emission from the initial or final lepton lines. This cannot be corrected experimentally because most of outgoing particles in SIDIS remain undetected. Therefore to account for these radiative effects one has to calculate radiative corrections (RC) theoretically.

We will follow Bardin-Shumeiko covariant approach [1] in which framework RC to the SIDIS have been obtained in Refs. [2, 3]. In the Ref. [2] the radiative effects were calculated for the three-dimensional cross section of polarized SIDIS (target and lepton were longitudinally polarized) and the FORTRAN code for numerical estimates was provided as a patch (named SIRAD) to POLRAD code [4]. While in the Ref. [3] RC for the unpolarized five-dimension cross-section has been computed and FORTRAN code HAPRAD has been developed. However in all of these papers RC do not include the radiative tail from the exclusive reaction at the threshold. This contribution is analogous to the elastic radiative tail in case of the inclusive DIS [5]. At very large Q^2 values this contribution is suppressed, but if Q^2 is in the range of HERMES and JLab experiments (few GeV^2) it can be of certain importance. This additional term of RC to SIDIS has not been investigated until now. Ignoring this effect in data analysis of modern and future experiments [6, 7] can result to essential distortions in observables. In the present article the contribution of the exclusive radiative tail in SIDIS RC is calculated for the first time. This is done using the result obtained in Ref. [8] and follow the notations of Ref. [3].

2. Kinematics and explicit expressions

Let's consider the radiative process that accompany the exclusive electroproduction where hadron h is measured in the final state in coincidence with the scattered electron e':

$$e(k_1) + p(p) \to e'(k_2) + h(p_h) + u(p_u) + \gamma(k).$$

Here k_1 (k_2) is the four-momentum of the initial (final) lepton, ($k_1^2 = k_2^2 = m^2$), p is the target four-momentum, ($p^2 = M^2$), p_h (p_u) is the four-momentum of the detected (undetected) hadron ($p_h^2 = m_h^2$, $p_u^2 = m_u^2$), and k is the emitted real photon four-momentum ($k^2 = 0$).

Similar to Ref. [3] for description of the real photon emission we will use the following three variables:

$$R = 2kp,\ \tau = \frac{2kq}{R},\ \phi_k$$

where $q = k_1 - k_2$ and ϕ_k is the angle between $(\mathbf{k_1}, \mathbf{k_2})$ and $(\mathbf{q}, \mathbf{k})$ planes in the target rest frame reference system $(\mathbf{p} = 0)$.

The set of variables describing the five-differential SIDIS cross section can be chosen as follows:

$$x = -\frac{q^2}{2qp},\ y = \frac{qp}{k_1 p},\ z = \frac{p_h p}{pq},\ t = (q - p_h)^2,\ \phi_h,$$

where ϕ_h is the angle between $(\mathbf{k_1}, \mathbf{k_2})$ and $(\mathbf{q}, \mathbf{p_h})$ planes in the target rest frame. We also will use the following invariants:

$$\begin{aligned}
&S = 2k_1 p,\ X = 2k_2 p = (1-y)S,\ Q^2 = -q^2 = xyS,\\
&W^2 = S_x - Q^2 + M^2,\ S_x = S - X,\ S_p = S + X,\ \lambda_q = S_x^2 + 4M^2 Q^2,\\
&V_{1,2} = 2k_{1,2}k = 2(a^{1,2} + b\cos\phi_k),\ \mu = \frac{2kp_h}{R} = 2(a^k + b^k\cos(\phi_k - \phi_h)),\\
&f = \frac{2k(p + q - p_h)}{R} = 1 + \tau - \mu.
\end{aligned}$$

The explicit expressions for $a^{1,2,k}$, b and b^k coefficients can be found in Appendix of Ref. [3].

According to Eq.33 of Ref. [8] RC to the exclusive hadron electroproduction can be expressed as the integral of the squared matrix element of real photon emission amplitudes over three variables: the inelasticity $v = (p + q - p_h)^2 - m_u^2$ and the photon solid angle. However, when we consider the contribution of the exclusive radiative process to five-dimensional SIDIS cross section the variable v and R are constrained through the following kinematic relation:

$$v = Rf = (1 - z)S_x + t + M^2 - m_u^2. \tag{1}$$

Taking into account Eq. 1 one could see that after some recombination of Eq.33 of Ref. [8] the contribution of the exclusive radiative tail to SIDIS cross section is given by:

$$\frac{d\sigma_R^{ex}}{dx dy dz dt d\phi_h} = -\frac{\alpha^3 S_x^2}{2^7 \pi^4 S \lambda_q} \int\limits_{\tau_{min}}^{\tau_{max}} d\tau \int\limits_0^{2\pi} d\phi_k \sum_{i=1}^{4} \theta_i(R, \tau, \phi_k) \frac{\mathcal{H}_i(\widetilde{W}^2, \widetilde{Q}^2, \widetilde{t})}{f\widetilde{Q}^4}, \tag{2}$$

where structure functions $\mathcal{H}_i$ are combinations of the exclusive photoabsorption cross sections given in Eq.19 of Ref. [8]. These structure functions depend on shifted variables modified by the real photon emission:

$$\begin{aligned}
&\widetilde{W}^2 = (p + q - k)^2 = W^2 - R(1 + \tau),\ \widetilde{Q}^2 = -(q - k)^2 = Q^2 + R\tau,\\
&\widetilde{t} = (q - p_h - k)^2 = t + R(\tau - \mu).
\end{aligned}$$

The integration limits over τ in Eq. 2 are given by $\tau_{min,max} = (S_x \pm \sqrt{\lambda_q})/2M^2$ and quantities $\theta_i(R, \tau, \phi_k)$ are defined in Appendix of Ref. [3].

3. Conclusions

To understand the impact of the obtained exclusive radiative tail in SIDIS we developed FORTRAN code for these corrections and made a few numerical estimates. The code

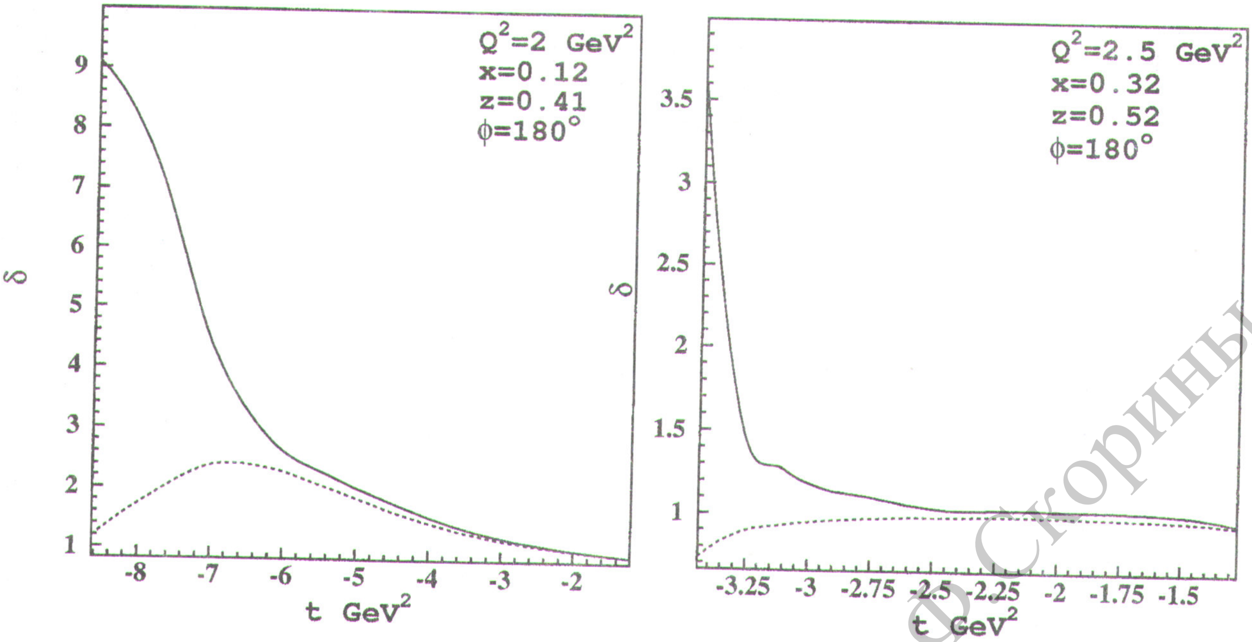

Figure 1: t-dependence of RC factor defined by Eq. 3 for the semi-inclusive π^+ electroproduction at HERMES [6] (left) and JLab [7] (right) kinematic conditions: solid lines show the total correction, dashed lines represent the correction excluding the exclusive radiative tail calculated in this article.

is using the parameterization of photoabsorption cross sections taken from **MAID 2003** [9] and extrapolated to higher W and Q^2 by means of the fit from Ref. [10]. Examples of RC including the exclusive radiative tail contribution are shown in Figs. 1 and 2, where

$$\delta = \frac{\sigma_{obs}}{\sigma_B}, \tag{3}$$

and σ_{obs} (σ_B) is the radiative corrected (Born) five-differential cross section of the semi-inclusive π^+ electroproduction. The absolute value of the exclusive radiative tail is increasing with inelasticity v (or missing mass of the detected lepton-hadron system) while the exclusive peak is located at $v = 0$. Fortunately, the SIDIS cross section is rising with v much faster making the relative contribution of the exclusive radiative tail negligible at large v. Meanwhile, at small v i.e. close to the threshold the situation changes to the opposite and the exclusive radiative tail exceeds the SIDIS cross section. In Fig. 1 this can be clearly seen in rapidly increasing exclusive radiative tail contribution at small t (at fixed Q^2, x and z according to Eq. 1 $t \sim v$). Moreover, one can see in Fig. 2 that exclusive radiative tail modifies significantly ϕ_h distributions at small t distorting usual $A + B\cos\phi_h + C\cos 2\phi_h$ behavior.

The calculated contribution of the exclusive radiative tail in SIDIS is very important in the region of small t and close to the threshold. This contribution modifies ϕ-asymmetries of SIDIS cross section and therefore has to be accounted for in extraction of azimuthal moments.

Acknowledgments. One of us (A.I.) would like to thanks the staff of Istituto Nazionale di Fisica Nucleare (Genova, Italy) for their generous hospitality during his visit.

Abstract. We consider the exclusive radiative tail contribution to unpolarized cross section of semi-inclusive deep inelastic scattering. The explicit expression for the five-differential

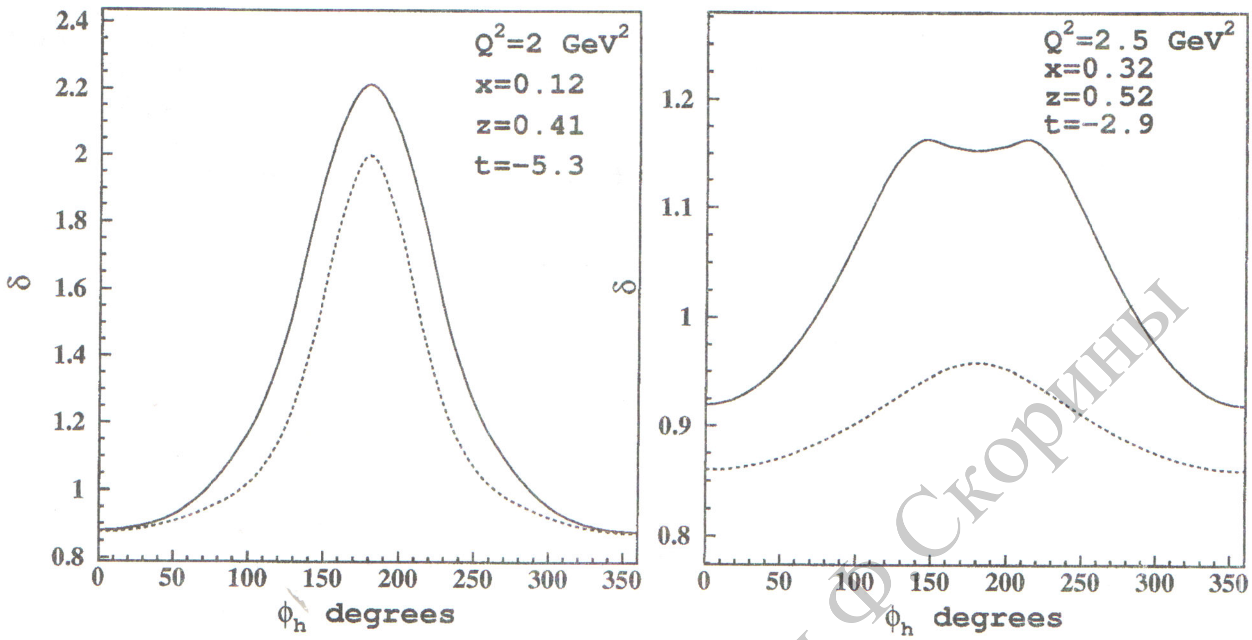

Figure 2: ϕ_h-dependence of RC factor defined by Eq. 3 for the semi-inclusive π^+ electroproduction at HERMES [6] (left) and JLab [7] (right) kinematic conditions: solid lines show the total correction, dashed lines represent the correction excluding the exclusive radiative tail calculated in this article.

cross section is obtained. Few numerical estimates are provided in HERMES and JLab kinematic conditions.

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Instituto Nazionale di Fisica Nucleare, Genoa, Italy
Duke University, Durham, NC 27708, USA
National Scientific and Educational Centre of Particle and
High Energy Physics of the Belarusian State University, Minsk, Belarus

Поступило 11.09.06