Partial two-particle equations in the relativistic configurational representation for scattering *p*-states in the case of a superposition of two δ-function potentials

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Abstract
Exact solutions of two-particle equations are considered in the relativistic configurational representation for scattering *p*-states in the case of superposition of two δ-function potentials. Scattering amplitudes and cross sections are determined. Comparative analysis is carried out with the case of scattering *s*-states.

Introduction

Partial wave equations in the relativistic configurational representation for scattering p-states (orbital angular momentum *l* = 1) of system of two particles are defined as [1]

$$\psi_{(j)}(r) = m\,\mathrm{sh}\,\chi_q p_1(\chi_q, r) + \int_0^\infty G_{(j)}(r, r') V(r') \psi_{(j)}(r')\,dr', \tag{1}$$

$$p_1(\chi_k, r) = r(mr+i)^{-1}\mathrm{sh}^{-2}\chi_k \left(\mathrm{ch}\,\chi_k \sin(\chi_k mr)(mr)^{-1} - \mathrm{sh}\,\chi_k \cos(\chi_k mr)\right). \tag{2}$$

In expressions (1) – (2) $\psi_{(j)}(r)$ – is the wave function, χ_q – is the rapidity connected with the system energy by the formula $2E = 2m\,\mathrm{ch}\,\chi_q$, m – is the mass of each particle, and the index j indicates the corresponding equation ($j=1$ – the modified Kadyshevsky equation, $j=2$ – the Logunov-Tavkhelidze equation, $j=3$ – the modified the Logunov-Tavkhelidze equation, $j=4$ – the Kadyshevsky equation). The explicit form of the partial Green's functions (GF) $G_{(j)}(r, r')$ was defined earlier [2].

When $r \to \infty$ the GF $G_{(j)}(r, r')$ take the following form:

$$G_{(j)}\left(r,r'\right)\Big|_{r\to\infty} \cong K_{(j)}\left(r'\right)e^{i\chi_q mr}, \tag{3}$$

$$K_{(1)}\left(r'\right)=K_{(3)}\left(r'\right)=-ir'\left(\left(mr'-i\right)\operatorname{sh}\chi_q\right)^{-1}\left(\cos\left(\chi_q mr'\right)-\left(mr'\right)^{-1}\operatorname{cth}\chi_q\sin\left(\chi_q mr'\right)\right),$$

$$K_{(2)}\left(r'\right)=K_{(4)}\left(r'\right)=-ir'\left(mr'-i\right)^{-1}\left(2\cos\left(\chi_q mr'\right)\left(\operatorname{sh}\left(2\chi_q\right)\right)^{-1}-\sin\left(\chi_q mr'\right)\left(mr'\operatorname{sh}^2\chi_q\right)^{-1}\right).$$

1. Solution of the equations for a superposition of two δ-function potentials

For the following superposition of two δ-function potentials:

$$V\left(r\right)=V_{01}\delta\left(r-a_1\right)+V_{02}\delta\left(r-a_2\right) \tag{4}$$

equation (1) takes the following form:

$$\psi_{(j)}\left(r\right)=m\operatorname{sh}\chi_q p_1\left(\chi_q,r\right)+G_{(j)}\left(r,a_1\right)V_{01}\psi_{(j)}\left(a_1\right)+G_{(j)}\left(r,a_2\right)V_{02}\psi_{(j)}\left(a_2\right). \tag{5}$$

In order to determine the values of $\psi_{(j)}\left(a_1\right)$ and $\psi_{(j)}\left(a_2\right)$ in (5), it is necessary to consider the wave function at the points $r=a_1$ and $r=a_2$, and then solve the resulting system of linear algebraic equations. Expressions for $\psi_{(j)}\left(a_1\right)$ and $\psi_{(j)}\left(a_2\right)$ can be then represented as

$$\psi_{(j)}\left(a_1\right)=\Delta_{1(j)}\left(\chi_q\right)/\Delta_{(j)}\left(\chi_q\right),\quad \psi_{(j)}\left(a_2\right)=\Delta_{2(j)}\left(\chi_q\right)/\Delta_{(j)}\left(\chi_q\right); \tag{6}$$

$$\Delta_{(j)}\left(\chi_q\right)=\prod_{n=1}^{2}\left[1-V_{0n}G_{1(j)}\left(\chi_q,a_n,a_n\right)\right]-V_{01}V_{02}\left(G_{1(j)}\left(\chi_q,a_1,a_2\right)\right)^2,$$

$$\Delta_{1(j)}\left(\chi_q\right)=m\operatorname{sh}\chi_q p_1\left(\chi_q,a_1\right)\left[1-V_{02}G_{1(j)}\left(\chi_q,a_2,a_2\right)\right]+V_{02}m\operatorname{sh}\chi_q p_1\left(\chi_q,a_2\right)G_{1(j)}\left(\chi_q,a_1,a_2\right),$$

$$\Delta_{2(j)}\left(\chi_q\right)=m\operatorname{sh}\chi_q p_1\left(\chi_q,a_2\right)\left[1-V_{01}G_{1(j)}\left(\chi_q,a_1,a_1\right)\right]+V_{01}m\operatorname{sh}\chi_q p_1\left(\chi_q,a_1\right)G_{1(j)}\left(\chi_q,a_2,a_1\right).$$

Taking into account the asymptotic expressions for GF (3), we obtain

$$\psi_{(j)}\left(r\right)\Big|_{r\to\infty}=m\operatorname{sh}\chi_q\left(p_1\left(\chi_q,r\right)\right)\Big|_{r\to\infty}+f_{1(j)}\left(\chi_q\right)\left(-i\right)e^{im\chi_q r}m\operatorname{sh}\chi_q. \tag{7}$$

In this case the relativistic scattering amplitude $f_{1(j)}\left(\chi_q\right)$ corresponding to the equation with the model potential (4) can be expressed analytically:

$$f_{1(j)}\left(\chi_q\right)=i\left(m\operatorname{sh}\chi_q\right)^{-1}\left(K_{(j)}\left(a_1\right)V_{01}\psi_{(j)}\left(a_1\right)+K_{(j)}\left(a_2\right)V_{02}\psi_{(j)}\left(a_2\right)\right). \tag{8}$$

The partial scattering cross section for the p-wave is expressed by the scattering amplitude $f_{1(j)}\left(\chi_q\right)$ as follows: $\sigma_{1(j)}\left(\chi_q\right)=12\pi\left|f_{1(j)}\left(\chi_q\right)\right|^2$. Figure 1 shows the graphs of the dependence of the scattering cross sections on the rapidity for the following parameter values: $a_1=3$, $V_{01}=1$, $a_2=4$, $V_{02}=2$, $m=1$.

2. Comparison of results for scattering s- and p-states

Earlier [3], the scattering cross sections were obtained for the equations under consideration in the case of an *s*-wave. Let us give the graphs of the scattering cross section for the Logunov-

Tavkhelidze equation in the case of $l = 0.1$ for $a_1 = 3$, $V_{01} = 1$, $a_2 = 4$, $V_{02} = 2$, $m = 1$ (Figure 2). Analyzing Fig. 2, we see that the local maxima of the scattering cross section for the p-state compared to the maxima for the s-state are shifted in the direction of high rapidity values. This feature is a consequence of the presence of a centrifugal barrier.

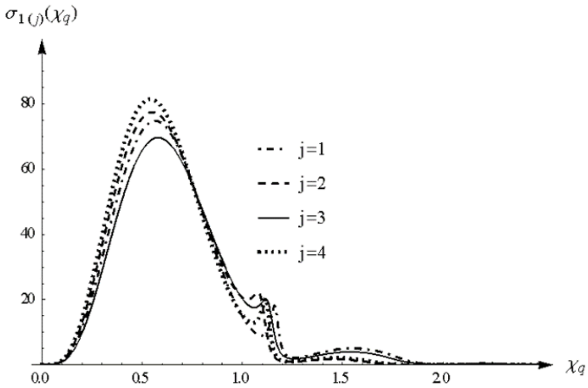

Fig. 1. Dependence of the partial scattering cross section $\sigma_{1(j)}$ *on* χ_q

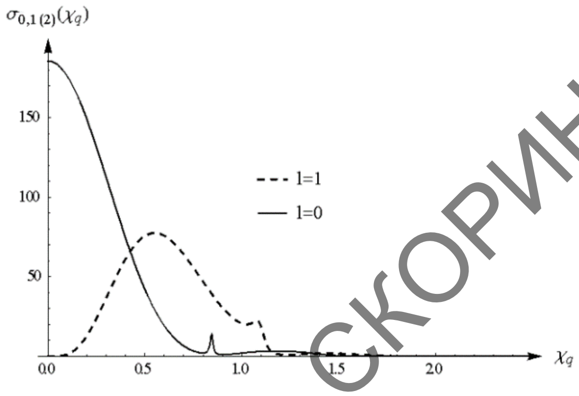

Fig. 2. Dependence of the partial scattering cross section $\sigma_{0,1(2)}$ *on* χ_q

Conclusion

Thus, in this paper, we consider the determination of the wave functions of partial two-particle equations in the relativistic configurational representation in case of a unit orbital angular momentum. On the basis of the obtained wave functions the partial scattering amplitudes and scattering cross sections for the superposition of two δ-function potentials are found.

References

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