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A NOTE ON THE *W*U-HEPERCENTER OF A FINITE GROUP

В работе исследуются свойства обобщенного гиперцентра и его влияние на строение конечных групп.

All groups considered here are finite. Let **X** be a class of groups. Recall [1, p. 6–8] that a chief factor H/K of a group G is called **X**-central if $[H/K](G/C_G(H/K)) \in \mathbf{X}$. A normal subgroup N of G is said to be **X**-hypercentral in G if $N = 1$ or $N \neq 1$ and every chief factor of G below N is **X**-central. The **X**-hypercenter $Z_{\mathbf{X}}(G)$ is the product of all normal **X**-hypercentral subgroups of G.

Recall [2] that a group G is called w-supersoluble if all Sylow subgroups of G are **P**-subnromal in G. The class $w\mathbf{U}$ of all w-supersoluble groups is a hereditary saturated formation with the Sylow tower of supersoluble type. Note that $\mathbf{U} \subseteq w\mathbf{U}$ where **U** is the class of all supersoluble groups.

In [3] R. Baer showed that a normal subgroup N of a group G is **U**-hypercentral in G if and only if it possesses the Sylow tower of supersoluble type and $N_G(P)/C_G(P)$ is strictly p-closed for every Sylow p-subgroup P of N and for every $p \in \pi(N)$.

Recall that a group G is stricly p-closed if G has the normal Sylow p-subgroup P and G/P is abelian of exponent dividing $p-1$. Let $\mathbf{N}_p(\boldsymbol{A}(p-1) \cap w\mathbf{U})$ be a class of all groups G such that all Sylow subgroups of $G/O_p(G)$ are abelian of exponent dividing $p-1$ and **P**-subnormal.

Theorem. Let N be a normal subgroup of a group G. The following statements are equivalent:

(1) N is w**U**-hypercentral in G.

(2) N possesses the Sylow tower of supersoluble type and

$$N_G(P)/C_G(P) \in \mathbf{N}_p(\boldsymbol{A}(p-1) \cap w\mathbf{U})$$

for every Sylow p-subgroup P of N and for every $p \in \pi(N)$.

References

1 Guo, W. Structure theory for canonical classes of finite groups / W. Guo. – Springer, 2015. – 359 p.

2 Vasilev, A. F. On the finite groups of supersoluble type / A. F. Vasil'ev, T. I. Vasil'eva, V. N. Tyutyanov // Sib. Math. J. – 2010. – V. 51, № 6. – P. 1004–1012.

3 Baer, R. Classes of finite groups and their properties / R. Baer // Illinois J. Math. – 1957. – № 1. – P. 115–187.