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# Multiple reflections method in the problem of inclined electromagnetic wave incidence on the layered planar biisotropic medium

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Expressions for the energy transmission and reflection coefficients are obtained in the method of multiple reflections for the case of a plane monochromatic electromagnetic wave inclined incidence on planar biisotropic medium, containing an arbitrary number of layers. These coefficients are expressed in terms of transmission and reflection coefficients at the interfaces of neighboring layers and phase factors. Numerical analysis of the expressions obtained is performed for the periodic structure example. The obtained dependences of the transmission and reflection coefficients on the angle of incidence are oscillatory in nature and contain a series of highs and lows.
**Keywords:** biisotropic medium, planar layered structure, inclined incidence, multiple reflections method, transmission and reflection coefficients, Pointing vector.

Методом многократных отражений получены выражения для энергетических коэффициентов прохождения и отражения плоской монохроматической электромагнитной волны при её падении на планарную биизотропную среду, содержащую произвольное количество слоёв. Эти коэффициенты выражены через коэффициенты прохождения и отражения на границах раздела смежных сред, и фазовые множители. Проведен численный анализ полученных выражений на примере периодической структуры. Полученные зависимости коэффициентов прохождения и отражения от угла падения имеют осциллирующий характер и содержат ряд максимумов и минимумов.
**Ключевые слова:** биизотропная среда, планарная слоистая структура, наклонное падение, метод многократных отражений, коэффициенты прохождения и отражения, вектор Умова-Пойнтинга.

**Introduction.** Composite and natural materials, in which magnetoelectric effect becomes apparent, provoke great interest in electrodynamics. Such materials can be natural, chromium sesquioxide ($Cr_2O_3$) is among them [1]. Its properties can be described using the model of uniaxial Tellegen media that possesses a nonreciprocity property. One of the major differences between biisotropic and other media is that only circularly polarized waves can propagate in such media. The media that possess the properties of nonreciprocity and chirality simultaneously are called biisotropic. Biisotropic media are characterized by the constitutive relations:

$$\vec{D} = \varepsilon\vec{E} + (\chi + i\alpha)\vec{H}; \quad \vec{B} = (\chi - i\alpha)\vec{E} + \mu\vec{H}. \tag{1}$$

The vectors $\vec{D}, \vec{B}$ in (1) are the vectors of electric and magnetic induction; $\vec{E}, \vec{H}$ are the vectors of electric and magnetic field intensity; $\varepsilon, \mu$ are dielectric and magnetic permeability; $\alpha, \chi$ are the parameters of chirality and nonreciprocity respectively.

Earlier the problem of transmission of a plane electromagnetic wave through planar boundaries in case of normal and inclined incidence was discussed in [2]–[3]. The problem of transmission through the biisotropic layer and through a system of biisotropic layers in case of normal incidence was described in [4]–[8]. In this article, the problem of transmission and reflection of electromagnetic wave in case of inclined incidence on a system of biisotropic layers is solved using the method of multiple reflections.

**1. The problem statement.** Let us consider multilayer system consists of biisotropic slabs, and let us choose a right-handed Cartesian coordinate system so that $Oz$-axis is orthogonal to the boundaries, and $Oy$-axis lays in the plane between media 1 and 2. Let number of biisotropic layers equals $N$ and thickness of the $p$-th layer is $d_p$ $(2 \le p \le N-1)$. Thus structure of the system is the next:

– the area $z \le 0$ is filled by the medium 1 that has the parameters $\varepsilon_1, \mu_1, \alpha_1, \chi_1$;

– position of the $p$-th layer that is filled by the biisotropic medium $p$ with parameters $\varepsilon_p, \mu_p, \alpha_p, \chi_p$ is defined by the conditions $\sum_{i=2}^{p-1} d_i < z \le \sum_{i=2}^{p} d_i$;

– the area $\sum_{i=2}^{N-1} d_i < z$ is filled by the biisotropic medium $N$ with parameters $\varepsilon_N$, $\mu_N$, $\alpha_N$, $\chi_N$.

The waves that propagate in the direction of increasing and decreasing of $z$-coordinate can be denoted by the symbols $\uparrow$ and $\downarrow$ in superindex to the left of the ordinal number of the medium.

The incident wave with polarization $\nu$ can be described by the equations

$$\vec{E}_\nu^{\uparrow 1} = \left(\vec{m} + i\nu \vec{l}_\nu^{\uparrow 1}\right) E_\nu^{\uparrow 1} e^{i\left(\vec{k}_\nu^{\uparrow 1}\vec{r} - \omega t\right)}; \qquad \vec{H}_\nu^{\uparrow 1} = -b_\nu^1 \vec{E}_\nu^{\uparrow 1}. \tag{2}$$

and the transmitted and reflected waves in the $p$-th medium look like this:

$$\vec{E}_\sigma^{\uparrow p} = \left(\vec{m} + i\sigma \vec{l}_\sigma^{\uparrow p}\right) E_\sigma^{\uparrow p} e^{i\left(\vec{k}_\sigma^{\uparrow p}\left(\vec{r} - \vec{d}_p\right) - \omega t\right)}; \quad \vec{H}_\sigma^{\uparrow p} = -b_\sigma^p \vec{E}_\sigma^{\uparrow p}; \qquad 2 \le p \le N \tag{3}$$

$$\vec{E}_\sigma^{\downarrow p} = \left(\vec{m} + i\sigma \vec{l}_\sigma^{\downarrow p}\right) E_\sigma^{\uparrow p} e^{i\left(\vec{k}_\sigma^{\downarrow p}\left(\vec{r} - \vec{d}_p\right) - \omega t\right)}; \quad \vec{H}_\sigma^{\downarrow p} = -b_\sigma^p \vec{E}_\sigma^{\downarrow p}; \qquad 1 \le p \le N-1, \tag{4}$$

the subscript is responsible for the wave polarization ($\sigma = +1$) – right-handed polarization, $\sigma = -1$ – left-handed polarization); the unit vectors

$$\vec{m} = (1;0;0); \quad \vec{l}_\sigma^{\uparrow p} = \left(0; \cos\theta_\sigma^p; -\sin\theta_\sigma^p\right); \quad \vec{l}_\sigma^{\downarrow p} = \left(0; -\cos\theta_\sigma^p; -\sin\theta_\sigma^p\right) \tag{5}$$

are directed perpendicularly to the direction of propagation of the corresponding wave. The angle $\theta_\sigma^p$ in (5) is the angle between $Oz$-axis and the direction of the wave propagation, which is determined by the refraction and reflection laws. In equations (2)–(4) we use the vectors

$$\vec{k}_\sigma^{\uparrow p} = \frac{2\pi}{\lambda} n_\sigma^p \cdot \left(0; \sin\theta_\sigma^p; \cos\theta_\sigma^p\right); \; \vec{k}_\sigma^{\downarrow p} = \frac{2\pi}{\lambda} n_\sigma^p \cdot \left(0; \sin\theta_\sigma^p; -\cos\theta_\sigma^p\right); \; \vec{d}_p = \sum_{i=2}^{p-1} d_p \cdot (0;0;1), \tag{6}$$

which are the wave vectors (the first and the second one) and the shift vector of the grid origin in the $p$–th medium; the coefficients in (2)–(6)

$$b_\sigma^p = \left(\chi_p + i\sigma\sqrt{\varepsilon_p \mu_p - \chi_p^{\,2}}\right) / \mu_p; \qquad n_\sigma^p = \sqrt{\varepsilon_p \mu_p - \chi_p^{\,2}} + \sigma\alpha_p \tag{7}$$

are the proportionality coefficients between electric and magnetic intensities ($b_\sigma^p$) and the refractive index ($n_\sigma^p$) for the wave which has polarization $\sigma$ and propagates in medium $p$. The coefficients $b_\sigma^p$ and $n_\sigma^p$ in (7) can be obtained by solving the Maxwell equations in biisotropic medium.

The scheme of the problem is presented in figure 1.

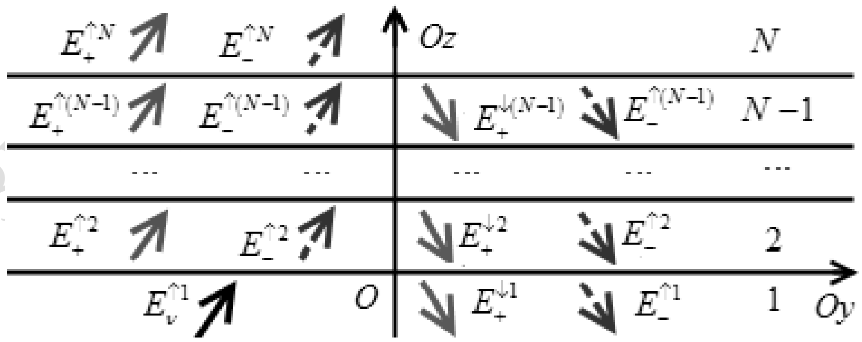


Figure 1 – The scheme of the electromagnetic wave propagation in case of inclined incidence on the layered biisotropic medium

Here and below the right-handed polarized waves are denoted by solid arrows and the left-handed polarized waves are denoted by dashed arrows. It is required to find the transmission and reflection coefficients of the system of biisotropic layers using the method of multiple reflections.

**2. The single layer.** Let us find the transmission and reflection coefficients for the first three media under an assumption of other media absence. The scheme is presented in figure 2a.

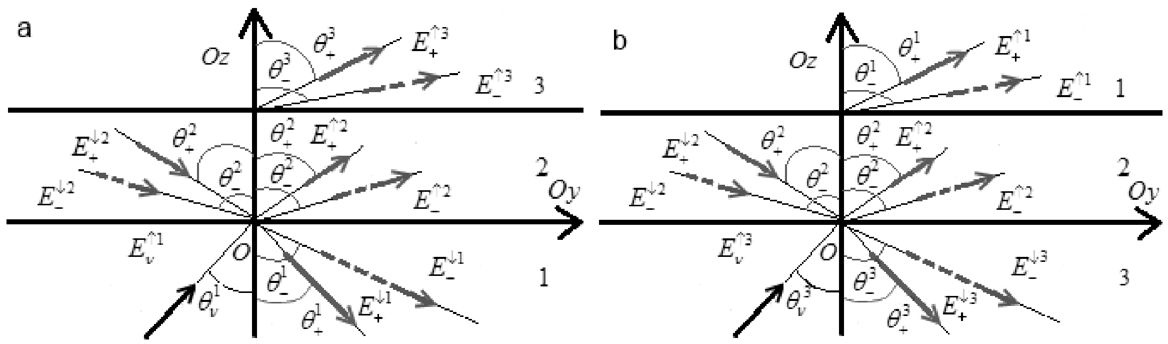


Figure 2 – The scheme of propagation of the electromagnetic wave through the first three media: a) from the medium 1 to 3; b) from the medium 3 to 1.

Let us write down the equations that connect amplitudes of the waves using the reflection and transmission coefficients:

$$\begin{cases} E_+^{\downarrow 2}/\kappa_+^2 = E_+^{\uparrow 2}\rho_{++}^{23}\kappa_+^2 + E_-^{\uparrow 2}\rho_{-+}^{23}\kappa_+^2; & E_+^{\uparrow 2} = E_+^{\downarrow 2}\rho_{++}^{21} + E_-^{\downarrow 2}\rho_{-+}^{21} + E_\nu^{\uparrow 1}\tau_{\nu+}^{12};\\ E_-^{\downarrow 2}/\kappa_-^2 = E_-^{\uparrow 2}\rho_{--}^{23}\kappa_-^2 + E_+^{\uparrow 2}\rho_{+-}^{23}\kappa_-^2; & E_-^{\uparrow 2} = E_-^{\downarrow 2}\rho_{--}^{21} + E_+^{\downarrow 2}\rho_{+-}^{21} + E_\nu^{\uparrow 1}\tau_{\nu-}^{12}, \end{cases} \tag{8}$$

where $\kappa_\sigma^2 = \exp\left(ik_\sigma^2 d_2 \cos\theta_\sigma^2\right)$ – the phase shift between lower and upper boundary.

Solving the equations set (8), one can obtain unknown variables $E_+^{\downarrow 2}, E_-^{\downarrow 2}, E_+^{\uparrow 2}, E_-^{\uparrow 2}$. Using them, one can calculate the other amplitudes of the waves:

$$E_\sigma^{\downarrow 1} = E_\nu^{\uparrow 1}\rho_{\nu\sigma}^{12} + E_+^{\downarrow 2}\tau_{+\sigma}^{21} + E_-^{\downarrow 2}\tau_{-\sigma}^{21}; \quad E_\sigma^{\uparrow 3} = E_+^{\uparrow 2}\kappa_+^2\tau_{+\sigma}^{23} + E_-^{\uparrow 2}\kappa_-^2\tau_{-\sigma}^{23}, \tag{9}$$

where $\sigma = \pm 1$. The formulae for the reflection and transmission coefficients are presented below:

$$T_{\nu\sigma}^{13} = E_\sigma^{\uparrow 3}/E_\nu^{\uparrow 1}; \;\; R_{\nu\sigma}^{13} = E_\sigma^{\downarrow 1}/E_\nu^{\uparrow 1}. \tag{10}$$

Reasoning in the same way, one can find the coefficients

$$T_{\nu\sigma}^{31} = E_\sigma^{\uparrow 1}/E_\nu^{\uparrow 3}; \;\; R_{\nu\sigma}^{31} = E_\sigma^{\downarrow 3}/E_\nu^{\uparrow 3} \tag{11}$$

for the wave that propagates from the third to the first medium. The scheme of this problem is presented in figure 2b. For this case the set of equations has the following form:

$$\begin{cases} E_+^{\downarrow 2}/\kappa_+^2 = E_+^{\uparrow 2}\rho_{++}^{21}\kappa_+^2 + E_-^{\uparrow 2}\rho_{-+}^{21}\kappa_+^2; & E_+^{\uparrow 2} = E_+^{\downarrow 2}\rho_{++}^{23} + E_-^{\downarrow 2}\rho_{-+}^{23} + E_\nu^{\uparrow 3}\tau_{\nu+}^{32};\\ E_-^{\downarrow 2}/\kappa_-^2 = E_-^{\uparrow 2}\rho_{--}^{21}\kappa_-^2 + E_+^{\uparrow 2}\rho_{+-}^{21}\kappa_-^2; & E_-^{\uparrow 2} = E_-^{\downarrow 2}\rho_{--}^{23} + E_+^{\downarrow 2}\rho_{+-}^{23} + E_\nu^{\uparrow 3}\tau_{\nu-}^{32}. \end{cases} \tag{12}$$

The other amplitudes of waves can be found after solving (12) using the next equations:

$$E_\sigma^{\downarrow 3} = E_\nu^{\uparrow 3}\rho_{\nu\sigma}^{32} + E_+^{\downarrow 2}\tau_{+\sigma}^{23} + E_-^{\downarrow 2}\tau_{-\sigma}^{23}; \quad E_\sigma^{\uparrow 1} = E_+^{\uparrow 2}\kappa_+^2\tau_{+\sigma}^{21} + E_-^{\uparrow 2}\kappa_-^2\tau_{-\sigma}^{21}. \tag{13}$$

The equations (13) are analogous to (9).

**3. The double layer.** Let us consider propagation of the electromagnetic wave from the first to the fourth medium. The scheme is presented in figure 3a. Let us replace the area between the third and the first medium with a conventional boundary, which has the transmission and reflection coefficients $T_\nu^{13}$, $R_\nu^{13}$, $T_\nu^{31}$, $R_\nu^{31}$. Then the scheme takes the form presented in figure 3b (the conventional boundary is denoted with the double line).

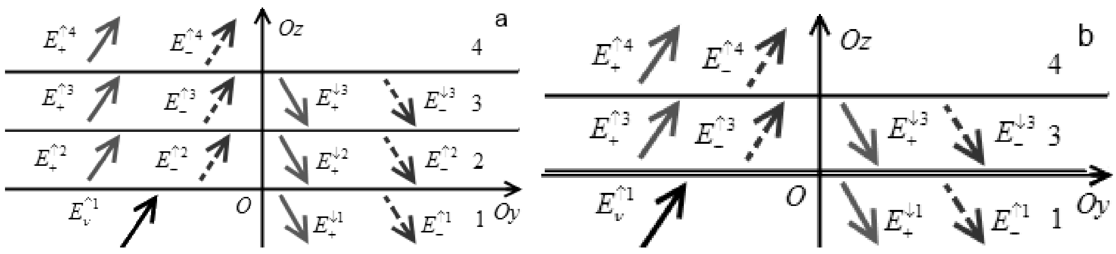


Figure 3 – The scheme of propagation of the wave from the first to the fourth medium in case of inclined incidence: a) full form; b) short form.

One can see that in figure 3b a problem arises, which is analogous to the one considered above. Reasoning in the same way, one can obtain the set of equations for new transmission and reflection coefficients:

$$\begin{cases} E_{+}^{\downarrow 3}/\kappa_{+}^{3}=E_{+}^{\uparrow 3}\rho_{++}^{34}\kappa_{+}^{3}+E_{-}^{\uparrow 3}\rho_{-+}^{34}\kappa_{+}^{3}; & E_{+}^{\uparrow 3}=E_{+}^{\downarrow 3}R_{++}^{31}+E_{-}^{\downarrow 3}R_{-+}^{31}+E_{\nu}^{\uparrow 1}T_{\nu +}^{13}; \\ E_{-}^{\downarrow 3}/\kappa_{-}^{3}=E_{-}^{\uparrow 3}\rho_{--}^{34}\kappa_{-}^{3}+E_{+}^{\uparrow 3}\rho_{+-}^{34}\kappa_{-}^{3}; & E_{-}^{\uparrow 3}=E_{-}^{\downarrow 3}R_{--}^{31}+E_{+}^{\downarrow 3}R_{+-}^{31}+E_{\nu}^{\uparrow 1}T_{\nu -}^{13}, \end{cases} \tag{14}$$

Having solved the equations set (14), one can write down the new reflection and transmission coefficients by analogy with the formulae (9)–(10):

$$E_{\sigma}^{\downarrow 1}=E_{\nu}^{\uparrow 1}R_{\nu\sigma}^{13}+E_{+}^{\downarrow 3}T_{+\sigma}^{31}+E_{-}^{\downarrow 3}T_{-\sigma}^{31};\; E_{\sigma}^{\uparrow 4}=E_{+}^{\uparrow 3}\kappa_{+}^{3}\tau_{+\sigma}^{34}+E_{-}^{\uparrow 3}\kappa_{-}^{3}\tau_{-\sigma}^{34}; \tag{15}$$

$$T_{\nu\sigma}^{14}=E_{\sigma}^{\uparrow 4}/E_{\nu}^{\uparrow 1}; \qquad R_{\nu\sigma}^{14}=E_{\sigma}^{\downarrow 1}/E_{\nu}^{\uparrow 1}. \tag{16}$$

Then one can find the coefficients $T_{\nu\sigma}^{41}$ and $R_{\nu\sigma}^{41}$ in the same way as in (11)-(13):

$$T_{\nu\sigma}^{41}=E_{\sigma}^{\uparrow 1}/E_{\nu}^{\uparrow 4}; \qquad R_{\nu\sigma}^{41}=E_{\sigma}^{\downarrow 4}/E_{\nu}^{\uparrow 4}. \tag{17}$$

The set of equations for this case are:

$$\begin{cases} E_{+}^{\downarrow 3}/\kappa_{+}^{3}=E_{+}^{\uparrow 3}R_{++}^{31}\kappa_{+}^{3}+E_{-}^{\uparrow 3}R_{-+}^{31}\kappa_{+}^{3}; & E_{+}^{\uparrow 3}=E_{+}^{\downarrow 3}\rho_{++}^{34}+E_{-}^{\downarrow 3}\rho_{-+}^{34}+E_{\nu}^{\uparrow 1}\tau_{\nu +}^{43}; \\ E_{-}^{\downarrow 3}/\kappa_{-}^{3}=E_{-}^{\uparrow 3}R_{--}^{31}\kappa_{-}^{3}+E_{+}^{\uparrow 3}R_{+-}^{31}\kappa_{-}^{3}; & E_{-}^{\uparrow 3}=E_{-}^{\downarrow 3}\rho_{--}^{34}+E_{+}^{\downarrow 3}\rho_{+-}^{34}+E_{\nu}^{\uparrow 1}\tau_{\nu -}^{43}, \end{cases} \tag{18}$$

We also need by analogy to (15) the next amplitudes:

$$E_{\sigma}^{\downarrow 1}=E_{\nu}^{\uparrow 1}\rho_{\nu\sigma}^{43}+E_{+}^{\downarrow 3}\tau_{+\sigma}^{34}+E_{-}^{\downarrow 3}\tau_{-\sigma}^{34}; \qquad E_{\sigma}^{\uparrow 4}=E_{+}^{\uparrow 3}\kappa_{+}^{3}T_{+\sigma}^{31}+E_{-}^{\uparrow 3}\kappa_{-}^{3}T_{-\sigma}^{31}. \tag{19}$$

The transmission and reflection coefficients can be found by substituting (19) into (17).

**4. The arbitrary system of layers.** Let us generalize the solution so that it can be used in case of several layers. Let $p$ be ordinal number of the medium. The scheme of this problem is presented in figure 4a.

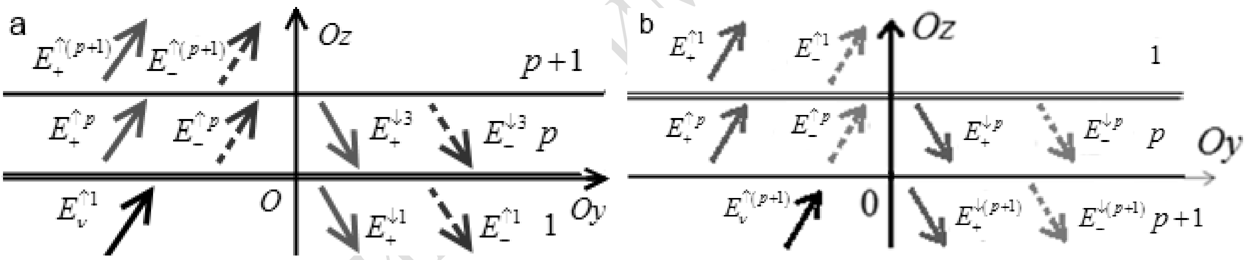


Figure 4 – The scheme of propagation from the $p$-th to the $(p+1)$-th medium

Then the set of equations for the electromagnetic wave that propagates from the first to the medium $(p+1)$ have the following form:

$$\begin{cases} E_{+}^{\downarrow p}/\kappa_{+}^{p}=E_{+}^{\uparrow p}\rho_{++}^{p(p+1)}\kappa_{+}^{p}+E_{-}^{\uparrow p}\rho_{-+}^{p(p+1)}\kappa_{+}^{p}; & E_{+}^{\uparrow p}=E_{+}^{\downarrow p}R_{++}^{p1}+E_{-}^{\downarrow p}R_{-+}^{p1}+E_{\nu}^{\uparrow 1}T_{\nu +}^{1p}; \\ E_{-}^{\downarrow p}/\kappa_{-}^{p}=E_{-}^{\uparrow p}\rho_{--}^{p(p+1)}\kappa_{-}^{p}+E_{+}^{\uparrow p}\rho_{+-}^{p(p+1)}\kappa_{-}^{p}; & E_{-}^{\uparrow p}=E_{-}^{\downarrow p}R_{--}^{p1}+E_{+}^{\downarrow p}R_{+-}^{p1}+E_{\nu}^{\uparrow 1}T_{\nu -}^{1p}, \end{cases} \tag{20}$$

Having solved the set of equations (20), one can find the other amplitudes of waves in the problem

$$E_{\sigma}^{\downarrow 1}=E_{\nu}^{\uparrow 1}R_{\nu\sigma}^{1p}+E_{+}^{\downarrow p}T_{+\sigma}^{p1}+E_{-}^{\downarrow p}T_{-\sigma}^{p1}; \quad E_{\sigma}^{\uparrow(p+1)}=E_{+}^{\uparrow p}\kappa_{+}^{p}\tau_{+\sigma}^{p(p+1)}+E_{-}^{\uparrow p}\kappa_{-}^{p}\tau_{-\sigma}^{p(p+1)}. \tag{21}$$

Then transmission and reflection coefficients can be found using formula

$$T_{\nu\sigma}^{1(p+1)}=E_{\sigma}^{\uparrow(p+1)}/E_{\nu}^{\uparrow 1}; \quad R_{\nu\sigma}^{1(p+1)}=E_{\sigma}^{\downarrow 1}/E_{\nu}^{\uparrow 1}. \tag{22}$$

The coefficients

$$T_{\nu\sigma}^{(p+1)1}=E_{\sigma}^{\uparrow 1}/E_{\nu}^{\uparrow(p+1)}; \quad R_{\nu\sigma}^{(p+1)1}=E_{\sigma}^{\downarrow(p+1)}/E_{\nu}^{\uparrow(p+1)} \tag{23}$$

can be found in the same way as in (17). The scheme of this case is presented in figure 4b.
Analogous set of equations is

$$\begin{cases} E_{+}^{\downarrow p}/\kappa_{+}^{p} = E_{+}^{\uparrow p} R_{++}^{p1}\kappa_{+}^{p} + E_{-}^{\uparrow p} R_{-+}^{p1}\kappa_{+}^{p}; & E_{+}^{\uparrow p} = E_{+}^{\downarrow p}\rho_{++}^{p(p+1)} + E_{-}^{\downarrow p}\rho_{-+}^{p(p+1)} + E_{\nu}^{\uparrow 1}\tau_{\nu +}^{(p+1)p}; \\ E_{-}^{\downarrow p}/\kappa_{-}^{p} = E_{-}^{\uparrow p} R_{--}^{p1}\kappa_{-}^{p} + E_{+}^{\uparrow p} R_{+-}^{p1}\kappa_{-}^{p}; & E_{-}^{\uparrow p} = E_{-}^{\downarrow p}\rho_{--}^{p(p+1)} + E_{+}^{\downarrow p}\rho_{+-}^{p(p+1)} + E_{\nu}^{\uparrow 1}\tau_{\nu -}^{(p+1)p}; \end{cases} \tag{24}$$

$$E_{\sigma}^{\downarrow 1} = E_{\nu}^{\uparrow 1}\rho_{\nu\sigma}^{(p+1)p} + E_{+}^{\downarrow p}\tau_{+\sigma}^{p(p+1)} + E_{-}^{\downarrow p}\tau_{-\sigma}^{p(p+1)}; \qquad E_{\sigma}^{\uparrow (p+1)} = E_{+}^{\uparrow p}\kappa_{+}^{p} T_{+\sigma}^{p1} + E_{-}^{\uparrow p}\kappa_{-}^{p} T_{-\sigma}^{p1}. \tag{25}$$

Having found $T_{\nu\sigma}^{1p}$, $T_{\nu\sigma}^{p1}$, $R_{\nu\sigma}^{1p}$, $R_{\nu\sigma}^{p1}$ for each $2 \le p \le N-1$, using then (20)-(25), one can come to calculation of the coefficients $T_{\nu\sigma}^{1N}$, $R_{\nu\sigma}^{1N}$, $T_{\nu\sigma}^{N1}$, $R_{\nu\sigma}^{N1}$. The energetic reflection and transmission coefficients can be obtained using the formula for absolute value of the Pointing vector:

$$\mathrm{T}_{\nu\sigma}^{1N} = \frac{\left|\vec{S}_{\sigma}^{\uparrow N}\right|}{\left|\vec{S}_{\nu}^{\uparrow 1}\right|} = \left|T_{\nu\sigma}^{1N}\right|^{2} \left|\frac{\mathrm{Im}(b_{\sigma}^{N})}{\mathrm{Im}(b_{\nu}^{1})} \frac{\mathrm{Re}\left(\cos\theta_{\sigma}^{N}\right)}{\mathrm{Re}\left(\cos\theta_{\nu}^{1}\right)}\right|; \quad \mathrm{R}_{\nu\sigma}^{1N} = \frac{\left|\vec{S}_{\sigma}^{\downarrow 1}\right|}{\left|\vec{S}_{\nu}^{\uparrow 1}\right|} = \left|R_{\nu\sigma}^{1N}\right|^{2} \frac{\left|\mathrm{Im}(b_{\sigma}^{1})\,\mathrm{Re}\left(\cos\theta_{\sigma}^{1}\right)\right|}{\left|\mathrm{Im}(b_{\nu}^{1})\,\mathrm{Re}\left(\cos\theta_{\nu}^{1}\right)\right|}. \tag{26}$$

Computational check shows that the formulae obtained here give the same results for particular case of normal incidence as the formulae, which were found by means of the matrix method [5].

**5. Computational analysis.** Let the layers of the biisotropic medium 2 with thickness $d_2$ are placed in the biisotropic medium 1 at a distance $d_1$ from each other. Let us plot dependence of the energetic reflection coefficients on the angle of incidence in case of two, three and four layers of the medium 2 (fragments a, b, c on the figure 5 respectively).

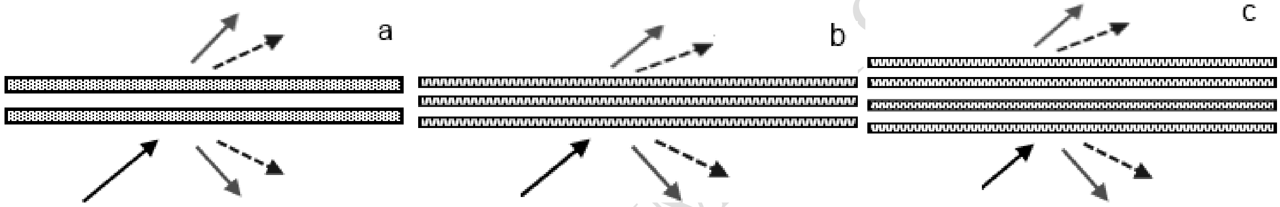


Figure 5 – The scheme for different number of layers of medium 2:
a) $N=5$ (two layers); b) $N=7$ (three layers); c) $N=9$ (four layers)

Results of computations are presented on the figures 6–9.

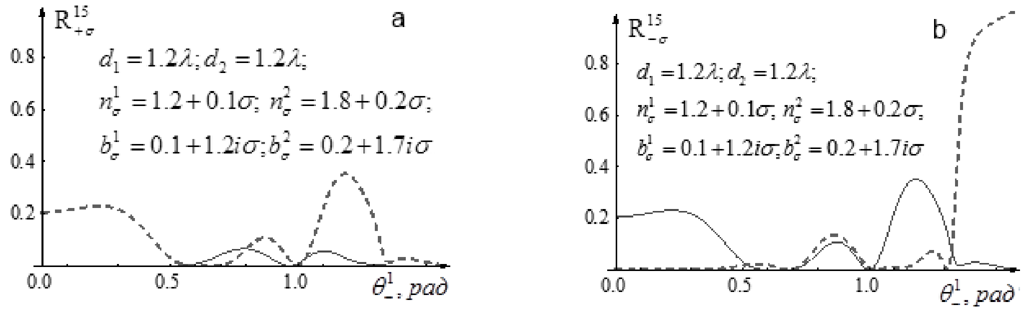


Figure 6 – The plots for the energetic reflection coefficients in case of two layers of the biisotropic medium 2: a solid line – $\sigma=+1$, a dashed line – $\sigma=-1$; polarization of the incident wave: a) $\nu=+1$; b) $\nu=-1$

The plots that reflect dependence of reflection coefficient on the angle of incidence are similar for different polarizations of incident wave. Figure 6 shows the dependence of the reflection coefficient on polarization of the incident wave. Number of layers on the figure equals two ($N=5$). Parameters of the biisotropic media are written on the plot. The angle by the abscissa axis is for left-handed polarized incident wave. The angle for right-handed polarized wave can be found using the Snell law. Solid lines denote reflection coefficients for the right-handed polarized reflected waves, dashed lines denote reflection coefficients for the left-handed polarized reflected waves. A lot of maximums and minimums can be seen on each plot. Other features can be seen in the next figures.

The figure 7 shows two plots for the cases when some of parameters of the both media are equal. On the plot (a), coefficient $b_{\sigma}$ is the same for the media 1 and 2. On the plot (b), coefficient

$n_\sigma$ is the same for the media 1 and 2. It can be seen that right-handed polarized reflected wave is absent in the first case and left-handed polarized wave can be registered only for $\theta_-^1 > 1.5\,rad$ .

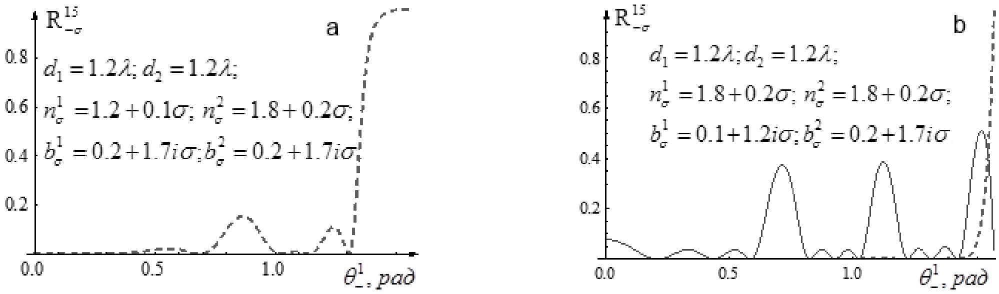


Figure 7 – The plots for the energetic reflection coefficients in case of two layers of the biisotropic medium 2: a solid line – $\sigma = +1$, a dashed line – $\sigma = -1$; a) $b_\sigma^1 = b_\sigma^2$; b) $n_\sigma^1 = n_\sigma^2$

The comparison of influence of $d_1$ and $d_2$ on reflection is presented on the figure 8. Increasing of $d_1$ or $d_2$ leads to increasing of number of maximums on the plots. There are two layers of the medium 2 and only one layer of the medium 1 in the structure. So increasing of $d_2$ leads to more significant change in the plot than increasing of $d_1$. It also can be seen that the envelops of the maximums have the similar form in both cases.

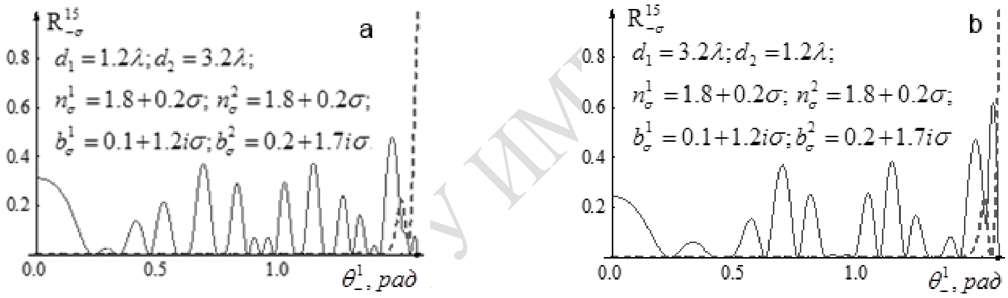


Figure 8 – The plots for the energetic reflection in case of two layers of the biisotropic medium 2: a solid line – $\sigma = +1$, a dashed line – $\sigma = -1$; a) $d_2 > d_1$; b) $d_1 > d_2$

Figure 9 shows the dependence of the reflection coefficient on the number of layers.

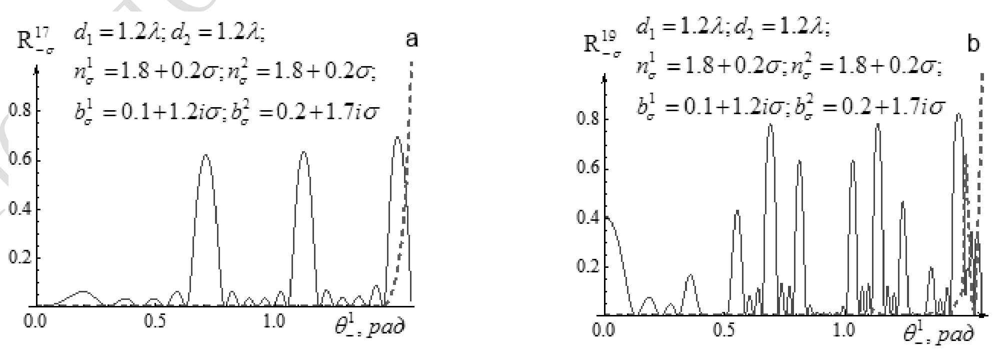


Figure 9 – The plots for the energetic reflection in case of three (a) and four (b) layers of the biisotropic medium 2: a solid line – $\sigma = +1$, a dashed line – $\sigma = -1$

It is noticeable that increasing of the number of the layers leads to increasing of maximum value of the reflection coefficient and the number of maximums on the plot. The envelops of the maximums have similar form in figures 8 and 9.

**Conclusions.** The obtained formulae for the reflection and transmission coefficients can be used to find the complex and the energetic reflection and transmission coefficients of the layered planar medium of any size. Advantage of this method is that it doesn't require knowing the parameters of the medium. It uses only the transmission and reflection coefficients on the boundaries and the phase coefficients. The merit of the method of multiple reflections in comparison with matrix method is that all computations can be performed with relatively small numbers. This method is also well applicable for calculation of periodic structures because the transmission and reflection coefficients calculated once for the boundary of two media can be used repeatedly for the same boundaries in a periodic structure. The final formulae were verified by the matrix method. These two methods give the same numerical results for the same biisotropic structures.

Numerical analysis showed that the dependence of reflection coefficient on the angle of incidence have a lot of maximums and minimums. Form of the envelop of maximums mostly depends on parameters of the media in the structure. Increasing of the number of layers or their thickness leads to growth of the number of maximums on the plot. Reflected wave of left-handed or right-handed polarization can be absent if some parameters of the different media are close.

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