FARADAY EFFECT AND MAGNETOGYRATION IN SUPERLATTICES IN THE LONG WAVELENGTH APPROXIMATION

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1. Introduction

The period of superlattices in practise is often from 1 to 10 nm [1].[1] This makes it possible to use the long wavelength approximation in analysis of superlattices, in a wide spectral range of electromagnetic radiation. In this case a superlattice can be considered as a homogeneous medium characterized by a set of effective parameters: dielectric, magnetic, non-linear optic and other (see References, e.g., [2, 3]). Natural gyrotropic properties of a superlattice satisfying the requirement of the long wavelength approximation were determined in [2, 3]. As we believe, the only realizable superlattices formed by naturally optically active crystals are structures which include layers of SiO_2 [4, 5]. At the same time practically all experimentally realized superlattices can possess induced gyrotropy properties whose phenomenological theory was not developed for superlattices, as far as we know.

The aim of this paper is the calculation (within the frame of the long wavelength approximation) of tensors characterizing usual and induced [6] (in presence of governing electric field) Faraday effects and magnetogyration in superlattices. The main features of these effects exhibited in superlattices are also investigated.

2. Calculation of Effective Tensors

In the long wavelength approximation one can assume that electromagnetic field existing in a superlattice does not change at distances of the order of the superlattice period $D = d^{(1)} + d^{(2)}$, where $d^{(r)}$ (here and thereafter

[1]Superlattices are periodically layered crystal structures.

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$r = 1, 2$) are the thicknesses of the monocrystal layers constituting the superlattice. Let us choose an orthogonal system of coordinates with the axis Z perpendicular to the boundaries of the layers forming the superlattice. From the condition of additivity of the electric and magnetic moments in the volume of superlattice [2, 7] we have the relations connecting the normal components of the field strengths $E_3^{(e)}, H_3^{(e)}$ and tangential components of the inductions $D_r^{(e)}, B_r^{(e)}$ of electromagnetic field in the superlattice and in the layers:

$$A^{(e)} = xA^{(1)} + (1-x)A^{(2)} \tag{1}$$

where $A = E_3, H_3, D_r, B_r, \quad x = d^{(1)}/D$, and indices $e, 1, 2$ define the quantities characterizing the effective medium, the first and the second monocrystal layers, respectively.

When defining the effective superlattice characteristics, we will suppose that the phenomenological material equations [8]

$$\mathbf{D}^{(s)} = \varepsilon^{(s)}\mathbf{E}^{(s)} + i\alpha^{(s)}\mathbf{H}^{(s)}, \quad \mathbf{B}^{(s)} = \mu^{(s)}\mathbf{H}^{(s)} - i\alpha_t^{(s)}\mathbf{E}^{(s)} \tag{2}$$

hold both for the monocrystals forming the superlattice and for the effective medium: $s = 1, 2, e$. Here, the index t denotes the transpose tensor, and $i^2 = -1$. Eqs. (2) are defined by the complex nonsymmetric tensors $\varepsilon^{(s)}, \mu^{(s)}, \alpha^{(s)}$ which describe media with various kinds of anisotropy, gyrotropy, and absorption.

As usually, components of the tensors of the permittivity and permeability ε, μ are significantly larger than the components of the pseudotensor α, which describes the effects of optical activity [8]. All the calculations below are made in the linear (in components of α) approximation.

According to the methods proposed in [7], we have from Eqs. (2)

$$\begin{aligned} E_3 &= \frac{1}{\varepsilon_{33}}\left[D_3 - \varepsilon_{31}E_1 - \varepsilon_{32}E_2 - i\left(\alpha_{31} - \frac{\mu_{31}}{\mu_{33}}\alpha_{33}\right)H_1 \right. \\ &\quad \left. - i\left(\alpha_{32} - \frac{\mu_{32}}{\mu_{33}}\alpha_{33}\right)H_2 - i\frac{\alpha_{33}}{\mu_{33}}B_3\right] \\ H_3 &= \frac{1}{\mu_{33}}\left[B_3 - \mu_{31}H_1 - \mu_{32}H_2 + i\left(\alpha_{13} - \frac{\varepsilon_{31}}{\varepsilon_{33}}\alpha_{33}\right)E_1 \right. \\ &\quad \left. + i\left(\alpha_{23} - \frac{\varepsilon_{32}}{\varepsilon_{33}}\alpha_{33}\right)E_2 + i\frac{\alpha_{33}}{\varepsilon_{33}}D_3\right] \end{aligned} \tag{3}$$

Substitution of expressions (3) into (1) and taking into account continuity of the quantities E_r, H_r, D_3, B_3 on the layer boundaries, defines the expressions

$$\frac{1}{\varepsilon_{33}},\quad \frac{\varepsilon_{3j}}{\varepsilon_{33}},\quad \frac{\varepsilon_{r3}}{\varepsilon_{33}},\quad \varepsilon_{rj}-\frac{\varepsilon_{r3}\varepsilon_{3j}}{\varepsilon_{33}} \tag{4}$$

$$\frac{1}{\mu_{33}},\quad \frac{\mu_{3j}}{\mu_{33}},\quad \frac{\mu_{r3}}{\mu_{33}},\quad \mu_{rj}-\frac{\mu_{r3}\mu_{3j}}{\mu_{33}} \tag{5}$$

$$\begin{gathered}\frac{\alpha_{33}}{\mu_{33}\varepsilon_{33}},\quad \frac{1}{\varepsilon_{33}}(\alpha_{3m}-\alpha_{33}\frac{\mu_{3m}}{\mu_{33}}),\quad \frac{1}{\mu_{33}}(\alpha_{m3}-\alpha_{33}\frac{\varepsilon_{m3}}{\varepsilon_{33}})\\ \alpha_{mn}-\alpha_{3n}\frac{\varepsilon_{m3}}{\varepsilon_{33}}-(\alpha_{m3}-\alpha_{33}\frac{\varepsilon_{m3}}{\varepsilon_{33}})\frac{\mu_{3n}}{\mu_{33}}\end{gathered} \tag{6}$$

Here the indices r, j, m, n take the values 1 and 2. Quantities (4)–(6) (summation over repeating indices is not made) when substituted instead of the parameter A in equation (1) define all the components of the tensors $\varepsilon^{(e)}$, $\mu^{(e)}$, $\alpha^{(e)}$ for arbitrary crystallographic symmetry of the layers. Note that nonsymmetry of the tensors $\varepsilon^{(s)}$, $\mu^{(s)}(s = 1, 2, e)$ leads to differences of the expressions (4)–(6) from the analogous expressions for symmetric tensors $\varepsilon^{(s)}$, $\mu^{(s)}$ [3, 7].

The influence of low-frequency magnetic field $\mathbf{H}^0$ on the material tensors of the layers and the superlattice can be described by the relations [9]

$$\varepsilon^*_{lp}=\varepsilon_{lp}+i\gamma_{lpk}H^0_k,\quad \mu^*_{lp}=\mu_{lp}+i\xi_{lpk}H^0_k,\quad \alpha^*_{lp}=\alpha_{lp}+i\nu_{lpk}H^0_k \tag{7}$$

where $l, p, k = 1, 2, 3$, and the asterisk marks the components of the tensors disturbed by the field $\mathbf{H}^0$. $\varepsilon_{lp}, \mu_{lp}$ are the nondisturbed parts of the tensors of permittivity and permeability, γ_{lpk}, ξ_{lpk} and ν_{lpk} are the axial and polar third-rank tensors describing the Faraday effect and the influence of magnetic field on optical activity (linear magnetogyration), respectively [9, 10].

Expressions (4)–(6), and (1) are correct both for disturbed and undisturbed tensors $\varepsilon^{(e)}, \mu^{(e)}, \alpha^{(e)}$. At the same time, the expansion of the quantities (4)–(6) into the Taylor series with an accuracy to the first order in small parameters $\varepsilon^*_{lp}-\varepsilon_{lp}, \mu^*_{lp}-\mu_{lp}, \alpha^*_{lp}-\alpha_{lp}$ and the following averaging (1) leads to the quantities:

$$\begin{gathered}(\varepsilon_{33}\mu^0_{33})^{-1}\gamma_{3r3},\quad \varepsilon_{33}^{-1}(\gamma_{3rj}-m^0_j\gamma_{3r3})\\ (\mu^0_{33})^{-1}(\gamma_{rj3}-e_r\gamma_{3j3}-e_j\gamma_{r33})\\ \gamma_{rjp}-e_r\gamma_{3jp}-e_j\gamma_{r3p}-m^0_p(\gamma_{rj3}-e_r\gamma_{3j3}-e_j\gamma_{r33})\end{gathered} \tag{8}$$

$$(\varepsilon_{33}\mu_{33}\mu_{33}^{0})^{-1}\nu_{333}, \quad (\varepsilon_{33}\mu_{33})^{-1}(\nu_{33r} - m_r^0\nu_{333})$$

$$(\varepsilon_{33}\mu_{33}^{0})^{-1}(\nu_{3p3} - m_p\nu_{333})$$

$$(\varepsilon_{33})^{-1}[\nu_{3pj} - m_p\nu_{33j} - m_j^0(\nu_{3p3} - m_p\nu_{333})]$$

$$(\mu_{33}\mu_{33}^{0})^{-1}(\nu_{p33} - e_p\nu_{333})$$

$$(\mu_{33})^{-1}[\nu_{p3j} - e_p\nu_{33j} - m_j^0(\nu_{p33} - e_p\nu_{333})] \tag{9}$$

$$(\mu_{33}^{0})^{-1}(\nu_{pr3} + e_pm_r\nu_{333} - e_p\nu_{3r3} - m_r\nu_{p33})$$

$$\nu_{prj} - m_j^0\nu_{pr3} + e_pm_r(\nu_{33j} - m_j^0\nu_{333})$$

$$- e_p(\nu_{3rj} - m_j^0\nu_{3r3}) - m_r(\nu_{p3j} - m_j^0\nu_{p33})$$

where $r, j, p = 1, 2,\ \ m_r = \mu_{r3}/\mu_{33},\ \ m_r^0 = \mu_{3r}^0/\mu_{33}^0,\ \ e_p = \varepsilon_{p3}/\varepsilon_{33}$ and summation over repeating indices is not made. For calculation of (8) and (9) we have taken into account the validity of the relations (1) for the field strength and induction of governing magnetic field, as well as the frequency dispersion: tensor μ corresponds to the frequency of the high-frequency wave, and tensor μ^0 corresponds to the frequency of the governing magnetic field (low-frequency wave). The expressions for the components of tensor $\xi^{(e)}$ follow from expressions (8) after substitutions $\varepsilon_{lp} \to \mu_{lp}, \varepsilon_r \to \mu_r, \gamma_{lpq} \to \xi_{lpq}$ (except the quantities m_j^0 which do not change). Quantities (8) and (9) define all the independent components of tensors $\gamma^{(e)}, \xi^{(e)}, \nu^{(e)}$ after appropriate averaging (1).

It follows from the relations derived above that symmetrical properties of the layers tensors: symmetry of tensors ε, μ and antisymmetry of γ, ξ in the first pair of indices [9] will take place also for the corresponding effective tensors of the superlattice. It makes it possible to express tensors γ, ξ through the second rank tensors (e_{lqp} is the Levi-Civita tensor)

$$\gamma_{lpk} = e_{lqp}\eta_{qk}, \quad \xi_{lpk} = e_{lqp}\rho_{qk} \tag{10}$$

describing the Faraday effect. Substitution of relations (10) in (8) gives

$$(\varepsilon_{33}\mu_{33}^{0})^{-1}\eta_{r3}, \quad \varepsilon_{33}^{-1}(\eta_{rj} - m_j^0\eta_{r3})$$

$$(\mu_{33}^{0})^{-1}(\eta_{33} + e_1\eta_{13} + e_2\eta_{23}) \tag{11}$$

$$\eta_{3p} + e_1\eta_{1p} + e_2\eta_{2p} - m_p^0(\eta_{33} + e_1\eta_{13} + e_2\eta_{23})$$

where $r, j, p = 1, 2$. The expressions for the quantities $\rho_{lp}^{(e)}$ follow from (11) after the above indicated substitutions and the substitution $\eta_{lp} \to \rho_{lp}$.

From expressions (4) and (11) one can get the expressions defining the components of the effective vector of magnetic gyration of the superlattice $\mathbf{G}^{(e)} = \eta^{(e)}\mathbf{H}^0$ ($\varepsilon^* = \varepsilon + i\mathbf{G}^{\times}$, where "$\times$" indicates a tensor dually conjugate to a vector,[2] $r = 1, 2$). After the averaging (1),

$$\varepsilon_{33}^{-1} G_r, \qquad G_3 + \varepsilon_{33}^{-1}(G_1\varepsilon_{13} + G_2\varepsilon_{23}) \tag{12}$$

Expressions (8), (9), (11), (12) characterize the linear Faraday effect and magnetogyration in a short-period superlattice subject to the long wavelength approximation.

For many important applications there is an interest in the research of possibility to govern the Faraday effect in crystal media with the help of external electric fields [6]. These tasks appear, in particular, when designing modulators of phase nonreciprocity for optical gyroscopes [11]. It was shown [6] that in crystal media the influence of electric field on the Faraday effect can be rather different. For example, there are conditions when the induced Faraday effect takes place but the usual one (without external electric field) is absent. Even more possibilities of governing the Faraday effect by electric field can be expected in superlattices.

When studying the Faraday effect in superlattices at the presence of constant (governing) electric field $\mathbf{E}^0$, we will proceed from the expansion of tensors of dielectric permittivity [6]

$$\varepsilon_{ij}^{(s)}(\mathbf{E}^0, \mathbf{H}^0) = \varepsilon_{ij}^{(s)} + \rho_{ijk}^{(s)} E_k^0 + \beta_{ijk}^{(s)} H_k^0 + \Delta_{ijkl}^{(s)} E_k^0 H_l^0 \tag{13}$$

where $s = 1, 2, e$; $i, j, k = 1, 2, 3$. In Eq. (13) tensors $\rho_{ijk}^{(s)}$ and $\beta_{ijk}^{(s)}$ describe linear electrooptic effect and the usual Faraday effect. According to the Onsager relations, we have for the tensors $\Delta_{ijkl}^{(s)}$ characterizing the influence of constant electric field on the Faraday effect [6],

$$\Delta_{ijkl}^{(s)} = -\Delta_{jikl}^{(s)} = e_{ijm}\chi_{mkl}^{(s)} \tag{14}$$

where $m = 1, 2, 3$, e_{ijm} is the Levi-Civita tensor, $\chi_{mkl}^{(s)}$ is a polar tensor of the third rank describing the studied effect. Carrying out the same expansion of relations (4) in Taylor series as that described above, and taking into account Eqs. (13) and (14), we get the expressions defining the

[2]In dyadic notation, this antisymmetric tensor is written using the cross product of the vector with the unit dyadic. *–(Editors)*

tensor $\chi^{(e)}$ through the known tensors [6] of the layers $\chi^{(1)}, \chi^{(2)}$ (using the averaging rule as in (1)):

$$\begin{aligned}
&\varepsilon_{33}^{-1}\chi_{ijp}, \quad (\varepsilon_{33}\varepsilon_{33}^{0})^{-1}\chi_{i3j}\\
&(\varepsilon_{33}\mu_{33}^{0})^{-1}\chi_{ij3}, \quad (\varepsilon_{33}\varepsilon_{33}^{0}\mu_{33}^{0})^{-1}\chi_{i33}\\
&\chi_{3ij}, \quad (\varepsilon_{33}^{0})^{-1}\chi_{33j}\\
&(\mu_{33}^{0})^{-1}\chi_{3j3}, \quad (\varepsilon_{33}\mu_{33}^{0})^{-1}\chi_{333}
\end{aligned} \tag{15}$$

where $i, j, p = 1, 2$. Expressions (15) are obtained for the cases of the layers with diagonal tensors ε, μ (for optical frequencies), ε^0, μ^0 (for low frequencies, corresponding to the fields $\mathbf{E}^0$,$\mathbf{H}^0$).

3. Analysis

Let us examine the Faraday effect in superlattices formed by class $\overline{4}3m$ crystals to which most of the realized superlattices belong. Let us assume $\mu(H^0) = 1$, which corresponds to the case of non-magnetic media. Then relations (11) give the expressions for the following non-zero components of tensor $\eta^{(e)}$

$$\begin{aligned}
&\eta_{11}^{(e)} = \eta_{22}^{(e)} = <\eta/\varepsilon> / <1/\varepsilon>\\
&\eta_{33}^{(e)} = <\eta/\mu^0> / <1/\mu^0>
\end{aligned} \tag{16}$$

where the angular brackets denote the averaging in view of (1), e.g., $<1/\varepsilon> = x/\varepsilon^{(1)} + (1-x)/\varepsilon^{(2)}$. According to relations (16), the general form of the tensor $\eta^{(e)}$ in this case at any x, $0 < x < 1$, coincides with the form of similar tensor for uniaxial crystals, except of classes $3, \overline{3}, 4, \overline{4}, 6, \overline{6}$ [12]. At $x = 0, 1$ from Eqs. (16) we have: $\eta_{11}^{(e)} = \eta_{22}^{(e)} = \eta_{33}^{(e)}$, which corresponds to the limiting transition to cubic monocrystal.

According to formulas (15), in case of the induced Faraday effect in superlattice from class $\overline{4}3m$ crystals tensor $\chi^{(e)}$ has the following non-zero components:

$$\begin{aligned}
&\chi_{132}^{(e)} = \chi_{231}^{(e)} = \frac{<\chi/(\varepsilon\varepsilon^0)>}{<1/\varepsilon><1/\varepsilon^0>}\\
&\chi_{123}^{(e)} = \chi_{213}^{(e)} = \frac{<\chi/(\varepsilon\mu^0)>}{<1/\varepsilon><1/\mu^0>}
\end{aligned} \tag{17}$$

$$\chi_{312}^{(e)} = \chi_{321}^{(e)} =< \chi >$$

Taking into account the form of the tensor χ for monocrystals [6], it follows from relations (17) that tensor $\chi^{(e)}$ with components (17) has the form which is analogous to that for class $\overline{4}2m$ crystals. At normal incidence on a superlattice of a linearly polarized electromagnetic plane wave, which has wave vector perpendicular to the external fields $\mathbf{E}^0, \mathbf{H}^0$, the usual Faraday effect is absent, and the angle of polarization plane rotation is determined only by the induced effect [6]:

$$\phi = (\pi L/\lambda)(x\chi^{(1)} + (1-x)\chi^{(2)})E^0 H^0 \sin(\alpha + \beta) \tag{18}$$

Here, L is the superlattice thickness, λ is the wavelength, $\chi^{(1)}$, $\chi^{(2)}$ are the components of the tensor χ in the first and the second layers [6], α (β) are the angles between the electric (magnetic) field vectors and the crystallographic axis X.

Let us consider the conditions when values of some effective magnetooptical parameters of the superlattice can exceed the analogous quantities of the superlattice monocrystal components. It follows from expressions (11) and (12), that satisfying the conditions $G_r^{(e)} > G_r^{(1)}, G_r^{(2)}$, where $r = 1, 2$, it is impossible at $0 \leq x \leq 1$ and for the layers of arbitrary crystallographic symmetry. Conditions $G_3^{(e)} > G_3^{(1)}, G_3^{(2)}$; $G_3^{(e)}(\varepsilon_{rr}^{(e)})^{-1/2} > G_3^{(1)}(\varepsilon_{11}^{(1)})^{-1/2}$, $G_3^{(2)}(\varepsilon_{11}^{(2)})^{-1/2}$ (the last two conditions are equivalent to the amplification of the Faraday rotation in the superlattice at wave normal parallel to the axis Z) are also not satisfied for diagonal tensors of the layers $\varepsilon^{(1)}, \varepsilon^{(2)}$. From relations (1), (4), (12) we get

$$G_3^{(e)} =< G_3 > +x(1-x)\theta[x\varepsilon_{33}^{(2)} + (1-x)\varepsilon_{33}^{(1)}]^{-1} \tag{19}$$

where $\theta = (\varepsilon_{31}^{(1)}) - \varepsilon_{31}^{(2)})(G_1^{(1)} - G_1^{(2)}) + (\varepsilon_{32}^{(1)} - \varepsilon_{32}^{(2)})(G_2^{(1)} - G_2^{(2)})$. From Eq. (19) it follows that the inequalities $G_3^{(e)} > G_3^{(1)} > G_3^{(2)}$ take place if the following conditions are simultaneously satisfied:

$$\begin{aligned} &\theta > \varepsilon_{33}^{(2)}(G_3^{(1)} - G_3^{(2)}), \\ &x > [1 - \frac{\varepsilon_{33}^{(2)}}{\varepsilon_{33}^{(1)}} + \frac{\theta}{\varepsilon_{33}^{(1)}(G_3^{(1)} - G_3^{(2)})}]^{-1}, \end{aligned} \tag{20}$$

that can take place for non-diagonal tensors $\varepsilon^{(1)}$, $\varepsilon^{(2)}$. It follows from Eq. (19) that inequalities $a^{(e)} > a^{(1)} > a^{(2)}$ are fulfilled simultaneously under conditions

$$x > e\nu(1-d)(1-e)^{-1}(1-\nu)^{-1} \quad (1-e)(1-\nu) > 0 \qquad (21)$$

where $e = \varepsilon^{(1)}/\varepsilon^{(2)}$, $d = a^{(2)}/a^{(1)}$, $\nu = b^{(1)}/b^{(2)}$ and $a = \chi_{132}$, $b = \varepsilon^0$ or $a = \chi_{123}$, $b = \mu^0$.

4. Conclusion

In this paper, the effective magneto-optical tensors of superlattices have been calculated within the frame of the long wavelength approximation. These tensors characterize completely the linear Faraday effect (usual and induced) and magnetogyration in superlattices. It follows from expressions (9), (11), (15) that on the basis of superlattices it is possible to obtain new materials which have wide range of magnetooptical characteristics. Values of some effective magneto-optical parameters of superlattices can exceed the analogous quantities of superlattice components, which can be used for amplification of the induced gyrotropy effects. This effect is due to creation of additional translational symmetry when superlattices are formed.

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