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Electromagnetic Waves in Artificial Chiral Structures with Dielectric and Magnetic Properties

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In this paper plane wave reflection and transmission phenomena in slabs of artificial anisotropic chiral media, taking into account dielectric and magnetic properties, are theoretically considered. The artificial medium is a superlattice of chiral layers and anisotropic magnetic layers. Biaxial symmetry of the medium is defined by the direction of spiral inclusions and the direction normal to the interfaces between layers. Normal incidence is assumed. The effect of compensation of magnetic and dielectric anisotropy is discussed. A new device based on this effect is proposed.

Keywords Chiral media, anisotropy, bianisotropic media, spiral, reflection, transmission

Introduction

In the last 10 years, interest in complex artificial media having chiral properties in the microwave region has been very high (Sihvola, Tretyakov, & Semchenko, 1993; Mariotte & Parneix, 1994; Sihvola et al., 1995; Priou et al., 1997; Weiglhofer, 1997; Jacob & Reinert, 1998; Barbosa & Topa, 2000; Cloete, 1997; Lafosse, 1994; Whites & Chang, 1997). Although the main initial motivation was in the design of novel absorbing materials, it has been established that chirality of microstructure does not lead to appreciable increase of absorption or better interface matching (Tretyakov, Sochava, & Simovski, 1996; Cloete, Bingle, & Davidson, 1999) (other complex materials with spatial dispersion might offer this possibility (Tretyakov & Sochava, 1993b)). However, as a result of very intensive investigations of chiral

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and, more generally, bianisotropic media, it has been found that more complicated properties of these materials can locate many more potential applications than in microwave absorbers. Limiting this discussion to chiral media, the main useful phenomenon there is polarization transformation of fields transmitted through chiral samples. This is, of course, a well-known phenomenon in optics, the new aspect in microwaves being that chiral media can be designed and fabricated, so that the chiral properties are orders of magnitude stronger as compared to those found in natural crystals and observed in the optical region. In addition (and this will be the emphasis in this paper), these properties can be electrically controlled, for example, with magnetic inclusions in chiral composite materials.

Arrays of highly conducting or lossy spirals, which exhibit very strong resonance chiral effects, can nowadays be quite easily and cheaply produced (Kuehl et al., 1997). To manufacture isotropic chiral media, random mixtures of spirals are needed, which is difficult and expensive to produce. That is one of the reasons for our interest in electromagnetic phenomena in regular arrays of spirals (Yatsenko, Tretyakov, & Sochava, 1998). If all the spirals have the same orientation (say, along unit vector **a**), the effective material is uniaxial: in its material relations all the dyadics can be written as combinations of the unit dyadic \overline{I} and the only physically defined direction of vector **a**. Eigenwaves in uniaxial bianisotropic media have been studied in the literature; see Semchenko et al. (1998); Lindell & Viitanen (1993, 1994); Lindell, Viitanen, & Koivisto (1993); Lindell, Tretyakov, & Viitanen (1993); Tretyakov & Sochava (1993a, 1994a). Reflection and transmission of such materials in slabs have been studied in Tretyakov & Sochava (1994b) for the case when the axis is orthogonal to the interfaces. Numerical techniques for planar structures of bianisotropic media have been developed in Graglia, Uslenghi, & Zich (1991) and Tsalamengas (1992). An extension of biaxiality to bianisotropic media was reported in (Weiglhofer & Lakhtakia, 1999).

Clearly, it is easier to manufacture slabs in such a way that the spiral direction is in the slab plane (Figure 1). However, this case is more difficult in the analysis, since there



Figure 1. Geometry of the problem. The axes of spirals are oriented along the x axis. The incident wave propagates along z axis. Calculations were made when the incident microwave electric field is both parallel (along vector **a**) and perpendicular (along vector **b**) to the axis of spirals.

are two special directions: the direction of the spiral axis and the direction normal to the slab. In Semchenko et al. (1998), the last case was considered for the normal incidence of waves and neglecting magnetic properties of the slab material. In this paper, the analysis is extended to account for magnetic properties of the material. The main motivation is that if in the composite structure magnetic layers are introduced (for example, as in Figure 1, layers separating spiral arrays), properties of the medium can be electrically controlled by an external bias magnetic field applied in the slab plane. The geometry of the problem is shown in Figure 1. We are interested in the reflection and transmission coefficients for plane electromagnetic waves in the case when the special axis defined by the unit vector **a** is in the plane of interfaces. Thicknesses of individual layers are assumed to be small compared to the wavelength, so that effective medium description is possible. However, even for thin layers material parameters can have resonances in the frequency area of interest, due to long enough sections of spirals in the each layer. Because of the magnetic layers, the symmetry of the structure is reduced: it is a structure with biaxial symmetry.

Anisotropic chiral media with pronounced magnetic properties can be realized as a periodic superlattice (period \mathcal{D}) in which every period consists of two layers. One layer is an array of parallel spirals, and this layer exhibits only anisotropic dielectric and chiral properties (manufacture of such media was described in Kuehl et al. (1997)). The other layer has anisotropic magnetic properties (it can be a weakly magnetized ferrite such that off-diagonal components of the permeability dyadic are small, but diagonal components are electrically controlled and losses can be neglected). When the condition $\mathcal{D} \ll \lambda$ is satisfied, the long-wave approximation can be used. Here \mathcal{D} is the structure period and λ is the wavelength.

In the present study, where we are only interested in waves travelling along axis *z* (orthogonal to the slab interfaces), all field components along that axis are zero. For the transverse field components with respect to that axis, the following effective material relations can be written:

$$\mathbf{D} = \epsilon_0 (\epsilon_b \mathbf{b} \mathbf{b} + \epsilon_a \mathbf{a} \mathbf{a} + \epsilon_z \mathbf{z}_0 \mathbf{z}_0) \cdot \mathbf{E} + j \sqrt{\epsilon_0 \mu_0} \, \kappa \mathbf{a} \mathbf{a} \cdot \mathbf{H},\tag{1}$$

$$\mathbf{B} = \mu_0(\mu_b \mathbf{b} \mathbf{b} + \mu_a \mathbf{a} \mathbf{a} + \epsilon_z \mathbf{z}_0 \mathbf{z}_0) \cdot \mathbf{H} - j \sqrt{\epsilon_0 \mu_0} \, \kappa \mathbf{a} \mathbf{a} \cdot \mathbf{E}.$$
 (2)

Clearly, because all the spirals are oriented along unit vector \mathbf{a} , the chirality dyadic has only one component \mathbf{aa} . On the other hand, dielectric and magnetic properties are anisotropic. Because of the special symmetry, eigenaxes of the permittivity and permeability dyadics coincide with the directions of \mathbf{a} and \mathbf{b} . Because of the magnetic layers, the symmetry of the structure is biaxial.

The effective parameters of such lattices can be approximately found as (Semchenko, 1990; Gaishun, Semchenko, & Serdyukov, 1993; Djafai Rouhani & Sapriel, 1986; Rytov, 1955)

$$\begin{aligned} \epsilon_{b} &= x\epsilon_{b}^{(1)} + (1-x)\epsilon_{b}^{(2)}, \\ \epsilon_{a} &= x\epsilon_{a}^{(1)} + (1-x)\epsilon_{a}^{(2)}, \\ \mu_{b} &= x\mu_{b}^{(1)} + (1-x)\mu_{b}^{(2)}, \\ \mu_{a} &= x\mu_{a}^{(1)} + (1-x)\mu_{a}^{(2)}, \\ \kappa &= x\kappa^{(1)} + (1-x)\kappa^{(2)}. \end{aligned}$$
(3)

Here x is the relative thickness of layers $x = d^{(1)}/\mathcal{D}$, $1 - x = d^{(2)}/\mathcal{D}$; $d^{(1,2)}$ are thicknesses of layers 1 and 2, $\mathcal{D} = d^{(1)} + d^{(2)}$; indices 1 and 2 correspond to the first and the second layers, respectively. In our case, in layer 1 we have $\mu_a = \mu_b = 1$, and in the second layer, $\kappa = 0$.

Eigenwaves in uniform bianisotropic uniaxial media have been studied in detail (Lindell & Viitanen, 1993, 1994; Lindell, Viitanen, & Koivisto, 1993; Lindell, Tretyakov, & Viitanen, 1993; Tretyakov & Sochava, 1994a). However, in most cases the study of anisotropic (mostly, uniaxial) chiral media was limited to the solution of problems of propagation in infinite or semi-infinite media. In view of the optical applications, usually only single reflections of waves from the first boundary of the medium were considered. Here, the exact solution of the associated boundary value problem for a sample of finite thickness is found. Thereby we take into account multiple reflections from the boundaries of a sample in the case of the normal incidence of plane waves.

Theory

Eigenwaves

Because the field components along the propagation direction are zero, it is convenient to split the fields into components parallel and orthogonal to the spiral axis:

$$\mathbf{E} = E_a \mathbf{a} + E_b \mathbf{b}, \qquad \mathbf{H} = H_a \mathbf{a} + H_b \mathbf{b}, \tag{4}$$

where **b** is the unit vector along $\mathbf{z}_0 \times \mathbf{a}$; see Figure 1. The eigennumbers for waves propagating along axis z in unbounded media modelled by constitutive relations (1) (2),

$$n_{\pm}^{2} = k_{0}^{2} \left[\frac{\epsilon_{a}\mu_{b} + \mu_{a}\epsilon_{b}}{2} \pm \sqrt{\left(\frac{\epsilon_{a}\mu_{b} - \mu_{a}\epsilon_{b}}{2}\right)^{2} + \epsilon_{b}\mu_{b}\kappa^{2}} \right], \tag{5}$$

were found in Lindell and Viitanen (1994). Here $k_0 = \omega \sqrt{\epsilon_0 \mu_0}$ is the free space wave number. Ellipticities of the eigenwaves (i.e., relations between E_a and E_b) are as follows:

$$E_b = e_{\pm} E_a = -\frac{jk_0 \kappa n_{\pm}}{n_{\pm}^2 - k_0^2 \epsilon_b \mu_a} E_a, \qquad H_b = h_{\pm} H_a = -\frac{jk_0 \kappa n_{\pm}}{n_{\pm}^2 - k_0^2 \epsilon_a \mu_b} H_a.$$
(6)

To gain more physical insight, let us consider the limiting case when the chirality parameter κ vanishes. The propagation factors (5) reduce to $n_+^2 = k_0^2 \epsilon_a \mu_b$ and $n_-^2 = k_0^2 \epsilon_b \mu_a$. Obviously, in the first case we have a plane eigenwave with $E_a \neq 0$, $H_a = 0$, and in the last case $H_a \neq 0$, $E_a = 0$. As is well known, for nonchiral media no polarization transformation occurs if the incident wave is polarized along the axis direction or orthogonal to that: these two polarizations are not coupled. On the other hand, if there is no anisotropy of permittivity and permeability ($\epsilon_a = \epsilon_b$, $\mu_a = \mu_b$), (6) simplifies as

$$\kappa E_b = \mp j \kappa \sqrt{1 \pm \frac{\kappa}{\sqrt{\epsilon\mu}}} E_a \tag{7}$$

and similarly for the magnetic field. Thus, in this limit the eigenwaves are elliptically polarized because of anisotropy of the chirality parameter (in isotropic chiral media, eigenwaves are circularly polarized). Finally, if the chirality parameter also vanishes, the last relation is satisfied identically for all fields, as should be.

In the works of Fedorov (e.g., Fedorov, 1976) the effect of compensation of birefringence in nonchiral crystals was predicted. This phenomenon was studied in more detail in (Gvozdev & Serdyukov, 1979) for chiral optical crystals. In our particular case, the conditions are

$$\kappa = 0, \qquad \epsilon_a \mu_b = \epsilon_b \mu_a. \tag{8}$$

As is obvious from (5), in that case $n_+ = n_-$. In nature there are no known crystals for which this condition is satisfied. But the modern technology allows us to create artificial materials with desired properties. In the composite structure that we study in this paper this may be possible since permeability of magnetic layers can be controlled independently from the properties of spiral arrays. In this paper we will suggest exploiting the regime when the second condition of (8) is fulfilled (effectively isotropic permittivity and permeability) but the first is not (chiral material). We can propose construction of a device that is based on this effect (see below).

Reflection and Transmission Coefficients

Electric field in the slab is the sum of four plane eigenwaves

$$E_a = A_+ e^{-jn_+z} + A_- e^{jn_+z} + B_+ e^{-jn_-z} + B_- e^{jn_-z},$$
(9)

where the axis z is along the propagation direction (orthogonal to the interfaces) and A_{\pm} and B_{\pm} are amplitude coefficients to be determined. The axial magnetic field is

$$\eta_0 H_a = X_+ \left(A_+ e^{-jn_+ z} + A_- e^{jn_+ z} \right) + X_- \left(B_+ e^{-jn_- z} + B_- e^{jn_- z} \right), \tag{10}$$

where

$$X_{\pm} = \frac{j\kappa\epsilon_b}{(\epsilon_a\mu_b - \mu_a\epsilon_b/2) \pm \sqrt{(\epsilon_a\mu_b - \mu_a\epsilon_b/2)^2 + \epsilon_b\mu_b\kappa^2}}$$
(11)

and $\eta_0 = \sqrt{\mu_0/\epsilon_0}$. The orthogonal components read

$$E_b = e_+ A_+ e^{-jn_+ z} - e_+ A_- e^{jn_+ z} + e_- B_+ e^{-jn_- z} - e_- B_- e^{jn_- z},$$
(12)

$$\eta_0 H_b = h_+ X_+ \left(A_+ e^{-jn_+ z} - A_- e^{jn_+ z} \right) + h_- X_- \left(B_+ e^{-jn_- z} - B_- e^{jn_- z} \right).$$
(13)

The field in the slab is excited by a plane electromagnetic wave

$$\mathbf{E}_{e} = (A_{1e}\mathbf{b} + A_{2e}\mathbf{a})e^{-jk_{0}z}, \qquad \eta_{0}\mathbf{H}_{e} = (A_{2e}\mathbf{b} - A_{1e}\mathbf{a})e^{-jk_{0}z}.$$
 (14)

A reflected wave

$$\mathbf{E}_r = (A_{1r}\mathbf{b} + A_{2r}\mathbf{a})e^{jk_0 z}, \qquad \eta_0 \mathbf{H}_r = (A_{1r}\mathbf{a} - A_{2r}\mathbf{b})e^{jk_0 z}$$
(15)

and a transmitted wave

$$\mathbf{E}_{\tau} = (A_{1\tau}\mathbf{b} + A_{2\tau}\mathbf{a})e^{-jk_0 z}, \qquad \eta_0 \mathbf{H}_{\tau} = (A_{2\tau}\mathbf{b} - A_{1\tau}\mathbf{a})e^{-jk_0 z}$$
(16)

also exist outside of the layer. From the condition of continuity of the tangential electric and magnetic field strength vectors on the interfaces where z = 0 and z = L we can obtain the system of eight algebraic linear equations:

$$A_{2e} + A_{2r} = A_{+} + A_{-} + B_{+} + B_{-},$$

$$A_{1e} + A_{1r} = e_{+}(A_{+} - A_{-}) + e_{-}(B_{+} - B_{-}),$$

$$-A_{1e} + A_{1r} = X_{+}(A_{+} + A_{-}) + X_{-}(B_{+} + B_{-}),$$

$$A_{2e} - A_{2r} = h_{+}X_{+}(A_{+} - A_{-}) + h_{-}X_{-}(B_{+} - B_{-}),$$

$$A_{2r}e^{-jk_{0}L} = A_{+}e^{-jn_{+}L} + A_{-}e^{jn_{+}L} + B_{+}e^{-jn_{-}L} + B_{-}e^{jn_{-}L},$$

$$A_{1r}e^{-jk_{0}L} = e_{+}A_{+}e^{-jn_{+}L} - e_{+}A_{-}e^{jn_{+}L} + e_{-}B_{+}e^{-jn_{-}L} - e_{-}B_{-}e^{jn_{-}L},$$

$$-A_{1r}e^{-jk_{0}L} = X_{+}(A_{+}e^{-jn_{+}L} - A_{-}e^{jn_{+}L}) + X_{-}(B_{+}e^{-jn_{-}L} - B_{-}e^{jn_{-}L}),$$

$$A_{2r}e^{-jk_{0}L} = h_{+}X_{+}(A_{+}e^{-jn_{+}L} - A_{-}e^{jn_{+}L}) + h_{-}X_{-}(B_{+}e^{-jn_{-}L} - B_{-}e^{jn_{-}L}).$$
(17)

Although all the coefficients in the above system of eight linear equations are known explicitly, the analytical solution is still too involved to write it down. However, the numerical solution is very simple and straightforward.

Effect of Birefringence Compensation

Using (3), the second condition in (8) can be represented in the following form:

$$\frac{x\epsilon_a^{(1)} + (1-x)\epsilon_a^{(2)}}{x\epsilon_b^{(1)} + (1-x)\epsilon_b^{(2)}} = \frac{x\mu_a^{(1)} + (1-x)\mu_a^{(2)}}{x\mu_b^{(1)} + (1-x)\mu_b^{(2)}},$$
(18)

From the last equation follows a quadratic equation for the relative thickness of the first layer *x*:

$$\begin{bmatrix} \left(\epsilon_{a}^{(1)} - \epsilon_{a}^{(2)}\right) \left(\mu_{b}^{(1)} - \mu_{b}^{(2)}\right) - \left(\epsilon_{b}^{(1)} - \epsilon_{b}^{(2)}\right) \left(\mu_{a}^{(1)} - \mu_{a}^{(2)}\right) \end{bmatrix} x^{2} \\ + \begin{bmatrix} \epsilon_{a}^{(2)} \mu_{b}^{(1)} + \epsilon_{a}^{(1)} \mu_{b}^{(2)} - 2\epsilon_{a}^{(2)} \mu_{a}^{(2)} - \epsilon_{b}^{(1)} \mu_{a}^{(2)} - \epsilon_{b}^{(2)} \mu_{a}^{(1)} + 2\epsilon_{b}^{(2)} \mu_{a}^{(1)} \end{bmatrix} x \\ + \epsilon_{a}^{(2)} \mu_{b}^{(2)} - \epsilon_{b}^{(2)} \mu_{a}^{(2)} = 0.$$
 (19)

Let us introduce the following notations:

$$\Delta \epsilon^{(1)} = \frac{\epsilon_b^{(1)} - \epsilon_a^{(1)}}{2}, \quad \langle \epsilon \rangle^{(1)} = \frac{\epsilon_b^{(1)} + \epsilon_a^{(1)}}{2}, \quad \Delta \mu^{(1)} = \frac{\mu_b^{(1)} - \mu_a^{(1)}}{2}, \quad \langle \mu \rangle^{(1)} = \frac{\mu_b^{(1)} + \mu_a^{(1)}}{2},$$

$$\Delta \epsilon^{(2)} = \frac{\epsilon_b^{(2)} - \epsilon_a^{(2)}}{2}, \quad \langle \epsilon \rangle^{(2)} = \frac{\epsilon_b^{(2)} + \epsilon_a^{(2)}}{2}, \quad \Delta \mu^{(2)} = \frac{\mu_b^{(2)} - \mu_a^{(2)}}{2}, \quad \langle \mu \rangle^{(2)} = \frac{\mu_b^{(2)} + \mu_a^{(2)}}{2},$$

$$\Delta \epsilon^{\text{eff}} = \frac{\epsilon_b^{\text{eff}} - \epsilon_a^{\text{eff}}}{2}, \quad \langle \epsilon \rangle^{\text{eff}} = \frac{\epsilon_b^{\text{eff}} + \epsilon_a^{\text{eff}}}{2}, \quad \Delta \mu^{\text{eff}} = \frac{\mu_b^{\text{eff}} - \mu_a^{\text{eff}}}{2}, \quad \langle \mu \rangle^{\text{eff}} = \frac{\mu_b^{\text{eff}} + \mu_a^{\text{eff}}}{2}.$$

Equation (19) is a standard quadratic equation. By drawing corresponding curves, it can be shown that this equation has a real root x within the interval (0, 1) when the following conditions are satisfied simultaneously:

$$\frac{\Delta\epsilon^{(1)}}{\langle\epsilon\rangle^{(1)}} < \frac{\Delta\mu^{(1)}}{\langle\mu\rangle^{(1)}}, \qquad \frac{\Delta\epsilon^{(2)}}{\langle\epsilon\rangle^{(2)}} > \frac{\Delta\mu^{(2)}}{\langle\mu\rangle^{(2)}}.$$
(20)

We have assumed that the structure is lossless. Inequalities (20) mean that the relative anisotropy of permittivity exceeds the relative anisotropy of permeability in one of the layers, and the opposite situation takes place for the other layer. When thickness x is a root of equation (19), the effective parameters of the structure become mutually proportional (that is, the second relation (8) is fulfilled). From condition (8) it follows that

$$\frac{\Delta\epsilon^{\rm eff}}{\langle\epsilon\rangle^{\rm eff}} = \frac{\Delta\mu^{\rm eff}}{\langle\mu\rangle^{\rm eff}},\tag{21}$$

which means that anisotropy of dielectric properties compensates for the anisotropy of the magnetic properties. However, if the chirality parameter is not zero, both eigenwaves are elliptically polarized and have different propagation factors.

Because of the frequency dispersion of the material parameters, condition (21) takes place only at a certain frequency. Therefore, such structures can be used as a device with selective transmittance. The scheme of the device is presented in Figure 2. Similar selective effects take place also in isotropic chiral slabs, since the chirality parameter is frequency dependent. We see from (5) that due to anisotropy of permittivity and permeability, selectivity can be improved. Indeed, at the point of anisotropy compensation, the difference between two wavenumbers has a minimum. When the balance (21) is not fulfilled (due to changed frequency), this difference increases sharply because of the first member under the square root.



Figure 2. The scheme of the device. (1) is the polarizer, (2) is the analyzer.

$$L = \frac{\pi}{2\theta},\tag{22}$$

where

$$\theta = \frac{k_0 \kappa}{2} \tag{23}$$

is the specific rotation of the polarization plane.¹ The angle between planes of the polarizer and the analyzer is $\pi/2$.

Numerical Results and Discussion

Calculations have been performed for linearly polarized incident waves of two main polarizations: the electric field is oriented along spirals (along vector **a**) and orthogonal to them (along vector **b**). The sample thickness L = 5.5 mm. The results demonstrate that in the general case both the transmitted and the reflected waves have elliptic polarizations. The frequency dependence of the rotation angle of the major axis of the polarization ellipsis of the transmitted wave is presented in Figures 3 and 4.

Analysis of the numerical results leads us to the conclusion that the chiral properties of the layer are anisotropic. This means that the rotation angle of the major axis of the polarization ellipse of the transmitted waves is different for different polarizations of the incident field (along vector \mathbf{a} or along vector \mathbf{b}). Such a peculiarity of the chiral properties was observed experimentally for nonmagnetic media and described in Kuehl et al. (1997). A possible explanation of such behavior was given in Semchenko et al. (1998). For magnetically anisotropic media, we observe (Figure 3) that the rotation angle depends on the frequency more strongly in anisotropic structures.

Dependence of the ellipticity of the transmitted wave on the frequency is presented in Figure 5. The ellipticity of the transmitted wave can reach zero at a certain frequency. The transmitted wave on this frequency is linearly polarized. The polarization rotation angle in a transmitted field depends on the azimuth of the incident wave polarization. Dependence of the ellipticity of the reflected wave on the frequency is presented in Figures 6 and 7. The ellipticity of the reflected wave shows a nonmonotonous frequency dependence, and in a certain interval of frequencies maxima and minima of the ellipticity take place. The dependence of the intensity of the transmitted and reflected wave on the frequency is presented in Figures 8 and 9. From Figure 9 we can see that magnetic anisotropy has negligible influence on the reflection and transmission coefficients when the incident wave is polarized along vector **b**. On a certain frequency that depends on the incident wave polarization, intensity of the reflected wave is close to zero. It is evidence of practically complete passage of the electromagnetic waves through the sample on this frequency.

In Figure 10 we demonstrate a typical dependence of the rotation angle on the effective permittivity. For the chosen parameter values, the compensation

¹It is well known that in uniform media the polarization rotation angle is proportional to the chirality parameter κ and the normalized path length $\kappa_0 L$; see, e.g., Fedorov (1976).



Figure 3. The rotation of the polarization ellipsis of the transmitted wave as a function of frequency. Incident wave polarization along the direction of the helix axes (vector **a**) $\bar{\epsilon} = 3.45$, $\Delta \epsilon = 0.45$, $\kappa_a = 0.45$. 1: $\mu_b = 1$. $\mu_a = 1$. 2: $\mu_b = 1.1$, $\mu_a = 1.2$. 3: $\mu_b = 1.05$, $\mu_a = 1.2$. 4: $\mu_b = 1$, $\mu_a = 1.2$.



Figure 4. The same as in Figure 3 for the orthogonal polarization (along vector **b**). $\overline{\epsilon} = 3.45$, $\Delta \epsilon = 0.45$, $\kappa_a = 0.45$. 1: $\mu_b = 1$, $\mu_a = 1$. 2: $\mu_b = 1$, $\mu_a = 1.2$. 3: $\mu_b = 1.05$, $\mu_a = 1.2$. 4: $\mu_b = 1.1$, $\mu_a = 1.2$.



Figure 5. The ellipticity of transmitted wave as a function of frequency. $\bar{\epsilon} = 3.45$, $\Delta \epsilon = 0.45$, $\kappa_a = 0.45$. 1: $\mu_b = 1$, $\mu_a = 1$. 2: $\mu_b = 1$, $\mu_a = 1.2$, for incident wave polarization along the direction of the helix axes (vector **a**). 3: $\mu_b = 1$. $\mu_a = 1$. 4: $\mu_b = 1$, $\mu_a = 1.2$ for orthogonal polarization (along vector **b**).



Figure 6. The ellipticity of reflected wave as a function of frequency. Incident wave polarization along the direction of the helix axes (vector **a**) $\overline{\epsilon} = 3.45$, $\Delta \epsilon = 0.45$, $\kappa_a = 0.45$. Solid line corresponds to $\mu_b = 1$, $\mu_a = 1$; dashed line corresponds to $\mu_b = 1$, $\mu_a = 1.2$.



Figure 7. The same as in Figure 6 for orthogonal polarization.



Figure 8. The intensity of transmitted and reflected waves as functions of frequency (incident wave polarized along vector **a**). 1: Transmitted wave when $\mu_b = 1$, $\mu_a = 1.2$. 2: Transmitted wave when $\mu_b = 1$, $\mu_a = 1.3$: Reflected wave when $\mu_b = 1$, $\mu_a = 1.2$. 4: Reflected wave when $\mu_b = 1$, $\mu_a = 1$.



Figure 9. The intensity of transmitted and reflected waves as functions of the frequency for the orthogonal polarization of the incident wave (along **b**). The solid line corresponds to the transmitted wave, the dashed line corresponds to the intensity of the reflected wave. The same parameters as in Figure 8 hold. The influence of magnetic anisotropy is negligible.



Figure 10. The rotation of the polarization ellipsis of the reflected wave as a function of μ_b . Incident wave polarization perpendicular to the direction of the helix axes (along vector **b**) $\epsilon_b = 3.9$, $\epsilon_a = 3$, $\kappa_a = 0.45$, $\mu_a = 1$, $\omega/(2\pi) = 14$ GHz.

Conclusion

The boundary value problem for artificial multilayered chiral structures with pronounced anisotropic magnetic and dielectric properties has been solved taking into account multiple reflections of electromagnetic waves from the sample boundaries. It has been shown that the rotation angle of the major axis of the polarization ellipse and ellipticity of transmitted wave essentially depend on the incident wave polarization. The intensity of the reflected wave has been estimated, and a possibility of complete transmittance for electromagnetic waves travelling through the plate at a certain frequency depending on the incident wave polarization has been demonstrated. The ellipticity of the reflected wave has also been calculated. It has been shown that the frequency dependence of the ellipticity has maxima and minima near the point of complete passage of waves through the plate. The effect of compensation of birefringence is discussed. It is found that in structures with anisotropic magnetic properties, the effect of compensation of anisotropy can be used to increase frequency selectivity of microwave and optical filters. A new device based on this effect is proposed.

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