Electromagnetic cloaking with a mixture of spiral inclusions

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Abstract

In this presentation a new realization for a metamaterial electromagnetic invisibility cloak is introduced. A cylindrical cloak can be implemented by an annular metamaterial region consisting of a racemic mixture of spiral-shaped, chiral inclusions. The geometry of the spirals makes it possible to create a composite material with equal permittivity and permeability along the spiral axis, which is a significant step towards the actual construction of a fully-functional electromagnetic cloaking device in practice. The work has been done as a student project work within the post-graduate course "Metamaterials in Electromagnetics and Radio Engineering", `http://www.tkk.fi/Yksikot/Sahkomagnetiikka/kurssit/S-96.4620/`

1. Introduction

Lately, a concept of an electromagnetic cloak has been introduced [1, 2]. A recent theory [1] shows that by adjusting the material parameters of an object in a certain way, the object can be used for electromagnetic cloaking. However, the construction of a cloak requires extremely complicated material design which has become possible only in the recent years by the research of artificial metamaterials. Also, the cloaking effect can be achieved only in a very narrow frequency band.

Some results have been already achieved for the cylindrical geometry (ρ, φ, z). The idea is based on a mathematical distortion of the coordinate system. The coordinates are stretched so that the space from inside the region $0 < \rho < b$ is compressed into the cylindrical shell $a < \rho < b$. This can be implemented effectively by manipulation of the material parameters. For the cylindrical cloak, the expressions of the components of the relative permittivity $\overline{\overline{\epsilon}}_r$ and permeability $\overline{\overline{\mu}}_r$ become [3]

$$\epsilon_\rho = \mu_\rho = \frac{\rho - a}{\rho}, \quad \epsilon_\varphi = \mu_\varphi = \frac{\rho}{\rho - a}, \quad \epsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{\rho - a}{\rho}. \tag{1}$$

All the corresponding components must be equal and they also have a radial dependence which is a very difficult task to realize. Relations (1) can, however, be simplified so that the ray trajectories inside the cloak remain the same as for the "full design" [3]. However, the cloak remains no longer non-reflective. A realization for cloaks which satisfy the simplified conditions for a TM-polarized wave [3] and a design for a TE-polarized wave [4] have already been reported.

One of the most challenging design requirements is realization of a medium with *equal* permittivity and permeability, that is, a design applicable for both polarizations.

In our project, we have chosen to write equations (1) as

$$\epsilon_\rho = \mu_\rho = \left(\frac{b}{b-a}\right)\left(\frac{\rho - a}{\rho}\right)^2, \quad \epsilon_\varphi = \mu_\varphi = \epsilon_z = \mu_z = \left(\frac{b}{b-a}\right). \tag{2}$$

Primarily, we focus on satisfying the condition for the radial parameters, $\epsilon_\rho = \mu_\rho$.

2. Properties of spiral particles

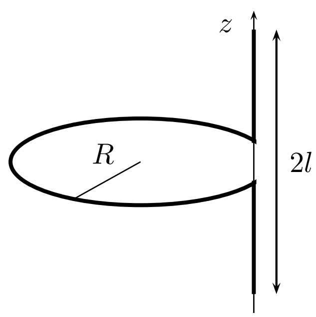


Figure 1: Geometry of the spiral

If the cloak is constructed by using spiral particles (see Fig. 1), the simplified conditions can be satisfied for ϵ and μ simultaneously. Currents induced in a small spiral generate an electric dipole (the wire) and a magnetic dipole (the loop). By adjusting the parameters l and R and the radius r_0 of the wire from which the spiral is made, the electric and the magnetic responses of the particle can be tuned to be identical [5–8]. The relative permittivity and permeability of a sparse mixture can be approximated using the Clausius-Mossotti formula [5]

$$\epsilon_r = 1 + \frac{n}{\epsilon_0 \mathrm{Re}\left\{\frac{1}{\alpha_{ee}^{zz}}\right\} - \frac{n}{3}}, \quad \mu_r = 1 + \frac{n}{\mu_0 \mathrm{Re}\left\{\frac{1}{\alpha_{mm}^{zz}}\right\} - \frac{n}{3}} \tag{3}$$

where n is the number of inclusions per unit volume and the polarizabilities [9]

$$\alpha_{ee}^{zz} = \left[\frac{\frac{\sin kl}{k} - l\cos kl}{1 - \cos kl} - \frac{1-\cos kl}{k \sin kl}\frac{Z_L}{Z_W + Z_L}\right]\frac{4\tan\left(\frac{kl}{2}\right)}{j\omega k Z_W} \tag{4}$$

and [10]

$$\alpha_{mm}^{zz} = -2\mu_0 \pi R^3 \frac{J_1(kR)}{A_0}\left[1 + \frac{j}{Y_L + Y_W}\frac{1}{\pi\eta A_0}\right] \tag{5}$$

where $Z_W = 1/Y_W$ and $Z_L = 1/Y_L$ are the impedances of the wire and the loop, respectively.

The operational frequency of the cloak should be near the resonant frequency of the spirals, otherwise they are weakly excited. By requiring $\mathrm{Re}\{\alpha_{ee}^{zz}/\epsilon_0\} = \mathrm{Re}\{\alpha_{mm}^{zz}/\mu_0\}$ at the resonant frequency, when $\mathrm{Im}\{Y_W\} = -\mathrm{Im}\{Y_L\}$, the optimal dimensions of the spiral can be solved numerically.

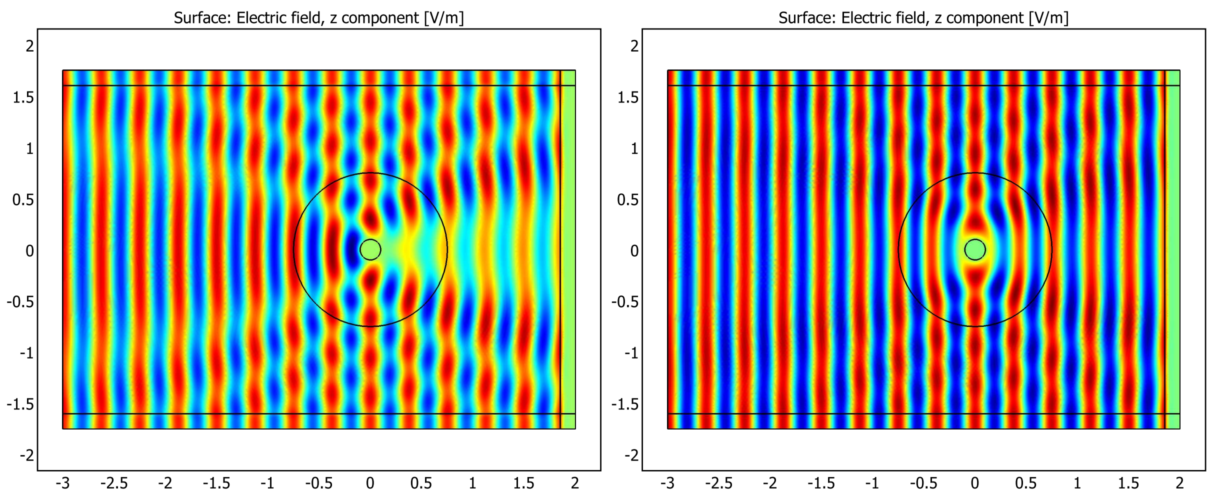
3. Simulations and measurements

In Fig. 2(a) we have simulated a TE polarized wave reflecting from a copper cylinder by Comsol Multiphysics$^{\mathrm{TM}}$. In Fig. 2(b), the same cylinder is surrounded by a cloak which satisfies the modified equations (2). The cloaking effect is noticeable.

We have been able to manufacture spirals by hand using copper wire with radius $r_0 = 0.1$ mm. It was also possible to measure the S parameters of the spiral at the frequency of $f = 1.5$ GHz. By applying the design rules we can find that the suitable spiral parameters for this frequency are $l \approx 17$ mm and $R \approx 11$ mm.

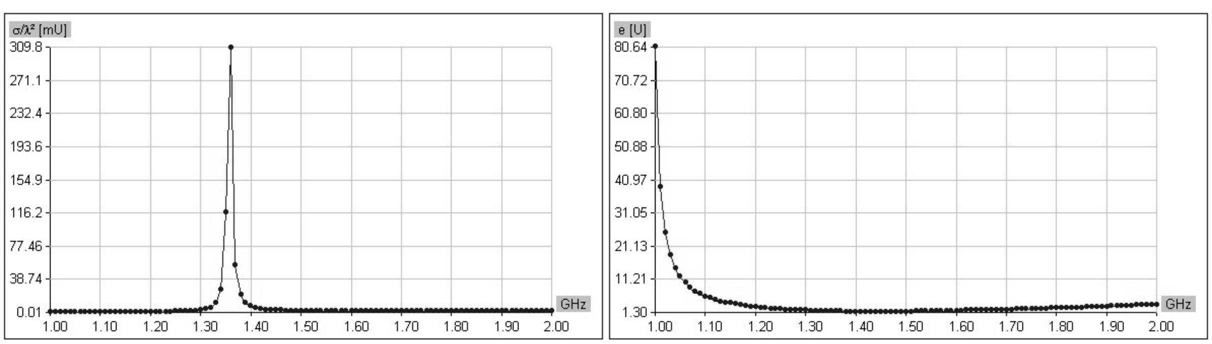
In Fig. 3 we have simulated the behavior of the spiral using WIPL-D$^{\mathrm{TM}}$. From 3(a) we see that the resonant frequency is a little lower than the theoretical value. Theory also implies that if $\alpha_{ee}^{zz}/\epsilon_0 = \alpha_{mm}^{zz}/\mu_0$ at the resonant frequency, the scattered electric field is circularly polarized. In Fig. 3(b) we have plotted the ellipticity of the scattered field. The ellipticity does not actually reach unity but we can see it has its minimum value at $f \approx 1.5$ GHz.

In Fig. 4(a) we have plotted the measured S_{21} parameter of the spiral. We can see that the actual resonant frequency is lower than the designed value. From the measured S_{11} parameters we were able to compute the normalized electrical polarizability $\alpha_{ee}^{zz}/\epsilon_0$ which is plotted in Fig. 4(b). The obtain curve follows nicely the Lorentz model. Unfortunately, we were not able to measure the magnetic response of the spiral.



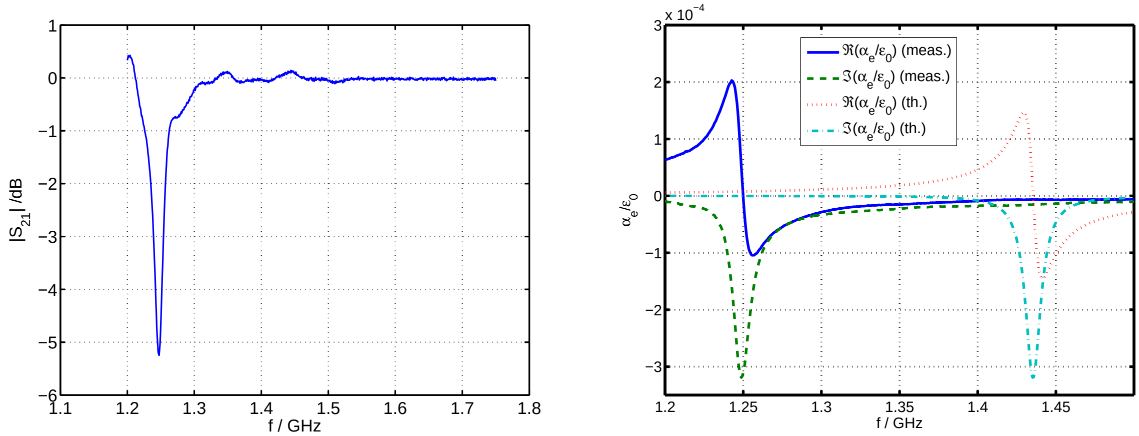
(a) (b)

Figure 2: An uncloaked copper cylinder (a) and the same cylinder with the cloak (b)



(a) (b)

Figure 3: The simulated radar cross section (a) and the ellipticity (b) of the scattered field of a spiral with $l = 17$ mm, $R = 11$ mm



(a) (b)

Figure 4: S_{21} parameter (a) and the measured α_e/ϵ_0 compared with the theoretical value (b) as a function of frequency

4. Conclusion

It is possible to tune both the permittivity and the permeability of a composite material using spiral inclusions. The effect of chirality must, however, be compensated by mixing the spirals racemically. Such materials can be used for constructing an electromagnetic cloak, as one of possible applications.

We have managed to manufacture some spirals by hand. We have also been able to perform measurements for a single spiral. The measurement results of the electric response were promising. However, the measured resonance was somewhat shifted into a lower frequency.

The construction of a completely functioning cloak requires satisfying all the equations (1) or (2). Therefore, the future work will focus on more efficient numerical design and simulations. Also, more advanced measurements must be performed.

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