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# PROPAGATION OF LIGHT ALONG ELLIPTIC SINGULAR DIRECTIONS OF ABSORBING GYROTROPIC CRYSTALS

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In [1] an invariant method [1, 2] was used to investigate the propagation of light along circular axes of nongyrotropic absorbing crystals. Following the method used in [1], we will consider certain features of the propagation of radiation along elliptic axes [3] in absorbing gyrotropic crystals. We will assume that there are no macroscopic currents and charges in the crystal, and that the medium itself possesses a magnetic structure or is in an external magnetic field. Then, in the optical band, its optical properties can be described by one complex unsymmetrical tensor of the reciprocal of the permittivity $\varepsilon^{-1}$, assuming, as in [4], that $\mu = 1$. The tensor $\varepsilon^{-1}$ can be represented in the invariant form [2]

$$\varepsilon^{-1} = \chi + i\mathbf{G}^{\times}, \tag{1}$$

where $\chi$ is a complex symmetrical tensor of the second rank, and $\mathbf{G}$ is the complex gyration vector.

Maxwell's equations for nonuniform plane harmonic waves of the form [1]

$$\mathbf{E} = \mathbf{E}(\xi)\, e^{i\omega t}, \quad \mathbf{H} = \mathbf{H}(\xi)\, e^{i\omega t}, \tag{2}$$

where $\xi = \omega\mathbf{nr}/c$, and $\mathbf{n}$ is the constant unit vector, lead to a second-order linear differential equation

$$\partial^2\mathbf{H}(\xi)/\partial\xi^2 = \hat{\varkappa}\mathbf{H}(\xi), \quad \varkappa = \mathbf{n}^{\times}\mathbf{n}^{\times}\,\tilde{\varepsilon}^{-1}/\mathbf{n}\overline{\overline{\varepsilon^{-1}}}\mathbf{n}. \tag{3}$$

From the equation of the normals [2]

$$1/n^4 + \mathbf{n}(\varepsilon^{-1} - \varepsilon_c^{-1})\,\mathbf{n}/n^2 + \mathbf{n}\overline{\varepsilon^{-1}}\mathbf{n} = 0, \tag{4}$$

which can also be written in the form [4]

$$(1/n^2 - 1/\mathring{n}_+^2)(1/n^2 - 1/\mathring{n}_-^2) = (\mathbf{nG})^2, \tag{4a}$$

it follows that when the refractive indices of the natural waves coincide

$$n^4 = 1/\mathbf{n}\overline{\varepsilon^{-1}}\mathbf{n}, \tag{5}$$

as in nongyrotropic crystals [1].

In [3] the limitations imposed on the components $\varepsilon_{ij}^{-1}$ were found, in order that singular elliptic axes should exist in gyrotropic absorbing crystals, which, taking (4a) into account, reduce to the condition

$$2i\mathbf{nG} = \pm(1/\mathring{n}_-^2 - 1/\mathring{n}_+^2). \tag{6}$$

In this case, both the natural uniform waves degenerate into one, i.e.,

$$\mathbf{h}_+ = \mathbf{h}_- = (\mathring{\mathbf{h}}_+ \mp \mathring{\mathbf{h}}_-)/\sqrt{2}, \tag{7}$$

where $\mathring{\mathbf{h}}_\pm$ are the orthonormalized ($\mathring{\mathbf{h}}_\pm^2 = 1$, $[\mathbf{n}\mathring{\mathbf{h}}_\pm] = \pm\mathring{\mathbf{h}}_\mp$) complex eigenvectors of the magnetic field of the corresponding wave equation ignoring gyrotropy

$$\mathbf{n}^{\times}\chi\mathbf{n}^{\times}\mathring{\mathbf{h}}_\pm = -\mathring{\mathbf{h}}_\pm/\mathring{n}_\pm^2, \tag{8}$$

the explicit form of which is well known [2]. Since $\mathbf{B} = \mathbf{H}$, we have $\mathbf{nH} = 0$ and the general solution of Eq. (3) will be sought in the form

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$$\mathbf{H}(\xi)=f_1(\xi)\,\mathbf{d}_1+f_2(\xi)\,\mathbf{d}_2,\quad \mathbf{d}_2=[\mathbf{d}_1\mathbf{n}], \tag{9}$$

where $\mathbf{d}_1=\mathbf{h}_\pm$ is the normalized elliptic polarization vector of the natural uniform wave (7), which can be excited along the elliptic axes [3]. We will assume, to be specific, that the vector $\mathbf{d}_1$ has a right direction of rotation. Unlike the axes investigated in [1, 5], the elliptic axes we are considering can only exist when there is gyrotropy. In addition, individual directions $\mathbf{n}$ cannot satisfy condition (6), but only a cone of directions.

We will write Eq. (3) in the matrix form

$$f_i''=\varkappa_{ij}f_j, \tag{10}$$

where $\varkappa_{ij}=\mathbf{d}_i\hat{\varkappa}\mathbf{d}_j,\ i,\ j=1,\ 2$ . For arbitrary vectors $\mathbf{a}$ and $\mathbf{b}$ lying in the phase plane of the wave ($[\mathbf{na}]=\mathbf{b}$),

$$\mathbf{b}\,(\mathbf{n}^\times\mathbf{n}^\times\tilde{\varepsilon}^{-1})\,\mathbf{a}=[\mathbf{na}]\,\mathbf{n}^\times\varepsilon^{-1}\,\mathbf{n}^\times\,[\mathbf{nb}]. \tag{11}$$

Calculating the components $\varkappa_{ij}$, taking (3)-(11) into account, we obtain

$$\hat{\varkappa}=-n^2(1+n^2 2in\mathrm{G}\,\rho),\quad \rho=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \tag{12}$$

for the case corresponding to the upper sign in (6) and (7), The general solution of differential equation (10) is

$$\mathbf{f}=e^{i\gamma\xi}\,\mathring{\mathbf{f}}_+ + e^{-i\gamma\xi}\,\mathring{\mathbf{f}}_-, \tag{13}$$

where $\mathbf{f}_+$ and $\mathbf{f}_-$ are arbitrary linearly independent two-dimensional vectors, orthogonal to the normal $\mathbf{n}$, and corresponds to two waves traveling in opposite directions, where $\gamma=\sqrt{\hat{\varkappa}}$, as in [1, 2]. Then, for one of the waves moving in the positive direction of $\mathbf{n}$, we obtain

$$\mathbf{f}=e^{i\gamma\xi}\,\mathring{\mathbf{f}}_+=e^{-i\omega\mathbf{mr}/c}\,(1+\mathbf{n}\mathrm{G}n^3\xi\rho)\,\mathring{\mathbf{f}}_+, \tag{14}$$

where $\mathring{\mathbf{f}}_+=(\mathring{f}_1,\ \mathring{f}_2)$ , whence we can write for the vector $\mathbf{H}$

$$\mathbf{H}=e^{i\varphi}\,[\mathring{f}_1\mathbf{d}_1+\mathring{f}_2\,(\mathbf{d}_2+\mathbf{n}\mathrm{G}\,n^3\xi\mathbf{d}_1)], \tag{15}$$

where $\varphi=\omega t-n\xi$. We can find the polarization of the remaining vectors of the field from Maxwell's equations for waves of the form (2a)

$$\mathbf{D}=e^{i\varphi}\,n\,\{-\mathring{f}_1\mathbf{d}_2+\mathring{f}_2\,[\mathbf{d}_1-in\mathrm{G}\,(1-in\xi)\,\mathbf{d}_2]\},\quad \mathrm{E}=\varepsilon^{-1}\mathbf{D}. \tag{16}$$

Equations (15) and (16) are formally identical with the corresponding expressions obtained in [1, 2] for circular axes in nongyrotropic media. However, for circular directions, the vectors $\mathbf{d}_1$ and $\mathbf{d}_2$ are circular, with opposite directions of rotation, whereas in (15) and (16) $\mathbf{d}_1$ and $\mathbf{d}_2$ are elliptical, with the same direction of rotation. In certain cases they may also be linear [3].

Hence, according to (15), together with the usual uniform elliptic wave $f_1\mathbf{d}_1$ along the singular directions investigated there can also be a Voigt-type wave, the elliptic polarization of the latter (with respect to the vector $\mathbf{H}$) $\mathring{f}_2(\mathbf{d}_2+in\mathrm{G}n^3\xi\mathbf{d}_1)$ continually changes with the distance traversed. Of course, in addition to this because of the usual absorption the amplitudes of both waves (the uniform and the Voigt) will decay exponentially with distance, which is taken into account by the factor $e^{i\varphi}$. As is well known [1, 2], as the Voigt wave propagates along the circular axes in nongyrotropic crystals a gradual transformation of its polarization from circular to circular with opposite direction of rotation occurs. Unlike the usual case, when light propagates along an elliptic axis the polarization of the Voigt wave will gradually change from the vector $\mathbf{d}_2$ to the vector $\mathbf{d}_1$, i.e., in the final analysis there will be a rotation of its principal axis of the ellipse by $\pi/2$, but the direction of rotation will not change. A similar consideration can also be applied to the left elliptic axis. The conditions for it to exist correspond to the lower sign in (6) and (7). In this case $\mathbf{d}_1$ becomes a left elliptic vector, and expression (12) reduces to $\hat{\varkappa}=-n^2(1-2in\mathrm{G}n^2\tilde{\rho})$. The right singular directions become left singular directions, according to (6), with the substitutions $\mathbf{n}\to(-\mathbf{n})$ or $\mathrm{G}\to(-\mathrm{G})$, i.e., when the direction of propagation of the light is reversed or the crystal is remagnetized.

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