JONES AND MÜLLER MATRICES FOR ABSORBING MAGNETICALLY ORDERED CRYSTALS
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In order to describe the interaction of polarized light with optical systems, together with other methods, the Jones or Müller matrix formalism is often used [1-5]. The Jones and Müller matrices were obtained in [6] in order to describe the normal reflection from and transmission of polarized light through a plane parallel plate made of transparent magnetically ordered crystal taking into account multiple reflections on its boundary. In so doing, the crystal was assumed to be gyroanisotropic.

In this paper, we take into account the absorbing properties of the crystal.
We will start from the material equations

$$
\begin{equation*}
\mathbf{E}=\varepsilon^{-1} \mathbf{D}, \mathbf{B}=\mathbf{H} \tag{1}
\end{equation*}
$$

assuming, in agreement with [7], that $\mu=1$ in the optical frequency range. Magnetooptical effects are described by a complex nonhermitian inverse dielectric permeability tensor $\varepsilon^{-1}$ and, in addition, the symmetrical and antisymmetrical parts of the tensor $\varepsilon^{-1}$ describe even and odd, with respect to magnetization and optical effects, respectively.

The propagation of plane monochromatic waves in a uniform crystal along an arbitrary direction with normal $n$ can be described by the two-dimensional tensor $\sigma$ [8, 9]:

$$
\begin{gather*}
\mathbf{H}(l)=\sigma \mathbf{H}(0)  \tag{2}\\
\sigma=e^{i \varphi_{+}} \mathbf{h}_{+} \cdot \tilde{\mathbf{h}}_{+}+e^{i \varphi_{-}} \mathbf{h}_{-} \cdot \tilde{\mathbf{h}}_{-} \tag{2a}
\end{gather*}
$$

where $H$ is the vector corresponding to the intensity of the magnetic field of the total wave; $h_{ \pm}\left(\tilde{h}_{ \pm}\right)$are the right (left) characteristic vectors of the tensor $\sigma$, characterizing the polarization of isonormal characteristic waves; $\varphi_{ \pm}=2 \pi n_{ \pm} l / \lambda$, are the increments of the phases of the characteristic waves over a distance 2 . The tensors $h_{ \pm}\left(\widetilde{h}_{ \pm}\right)$represent dyads, which have the following properties [9]:

$$
\begin{equation*}
\mathbf{h}_{ \pm} \tilde{\mathbf{h}}_{\mp}=0, \quad \mathbf{n} \mathbf{h}_{ \pm}=\mathbf{n} \tilde{\mathbf{h}}_{ \pm}=0, \quad \mathbf{h}_{ \pm} \tilde{\mathbf{h}}_{ \pm}=1 \tag{3}
\end{equation*}
$$

The polarization of plane waves in absorbing magnetically ordered crystals was studied in [10]. According to the data in [10],

$$
\begin{gather*}
\mathbf{h}_{ \pm} \sim\left(\circ_{( }^{h_{ \pm}}+x_{ \pm} \mathrm{h}_{\mp}\right)  \tag{4}\\
x_{ \pm}=i\left(\gamma \pm \gamma_{0}\right) /\left(1 \pm \gamma \gamma_{0}\right) \tag{4a}
\end{gather*}
$$

Here, $h_{ \pm}^{\circ}$ are orthonormal vectors, situated in the phase plane of the wave, giving the main directions of the plate; $\gamma$ and $\gamma_{0}$ are parameters of the medium that depend on the components of the tensor $\varepsilon^{-1}$ and the normal $n$ of the wave. In order to describe the polarization of characteristic waves of an absorbing crystal, it is also convenient to introduce the complex angles $\alpha$ and $\beta *$ :

$$
\begin{equation*}
\gamma=\operatorname{th}(\alpha / 2), \quad \gamma_{0}=\operatorname{th}(\beta / 2) \tag{5}
\end{equation*}
$$

then

$$
\begin{equation*}
x_{ \pm}=i \operatorname{th}[(\alpha \pm \beta) / 2] \tag{6}
\end{equation*}
$$

and, according to $[11,8]$ from (4)-(6) it immediately follows that the principle axes of the polarization ellipses of the characteristic waves in an absorbing magnetically ordered crystal

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are rotated at equal and opposite angles $\stackrel{\circ}{\varphi}_{ \pm}$relative to the directions of the vectors $\dot{\circ}_{+}$and $h_{-}$, while their ellipticities $\gamma_{ \pm}$(ratio of the semiaxes of the ellipses) are different [10]:

$$
\begin{equation*}
\stackrel{\circ}{\varphi}_{ \pm}= \pm \alpha^{\prime \prime} / 2, \quad \gamma_{ \pm}=\operatorname{th}\left[\left(\alpha^{\prime} \pm \beta\right) / 2\right] . \tag{7}
\end{equation*}
$$

Taking into account the Eqs. (3)-(4a), we find $\widetilde{\mathbf{h}}_{ \pm}$, which are obtained from $\mathbf{h}_{ \pm}$by the substitution $\gamma \rightarrow(-\gamma)$. Then, substituting the expression for the vectors $h_{ \pm}$and $\widetilde{h}_{ \pm}$into (2a), after transformations, we find the Jones matrix for an unbounded uniform absorbing magnetically ordered crystal with arbitrary symmetry

$$
\hat{D}=\frac{e^{i \frac{\varphi_{+}+\varphi}{2}}}{\operatorname{ch} \alpha}\left[\begin{array}{l}
\operatorname{ch} \alpha \cos \frac{\Delta}{2}-i \operatorname{ch} \beta \sin \frac{\Delta}{2} ;(\operatorname{sh} \beta-\sin \alpha) \sin \frac{\Delta}{2}  \tag{8}\\
(\operatorname{sh} \beta+\sin \alpha) \sin \frac{\Delta}{2} ; \operatorname{ch} \alpha \cos \frac{\Delta}{2}+i \operatorname{ch} \beta \sin \frac{\Delta}{2}
\end{array}\right],
$$

where the complex phase shift

$$
\begin{equation*}
\Delta=\varphi_{-}-\varphi_{+} . \tag{8a}
\end{equation*}
$$

Here, the $x$ and $y$ axes are oriented along the principal directions $\circ_{+}$and $\stackrel{\circ}{h}_{-}$. Equation (8) is quite general and takes into account the anisotropy, gyrotropy, and absorption of the medium. The parameters $\alpha$ and $\Delta$, in general, will be complex. The term $\exp \left[i\left(\varphi_{+}^{\prime}+\varphi_{-}^{\prime}\right) / 2\right]$ is a phase factor, the parameter $\exp \left[-\left(\varphi_{+}^{\prime \prime}+\varphi_{-}^{\prime \prime}\right) / 2\right]$ is the average absorption coefficient of the crystal for a given direction $n$, and the parameter $\Delta^{\prime}$ describes birefringence, while $\Delta^{\prime \prime}$ describes dichroism. It should be kept in mind [8] that crystals, generally speaking, do not have linear or circular but elliptical birefringence and dichroism.

The results obtained can be extended to the case of normal transmission (or reflection) of light through a plane parallel plate, i.e., to take into account interference phenomena in the plate. For a plate, the tensor $\sigma$ has the form [8, 9]

$$
\begin{equation*}
\sigma=D_{+} \mathbf{h}_{+} \cdot \tilde{\mathbf{h}}_{+}+D_{-} \mathbf{h}_{-} \cdot \tilde{\mathbf{h}}_{-}, \tag{9}
\end{equation*}
$$

where $D_{ \pm}$are known [12-14] amplitude coefficients for transmission of characteristic waves through the plate. We introduce the increase in the complex phases $\rho_{ \pm}$of the characteristic waves after passing through the plate taking into account multiple reflection by the expressions

$$
\begin{equation*}
D_{ \pm}=\exp \left(i \varphi_{ \pm}\right) . \tag{10}
\end{equation*}
$$

Then, $\varphi_{ \pm}^{\prime}$ and $\varphi_{ \pm}^{\prime \prime}$ have the same meaning as above, so that Eqs. (8) and (8a) for the Jones inatrix remain in force for the plate as well.

Knowing the Jones matrix, it is possible to calculate the components $M_{i j}$ of the corresponding Mïler matrix $M$ using known rules [4]:

$$
\begin{gather*}
M_{i j}=\frac{2 \exp \left[-\left(\varphi_{+}^{\prime \prime}+\varphi_{-}^{\prime \prime}\right)\right]}{\operatorname{ch} 2 \alpha^{\prime}+\cos 2 \alpha^{\prime \prime}} m_{i j}(i, j=1,2,3,4) ;  \tag{11}\\
2 m_{11,44}=\left(\operatorname{ch} 2 \alpha^{\prime} \pm \operatorname{ch} 2 \beta\right) \operatorname{ch} \Delta^{\prime \prime}+\left(\cos 2 \alpha^{\prime \prime} \mp \operatorname{ch} 2 \beta\right) \cos \Delta^{\prime} ; \\
2 m_{22,33}=\left(\cos 2 \alpha^{\prime \prime} \pm 1\right) \operatorname{ch} \Delta^{\prime \prime}+\left(\operatorname{ch} 2 \alpha^{\prime} \mp 1\right) \cos \Delta^{\prime} ; \\
2 m_{23,32}= \pm\left(\operatorname{sh} \Delta^{\prime \prime} \sin 2 \alpha^{\prime \prime}-\sin \Delta^{\prime} \operatorname{sh} 2 \alpha^{\prime}\right) ;  \tag{11a}\\
2 m_{41,14}=\sin \Delta^{\prime} \sin 2 \alpha^{\prime \prime}+\operatorname{sh} \Delta^{\prime \prime} \operatorname{sh} 2 \alpha^{\prime} \pm\left(\operatorname{ch} \Delta^{\prime \prime}-\cos \Delta^{\prime}\right) \operatorname{sh} 2 \beta ; \\
m_{12,24}=\left(\operatorname{sh} \Delta^{\prime \prime} \cos \alpha^{\prime \prime} \operatorname{ch} \alpha^{\prime}-\sin \Delta^{\prime} \sin \alpha^{\prime \prime} \operatorname{sh} \alpha^{\prime}\right) \operatorname{ch} \beta \pm \\
\pm\left(\operatorname{ch} \Delta^{\prime \prime}-\cos \Delta^{\prime}\right) \operatorname{sh} \beta \cos \alpha^{\prime \prime} \operatorname{sh} \alpha^{\prime} ; \\
m_{43,34}=\operatorname{sh} \beta \sin \alpha^{\prime \prime} \operatorname{ch} \alpha^{\prime}\left(\operatorname{ch} \Delta^{\prime \prime}-\cos \Delta^{\prime}\right) \pm \\
\pm\left(\sin \Delta^{\prime} \cos \alpha^{\prime \prime} \operatorname{ch} \alpha^{\prime}+\operatorname{sh} \Delta^{\prime \prime} \sin \alpha^{\prime \prime} \operatorname{sh} \alpha^{\prime}\right) \operatorname{ch} \beta ; \\
n_{13,31}=\left(\sin \Delta^{\prime} \cos \alpha^{\prime \prime} \operatorname{ch} \alpha^{\prime}+\operatorname{sh} \Delta^{\prime \prime} \sin \alpha^{\prime \prime} \operatorname{sh} \alpha^{\prime}\right) \operatorname{sh} \beta \pm\left(\operatorname{ch} \Delta^{\prime \prime}-\cos \Delta^{\prime}\right) \operatorname{ch} \beta \sin \alpha^{\prime \prime} \operatorname{ch} \alpha^{\prime} ; \\
m_{42,24}=\left(\operatorname{ch} \Delta^{\prime \prime}-\cos \Delta^{\prime}\right) \operatorname{ch} \beta \cos \alpha^{\prime \prime} \operatorname{sh} \alpha^{\prime} \pm\left(\operatorname{sh} \Delta^{\prime \prime} \cos \alpha^{\prime \prime} \operatorname{ch} \alpha^{\prime}-\sin \Delta^{\prime} \sin \alpha^{\prime \prime} \operatorname{sh} \alpha^{\prime}\right) \operatorname{sh} \beta .
\end{gather*}
$$

Thus, we have calculated the components of the Müller matrix describing the normal transmission of polarized light through a plane parallel plate consisting of an absorbing mag-
netically ordered crystal, which simultaneously has elliptical birefringence and elliptical dichroism. In order to go over to a transparent crystal, it is enough to set $\gamma_{0}=\gamma^{\prime \prime}=0$, i.e., $\beta=\alpha^{\prime \prime}=0$. Then Eqs. (8), (8a) and (11), (11a) go over into the corresponding Eqs. (3) and (4), (4a), obtained previously in [6]. In so doing, it should be kept in mind that in [6] a different representation was used for the parameters $\gamma$ and $D_{ \pm}$. In contrast to (5), (8a), and (10) in [6] the notation $D_{-} / D_{+}=k e^{i \Delta}, \gamma=\tan (\eta / 2)$ is introduced. For this reason, if the substitutions sinh $\alpha \rightarrow \tan \eta, \cosh \alpha^{\prime} \rightarrow \sec \eta, \sinh \Delta^{\prime \prime} \rightarrow \cot \eta_{k}$ and $\cosh \Delta^{\prime \prime} \rightarrow \csc \eta_{k}$ are made in (8), (8a), (11), and (11a), then we obtain the corresponding Eqs. (3), (4), and (4') in [6].

Although Eqs. (11) and (1la) are quite cumbersome, in most practically important cases they greatly simplify. Thus, most often, it is possible to neglect multiple reflection. In addition, if the light propagates in the plane of symmetry of the crystal, then it is possible to set $\gamma_{0}=0$, i.e., $\beta=0$.

Starting from the solution of the boundary value problem of the transmission of light in directions far away from the optical axis through an absorbing uniaxial crystal, which has natural optical activity, the Müller matrix is calculated for the crystal in [15]. In so doing, however, the birefringence was assumed to be small and losses due to reflection were not taken into account.

In $[16,17]$, the Jones and Müller matrices were calculated by separating the medium into infinitely thin layers and parameterizing several magnetooptical effects for a medium that exhibits similtaneously the Faraday and Voight effects and linear and circular dichroism; multiple reflections inside the plane parallel specimen are also taken into account. This approach is similar to Jones' technique, in which the so-called Jones differential $N$ matrices were introduced [18]. The inadequacies of this approach are discussed in [14], where it is noted that in developing his treatment of crystallooptical phenomena Jones neglected boundaries between media. In addition, as indicated in [14], this approach is applicable, strictly speaking, only in those cases when the $N$ matrix (or the exponential operator according to Barkovskii) can be represented as a product of several, depending on the same $Z$ ( $Z$ being the thickness of the crystal), commuting matrices (operators), each of which is responsible for some single optical property of the crystal (for example, for the circular or linear dichroism, for circular or linear birefringence). In general, it is impossible to separate Jones' N matrix into such factors, so that it has limited applicability.

Indeed, in general, an absorbing anisotropic gyrotropic medium, as already emphasized [8], does not have linear or circular, but elliptical birefringence and dichroism. The manifestation of such linear and circular dichroism corresponds to the particular case when $\beta=0$, which is satisfied, as a rule, only for crystals with quite high symmetry. This leads to additional symmetry properties and decreases the number of independent components of the Jones matrix $d_{i j}$ and Müler matrix $M_{i j}$. Then, in Jones' matrix $d_{21}=\mathcal{d}_{22}$, while the characteristic waves in the medium have the same ellipticity and different directions of rotation. The work in [16] and [17] is limited to this case. For the Müller matrix, in addition to the condition $m_{32}=-\mathrm{m}_{23}$, we then obtain

$$
\begin{gather*}
m_{12}=m_{21}, m_{13}=-m_{31}, m_{14}=m_{41}, m_{24}=m_{42} \\
m_{34}=-m_{43}, m_{11}-m_{44}=m_{22}-m_{33} \tag{12}
\end{gather*}
$$

In addition, interference phenomena inside the plane parallel specimen are not quite correctly taken into account in [16] and [17], since for this purpose, the authors of these works introduce the coefficient of reflection of light $R$ from the plane of the specimen, which is the same for both characteristic waves.

The artificial nature of the method of separating magnetooptical effects, described by a single dielectric permeability tensor in the optical frequency range into eight (!) different effects and parameterizing them with the help of eight magnetooptical parameters, as done in [19], is obvious. As a result, the equations obtained, in the words of the authors of [19], are "very complicated and difficult to understand and use." In addition, the effect of boundaries of the plane parallel specimen were not taken into account in [19].

The Jones and Müller matrices obtained in this paper make it possible to calculate the change in the state of a light beam by the plate in the general case. For this, it is first necessary to give the Stokes vector of the incident beam $S_{0}=\left\{S_{0_{1}}, S_{02}, S_{03}, S_{04}\right\}$, written in the form [1, 20]

$$
\begin{gather*}
S_{01}=I_{0}, S_{02}=I_{0} p_{0} \cos \eta_{0} \cos 2 \psi_{0}, \\
S_{03}=I_{0} p_{0} \cos \eta_{0} \sin 2 \psi_{0}, S_{04}=I_{0} p_{0} \sin \eta_{0} . \tag{13}
\end{gather*}
$$

Here, $I_{0}$ is the intensity of the light wave; $p_{0}\left(0 \leqslant p_{0} \leqslant 1\right)$ and $\psi_{0}\left(0 \leqslant 2 \psi_{0} \leqslant 2 \pi\right)$ are, respective$1 y$, the degree and azimuth of its polarization. The ellipticity $\gamma_{0}=\tan \left[\left(\arcsin \eta_{0}\right) / 2\right]$ (ratio of semiaxes of the polarization ellipse) is given by the parameter $\eta_{0}\left(-\pi / 2 \leqslant \eta_{0} \leqslant \pi / 2\right)$.

In order to have a unified approach, in correspondence with [7], we will further give the ellipticity $\gamma_{0}$ by another parameter $\alpha_{0}$ :

$$
\begin{equation*}
\gamma_{0}=\operatorname{th}\left(\alpha_{0} / 2\right)\left(-\infty<\alpha_{0}<+\infty\right), \tag{14}
\end{equation*}
$$

then

$$
\begin{gather*}
S_{01}=I_{0}, S_{02}=I_{0} p_{0} \cos 2 \psi_{0} \operatorname{sch} \alpha_{0}, \\
S_{03}=I_{0} p_{0} \sin 2 \psi_{0} \operatorname{sch} \alpha_{0}, S_{04}=I_{0} p_{0} \text { th } \alpha_{0} . \tag{15}
\end{gather*}
$$

As is well known, Stokes' vector $S=\left\{S_{1}, S_{2}, S_{3}, S_{4}\right\}$ for the transmitted light equals

$$
\begin{equation*}
\mathbf{S}=\hat{M} S_{0}, \tag{16}
\end{equation*}
$$

while the degree of polarization of the beam at the exit [20] is

$$
\begin{equation*}
p=\left[S_{2}^{2}+S_{3}^{2}+S_{4}^{2}\right]^{1 / 2} / S_{1} . \tag{17}
\end{equation*}
$$

In calculating (17), a quite cumbersome expression is obtained, which, however, can be simplified if we take into account the fact that the plate is a nondepolarizing system and, for this reason [1], its Müller matrix $\hat{M}$ must have only seven independent components. Keeping in mind the relation between the components of the Jones and Milller matrices [4], it is possible to obtain relations (invariants), relating the components $M_{m n}$, of the Müller matrix of a nondepolarizing system

$$
\begin{equation*}
M_{i 1} M_{j_{1}}-\sum_{k=2} M_{i k} M_{j_{k}}=L_{+} L_{-} \delta_{i j}(i, j=1,2,3,4), \tag{18}
\end{equation*}
$$

where $L_{ \pm}=\exp \left(-2 \varphi_{ \pm}^{\prime \prime}\right)$. are the energy coefficients of transmission of characteristic waves through the system; $\delta_{i j}$ is the four-dimensional Kronecker symbol. We note that (18) remain valid also after transposing in them the components $M_{m n}$.

Taking into account (18), the degree of polarization of the transmitted light takes the form

$$
\begin{equation*}
p=\left[1-L_{+} L_{\ldots}\left(1-p_{0}^{2}\right) / L^{2}\right]^{1 / 2}, \tag{19}
\end{equation*}
$$

where the coefficient of transmission of light (of the total wave) L of the plate equals

$$
\begin{equation*}
L=M_{11}+p_{0}\left[\operatorname{sch} \alpha_{0}\left(M_{13} \sin 2 \psi_{0}+M_{12} \cos 2 \psi_{0}\right)+M_{14} \text { th } \alpha_{0}\right] . \tag{20}
\end{equation*}
$$

Further, it is possible to calculate the ellipticity and azimuth of polarization of light at the exit from the crystal using the equations [20]

$$
\begin{equation*}
\operatorname{tg} \alpha=S_{4} /\left(S_{1} p\right), \operatorname{tg} 2 \psi=S_{3} / S_{2} . \tag{21}
\end{equation*}
$$

Without writing out Eqs. (21), we note that in the latter case expressions are obtained that extend the corresponding equations proposed in [21] to describe the Faraday effect in anisotropic absorbing media with linear dichroism and for Faraday rotation in absorbing orthoferrite along the crystallographic $z$ axis [22], as well as the results of the investigation of normal reflection and transmission of completely [23] and partially [24] polarized radiation through a plane parallel plate made of a transparent magnetically ordered crystal.

In order to have an invariant description of the light beam, it was proposed in [25] that the coherence tensor be used. This approach is successfully developed in [9, 26], where, in particular, an algorithm was proposed for carrying out polarization calculations of parametrically controlled transparent and absorbing unbounded crystals, characterized by a complex unsymmetrical dielectric permeability tensor. The approach that we used, based on the description of a light wave with the help of Stokes' parameters, is closer to the traditional approach and, as indicated above, takes into account the effect of interference phenomena in thin plates.

Let us consider further the problem of the intensity of light passing through a plane parallel plate consisting of an absorbing magnetically ordered crystal, placed between elliptical polarizer and analyzer. Calculations show that the intensity of light sought equals

$$
\begin{equation*}
I=\tilde{S}_{a} \hat{M} \mathbf{S}_{t}, \tag{22}
\end{equation*}
$$

where $S_{p}$ is Stokes' vector for light passing through the polarizer; $\widetilde{\mathbf{S}}_{\alpha}$ is the transposed Stokes' vector, characterizing radiation passing through the analyzer. Substituting into (22) Müller's matrix for a transparent magnetically ordered crystal, we obtain the result obtained previously in [27].

In conclusion, we note that the results obtained in this work can be used to describe the reflection of light and for other types of crystals as well, for example, with natural optical activity. It is only necessary to choose a system of coordinates in which the polarization ellipses of the characteristic isonormal waves in the crystal, moving along the $z$ axis, are rotated by identical but opposite angles relative to the $x$ and $y$ axes.

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LOWERING OF THE DETECTION LIMIT IN SPECTROGRAPHIC DETERMINATION
OF TRACES OF ORGANIC SUBSTANCES
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UDC 543.422 .063

In many scientific and technical applications, it is important to determine traces of organic substances. One such application is monitoring of the purity of electronic, optical, and other products.

It is not always justified to use the mass-spectral procedure for this purpose [1] because there is often no need to know the molecular composition of the contaminating organic substances. An efficient tool for detection of total hydrocarbons is a field-emission projector which shows the presence of total carbon evolved during pyrolysis of hydrocarbons on a hot tungsten emitter [1, 2]. However, the coefficients of capture of various hydrocarbons by the emitter are not identical. They are close to unity only for compounds with molecular weights $M>78$. For low $M$ values, they are much less than unity.

The methods of emission atomic spectroscopy make it possible to determine total carbon both according to spark spectra [3-5] and according to arc spectra [6, 7]. In addition, the use of spark spectra is preferable because of the higher accuracy of the measurement results.

The presence of carbon dioxide in air hinders the determination of minimal carbon concentrations. The use of a discharge in $\mathrm{CO}_{2}$-free inert gas [8] makes it more expensive and complicated to obtain measurement results. In the present paper, we investigated the possibility of detecting carbon traces in an atmosphere of $\mathrm{CO}_{2}$-free air.

The investigations were carried out with a setup which consisted of an IG-3 spark generator, specially developed electrode holders (see Fig. 1), an air-cleaning system, and an ISP-28 spectrograph.

The analytical spectral line of C III at 229.6 nm was excited stably with switching-in of an IG-3 spark generator in a circuit with an auxiliary discharger whose size was established as equal to 3 mm with switched-out inductance and a capacitance of $0.01 \mu \mathrm{~F}$ and a current of 1.6 A .

High-purity aluminum electrodes ground flat with a diameter of 2 mm were used as the upper electrodes. The lower electrodes were aluminum rods with a diameter of 4 mm . The upper electrode was placed in an electrode holder, and it was blown with a stream of $\mathrm{CO}_{2}$-free air directed coaxially to the discharge downward toward the lower electrode. The airstream reliably protected the spark discharge from contact with carbon dioxide molecules from the surrounding atmosphere.

The air-cleaning system consisted of a compressor from a PFM flame photometer of type U 4.2, pumping air through an RS-3A rotameter into three Drexel bottles connected in series and filled with barium hydroxide, silica gel, and cotton.

The electrode holder was located at a distance of 90 mm from the spectrograph slit without the use of condenser lenses. As shown by the results of investigations [9] and [10], bringing the spectral radiation source closer to the slit of the spectral device made it possible to significantly increase the 1 uminosity of the spectrum and, thus, the efficiency of the spectral device. The spectrograph slit width was $20 \mu \mathrm{~m}$, and the photographic plates "for scientific purposes" were type III with a sensitivity of 6 GOST units.

To obtain quantitative data on the detection limit of organic substances, we prepared a

[^1]
[^0]:    *Here and in what follows, we will indicate by a single prime the real parts and by double primes the imaginary parts of the corresponding complex parameters.

[^1]:    Translated from Zhurnal Prikladnoi Spektroskopii, Vol. 35, No. 4, pp. 721-723, October, 1981. Original article submitted December 2, 1980.

