

8. G. P. Bepamyatov, K. K. Bogushevskaya, A. V. Bepamyatova, Yu. A. Krotov, L. A. Zelenskaya, V. F. Plekhotkin, and G. G. Smirnov, Maximum Permissible Concentrations of Harmful Substances in Air and in Water [in Russian], Khimiya, Moscow (1975).
9. A. M. Obukhov, *Izv. Akad. Nauk SSSR, Ser. Geofiz.*, No. 3, 432-439 (1960).
10. S. Butcher and R. Charlson, *Introduction Air Chemistry*, Academic Press (1972).
11. M. Mac Ewen and L. Phillips, *Atmosphere Chemistry* [Russian translation], Mir, Moscow (1978).
12. R. T. Manziev and M. S. Shumate, *Appl. Opt.*, **15**, No. 9, 2080 (1976).
13. C. R. Sreedharan, A. N. Chopra, K. K. Kutty, and S. Rangurajan, in: *Proc. Joint. Symp. Atmos. Ozone, Dresden (1976)*; Vol. 3, Berlin (1977), p. 249.
14. J. Samson Perry, *Atmos. Environ.*, **12**, No. 4, 951 (1978).
15. M. F. Frequeira, in: *Proc. Joint Symp. Atmos. Ozone, Dresden (1976)*; Vol. 3, Berlin (1977), p. 269.
16. A. Kh. Khrgian, *Physics of Atmospheric Ozone* [in Russian], Gidrometizdat, Moscow (1973).
17. G. V. Rozenberg, Yu. S. Georgievskii, V. N. Kapustin, Yu. S. Lyubovtseva, M. S. Malkevich, S. M. Pirogov, A. I. Chavro, and A. Kh. Shukurov, *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana*, **13**, No. 11, 1183 (1977).
18. L. M. Sverdlov, M. A. Kovner, and E. P. Krainov, *Vibration Spectra of Polyatomic Molecules* [in Russian], Nauka, Moscow (1970).

JONES AND MUELLER MATRICES FOR A PLANE-PARALLEL PLATE OF MAGNETICALLY ORDERED MATERIAL

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Jones and Mueller matrices are frequently used to describe the interaction of light with optical systems [1-4].

The Mueller and Jones matrices are now known for nongyrotropic crystals [1, 2] and for gyrotropic ones (crystals with natural optical activity and magnetically ordered ones) [5, 6]. Much less is known about semiinfinite media, evidently because the calculations are very complicated. A rigorous solution to the boundary-value problem of reflection and transmission of radiation by a nongyrotropic plate in normal incidence was given only in 1963 [7], where the Jones matrix was actually determined. In [8] we find Jones and Mueller matrices for a uniaxial nongyrotropic plate in normal incidence.

The Mueller matrix for an unbounded gyrotropic magnetically ordered crystal has been derived [9, 10] and the Mueller matrix for a uniaxial plate made of crystal with natural optical activity has been computed [6].

Later, Jones matrices were derived [5, 10-13] for coherent light and Mueller matrices for incoherent light for the linear and quadratic magnetooptic effects in some important cases; the parameters of the plate that describe the anisotropy and gyrotropy were introduced purely formally and were not related to the dielectric-constant tensor ϵ . Therefore, the resulting expressions did not contain explicitly the components of that tensor, which hinders the use of the formulas. In the calculations of [5] it was assumed for simplicity that the reflection and transmission coefficients of both isonormal waves in the crystal are identical, which is not true. Consequently, the expressions for the Jones matrix and the Mueller matrix for a plane-parallel magnetically ordered plate are not correct for the general case.

It is usual to employ much simpler matrices derived for infinite media instead of the rigorous Jones and Mueller matrices in experiments and in calculations on optical instruments and devices. It is assumed [6] that the maximum errors for most crystals will not be more than 5%. Here we show that this is not always so.

We derive explicit general expressions for the Jones and Mueller matrices for a plate cut in any fashion from a transparent magnetically ordered crystal of any symmetry. The following is the matrix that describes the normal transformation of polarization by a transparent crystalline plate with a hermitian tensor for the dielectric constant, which incorporates multiple reflection at the boundaries [7, 14, 15]:

$$\alpha = D_+ h_+ \cdot h_+^* + D_- h_- \cdot h_-^* \quad (1)$$

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Here D_{\pm} are known [15] amplitude transmission coefficients for the eigenwaves h_{\pm} of the crystal, while h_{\pm} are normalized magnetic-field vectors that describe the polarization of the eigenwaves excited in the plate.

The matrix of (1) is applicable also to magnetically ordered crystals, for which h_{\pm} can be put in the following form [16]:

$$h_{\pm} = (h_{\pm}^{\circ} + i\gamma h_{\mp}^{\circ})/\sqrt{1 + \gamma^2}, \quad (2)$$

where h_{\pm}° are orthonormalized vectors that define the principal semiaxes of the ellipses of polarization for the vectors h_{\pm} , while γ is the ellipticity of these.

Matrix α in fact is a Jones matrix if it is written in the more usual 2×2 form. We set the z axis normal to the plate, with the x and y axes along the principal semiaxes of the ellipses of polarization for the eigenwaves in the crystal h_{+}° and h_{-}° , which gives us a natural coordinate system in which the Jones matrix takes the form

$$\hat{D} = \frac{D_{+}e^{i\Delta/2}}{1 + \gamma^2} \begin{bmatrix} e^{-i\Delta/2} + ke^{i\Delta/2}; & i\gamma (ke^{i\Delta/2} - e^{-i\Delta/2}) \\ -i\gamma (ke^{i\Delta/2} - e^{-i\Delta/2}); & \gamma^2 e^{-i\Delta/2} + ke^{i\Delta/2} \end{bmatrix}. \quad (3)$$

We have explicitly isolated the modulus k of the ratio of the amplitude transmission coefficients and the phase shift Δ for the eigenwaves in $D_{-}/D_{+} = ke^{i\Delta}$.

On standard rules [2], the Jones matrix can be transformed to the Mueller matrix; we then get the following Mueller matrix:

$$M_{ij} = (|D_{+}|^2 + |D_{-}|^2) a_{ij}/2, \quad (4)$$

where the components a_{ij} are

$$\begin{aligned} a_{12} &= a_{21} = \cos \eta \cos \eta_k, & a_{14} &= a_{41} = \sin \eta \cos \eta_k, \\ a_{32} &= -a_{23} = \sin \eta_k \sin \eta \sin \Delta, & a_{13} &= a_{31} = 0, \\ a_{24} &= a_{42} = \sin \eta \cos \eta (1 - \sin \eta_k \cos \Delta), & a_{11} &= 1, \\ a_{43} &= -a_{34} = \sin \eta_k \cos \eta \sin \Delta, & a_{33} &= \sin \eta_k \cos \Delta, \\ a_{44} &= \sin \eta_k \cos^2 \eta \cos \Delta + \sin^2 \eta, & a_{22} &= \sin^2 \eta \sin \eta_k \cos \Delta + \cos^2 \eta. \end{aligned} \quad (4')$$

Here we have introduced the trigonometric functions $\cos \eta$ and $\sin \eta$ in the usual way, which describe the ellipticity γ of the eigenwaves excited in the crystal:

$$\sin \eta = 2\gamma/(1 + \gamma^2), \quad \cos \eta = (1 - \gamma^2)/(1 + \gamma^2). \quad (5)$$

Expressions for $\sin \eta_k$ and $\cos \eta_k$ are derived from (5) by the substitution $\gamma \rightarrow k$.

We thus have general expressions (3)-(5) for the Mueller matrix that describe the transmission in normal incidence for polarized light in a plane-parallel plate cut from a transparent magnetically ordered crystal of any symmetry.

The same expressions can be used to describe the transformation of polarized light reflected at right angles; it is sufficient to replace D_{\pm} by R_{\pm} in (3)-(5), which are the amplitude coefficients for normal reflection of the eigenwaves from the plate.

Here we have $R_{-}/R_{+} = k'e^{i\Delta}$, $k' = |R_{-}/R_{+}|$.

The following conclusions are drawn.

If we put $k = 1$ in (3)-(4'), these expressions become the corresponding Mueller and Jones matrices for unbounded gyrotropic and nongyrotropic media [1-3, 5, 6, 8, 11].

The phase shift Δ and the modulus of the ratio of the amplitude transmission coefficients $k = |D_{-}/D_{+}|$ appearing in the elements of the Mueller and Jones matrices oscillate as the thickness of the crystal varies, and the oscillations are dependent in the main on the ratio of the refractive indices n_{\pm} of the crystal to the refractive index n of the isotropic medium, and they may be considerable. For example, k varies over the range from 1.08 to 0.92 for $n_{\pm}/n = 1.5$, and the maximum deviation of k from 1 is as much as 20% for $n_{\pm}/n = 2$.

If the plates are not very thin ($l \gg \lambda$) one can neglect such oscillations, and in that case the mean values \bar{k} and $\bar{\Delta}$ are dependent in the main on the anisotropy. If the anisotropy is small, or if the cut of the crystal has low birefringence, one can take $\bar{k} \approx 1$ as an approximation and use the simpler Jones and Mueller matrices for unbounded media.

The phase shift is $\Delta' = \Delta$ for the light reflected from a plate, whereas even the average value $k' = |R_-/R_+|$ may differ considerably from 1 when $n_+ \approx n$, $n_- \approx n$ in contrast to k , and then it is obligatory to consider the effects of the plate boundaries.

LITERATURE CITED

1. W. A. Shurcliff, Polarized Light, Production and Use [Russian translation], Mir, Moscow (1965).
2. A. Gerrard and J. M. Burch, Introduction to Matrix Methods in Optics, Wiley-Interscience (1975).
3. G. V. Rozenberg, "The Stokes vector parameter," Usp. Fiz. Nauk, 16, No. 1, 77-110 (1955).
4. M. M. Gorshkov, Ellipsometry [in Russian], Sov. Radio, Moscow (1974).
5. V. D. Tron'ko and G. E. Dovgalenko, "The passage of coherent radiation through an optically active birefringent medium in an arbitrary direction: Jones matrices," Opt. Spektrosk., 34, No. 6, 1157-1164 (1973).
6. B. N. Grechushnikov and A. F. Konstantinova, "Mueller matrices for optically active crystals," Kristallografiya, 16, No. 2, 448-449 (1971).
7. A. M. Goncharenko and F. I. Fedorov, "Optical parameters of crystal plates," Opt. Spektrosk., 14, No. 1, 94-99 (1963).
8. V. A. Zamkov, A. S. Kondrat'ev, and A. E. Kuchina, "Jones and Mueller matrices for a uniaxial plane-parallel plate," Opt. Spektrosk., 38, No. 5, 1027-1029 (1975).
9. W. J. Tabor and F. S. Chen, "Electromagnetic propagation through materials possessing both Faraday rotation and birefringence experiments with ytterbium orthoferrite," J. Appl. Phys., 40, No. 7, 2760-2765 (1969).
10. V. D. Tron'ko, "Transmission of a light beam by a medium with linear and quadratic magneto-optic effects," Opt. Spektrosk., 29, 354-359 (1970).
11. V. D. Tron'ko, "The Faraday effect and the influence of multiple reflection within a plane-parallel magnetically ordered plate," Opt. Spektrosk., 26, No. 3, 484-487 (1969).
12. V. D. Tron'ko, "Magneto-optic effects in birefringent media," Opt. Spektrosk., 30, No. 4, 739-744 (1971).
13. V. V. Sporik, V. D. Tron'ko, and V. I. Tsimbarevich, "Transmission of a light beam by an optically active medium with birefringence," Fiz. Tverd. Tela, 16, No. 5, 1517-1519 (1974).
14. F. I. Fedorov, Theory of Gyrotropy [in Russian], Nauka i Tekhnika, Minsk (1977).
15. I. M. Barkovskii, "Optics of a crystal with a hermitian dielectric-constant tensor," Opt. Spektrosk., 34, No. 6, 1193-1197 (1973).
16. V. B. Bokut' and S. S. Girgel', "Polarization of electromagnetic waves in transparent magnetically ordered crystals," Kristallografiya, 21, No. 2, 264-268 (1976); "Polarization of electromagnetic waves in transparent magnetically ordered crystals," Ibid., 269-274.