The Relativistic-Invariant Representation of Compton Scattering Amplitude on a Nucleon Taking into Account Polarizabilities

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Abstract

In the present paper the relativistic Compton scattering amplitude with the contribution of polarizabilities using the Lagrangian has been obtained. The physical interpretation of invariant functions of this amplitude has been presented as well.

For the purpose of more precise measurement nucleon's electrical and magnetic polarizabilities in Compton scattering on a nucleon with the energy of photon close to the threshold photoproduction of pions the important role play recoil effects of a nucleon[1]. In order consecutively to consider recoil effects relating to the induced dipole moments of a nucleon in a static limit, it is necessary to use the Lagrangian of electromagnetic field interaction with a nucleon in relativistic field-theory approach taking into account polarizabilities [2, 3].

The effective field-theory Lagrangian with contribution of induced dipole moments in a static limit has the following form [2]

$$\mathcal{L} = -\frac{2\pi}{m}\left[\alpha F_{\mu\sigma}F_{\nu}{}^{\mu} + \beta\widetilde{F}_{\mu\sigma}\widetilde{F}_{\nu}{}^{\mu}\right]\Theta^{\sigma\nu}. \qquad (1)$$

In this expression α - and β - electric and magnetic polarizabilities of a nucleon, $F_{\mu\nu}$ - and $\widetilde{F}_{\mu\nu}$ - are usual and dual tensors of electromagnetic field, $\Theta^{\sigma\nu}$ - is the energy-momentum tensor of spinor field of a nucleon:

$$\Theta^{\sigma\nu} = \frac{i}{2}\overline{\psi}\gamma^{\sigma}\overleftrightarrow{\partial^{\nu}}\psi.$$

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Using the expression

$$\tilde{F}_{\mu\sigma}\tilde{F}^{\mu\nu} = F_{\mu\sigma}F^{\mu\sigma} - \frac{1}{2}\delta^{\nu}_{\sigma}F_{\mu\rho}F^{\mu\rho}$$

the Lagrangian (1) it's possible to present by the way:

$$\mathcal{L} = -\frac{2\pi}{m}\left[(\alpha+\beta)F_{\mu\sigma}F^{\mu\sigma}\Theta^{\sigma}_{\nu} - \frac{\beta}{2}\Theta^{\sigma}_{\sigma}F_{\mu\nu}F^{\mu\nu}\right]. \quad (2)$$

The field-theory representation of Lagrangian (2) is different from Lagrangian of the paper [1]. The Lagrangian of paper [1] usually is used for definition of the Compton scattering amplitude on scalar particles [4].

In the expression (2) spin properties of the nucleon as particle of spin 1/2 are taken into account. In the rest frame of a nucleon where are used approximations $\partial_i\psi = 0$, $i\gamma^0\partial_0\psi = m\psi$ and $i(\partial_0\bar{\psi})\gamma^0 = -m\bar{\psi}$ the Lagrangian (2) and Lagrangian of the paper [1] are result one and the same form:

$$\mathcal{L} = -H = 2\pi(\alpha\vec{E}^2 + \beta\vec{H}^2).$$

For the determination of electric and magnetic polarizabilities contribution to relativistic Compton scattering amplitude on a nucleon let's use by the Green's function method [5, 6].

Using the Lagrangian (2) the equation of motion of a nucleon in the electromagnetic field it's possible to present by the way:

$$(i\hat{\partial} - m)\psi = e\hat{A}\psi - \frac{i}{2}[\partial^{\nu}(K_{\sigma\nu}\gamma^{\sigma}\psi) + K_{\sigma\nu}\gamma^{\sigma}\partial^{\nu}\psi], \quad (3)$$

where

$$K_{\sigma\nu} = \frac{2\pi}{m}[\alpha F_{\sigma\mu}F^{\mu}_{\nu} + \beta\tilde{F}_{\sigma\mu}\tilde{F}^{\mu}_{\nu}].$$

Receiving the equation (3) the charge of a nucleon and polarizabilities were taken into account only.

Let's present the equation (3) in which will take into account only the contribution of polarizabilities in integral form:

$$\psi(x) = \psi^{(0)}(x) + \int S_F(x - x')V^{(\alpha,\beta)}(x')dx', \quad (4)$$

where $V^{(\alpha,\beta)}(x') = -\frac{i}{2}[\partial^{\nu}(K_{\sigma\nu}(x')\gamma^{\sigma}\psi(x')) + K_{\sigma\nu}(x')\gamma^{\sigma}\partial^{\nu}\psi(x')]$.

The Green's function $S_F(x-x^{'})$ is satisfies to the equation:

$$(i\hat{\partial}-m)S_F(x-x^{'})=\delta(x-x^{'}).$$

Now defining the S-matrix element of the photon's scattering on a nucleon following to articles [5, 6]. For this purpose we will turn (4) with $\bar{\psi}_{p_2}^{(r_2)}(x)$ at $t\to+\infty$ and use by the relation

$$\int\bar{\psi}_{p_2}^{(r_2)}(x)S_F(x-x^{'})d^3(x)|_{t\to+\infty}=(-i)\bar{\psi}_{p_2}^{(r_2)}(x^{'}),$$

where

$$\bar{\psi}_{p_2}^{(r_2)}=\frac{1}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_2}}\bar{U}^{(r^2)}(p_2)e^{ip_2x^{'}}.$$

As a result we obtain

$$S_{fi}=(-i)\int\bar{\psi}_{p_2}^{(r_2)}(x^{'})V^{(\alpha,\beta)}(x^{'})d^4x^{'}. \qquad (5)$$

Integrating by parts and using the condition of the crossing symmetry we obtain:

$$S_{fi}=\frac{im\delta(k_1+p_1-k_2-p_2)}{(2\pi)^2\sqrt{4\omega_1\omega_2E_1E_2}}M. \qquad (6)$$

In the (6) the amplitude M is represented by the following way:

$$M=\frac{2\pi}{m}\bar{U}^{(r_2)}(\vec{p}_2)\left\{[\hat{k}_2e_\mu^{(\lambda_2)}-k_{2\mu}\hat{e}^{(\lambda_2)}]\,[k_1^\mu(e^{(\lambda_1)}P)-(k_1P)e^{(\lambda_1)\mu}]+\right. \qquad (7)$$

$$\left.+\,[k_2^\mu(e^{(\lambda_2)}P)-(k_2P)e^{(\lambda_2)\mu}][\hat{k}_1e_\mu^{(\lambda_1)}-k_{1\mu}\hat{e}^{(\lambda_1)}]\right\}(\alpha+\beta)U^{(r_1)}(\vec{p}_1)+$$

$$+\frac{2\pi}{m}\beta\bar{U}^{(r_2)}(\vec{p}_2)\left\{[k_{2\mu}e_\nu^{(\lambda_2)})-k_{2\nu}e_\mu^{(\lambda_2)}][k_1^\mu e^{(\lambda_1)\nu}-k_1^\nu e^{(\lambda_1)\mu}]\right\}U^{(r_1)}(\vec{p}_1).$$

In this expression $e_\mu^{(\lambda_1)}$ and $e_\mu^{(\lambda_2)}$ –are the polarization vectors of the initial and final photons, $P=\frac{1}{2}(p_1+p_2)$, $\hat{k_2}=k_{2\mu}\gamma^\mu$, k_1, p_1 and k_2, p_2 – are momentums of the initial and final photons and nucleons, $U^{(r_1)}(\vec{p}_1)$ and $\bar{U}^{(r_2)}(\vec{p}_2)$ – are bispinors initial and final nucleons.

As seen from (7) relativistically covariant definition of the amplitude of Compton scattering taking into account the spin of a nucleon is different from that of the amplitude in the paper [1]. However, representing the amplitude in the rest frame of the target, considering the contribution of scattering on the nucleon is electrically charged and limited by the decomposition of the amplitude by the members are not higher than second order, then we obtain the amplitude in the same form that follows from the work [1] as well:

$$M = \chi^{(r_2)+} \left[\left(-\frac{e^2}{m} + 4\pi\omega^2\alpha \right) (\vec{e}^{(\lambda_2)}\vec{e}^{(\lambda_1)}) + \right. \tag{8}$$

$$\left. + 4\pi\omega^2\beta([\vec{n}_2\vec{e}^{(\lambda_2)}][\vec{n}_1\vec{e}^{(\lambda_1)}]) \right] \chi^{(r_1)},$$

where $\vec{n}_1$ and $\vec{n}_2$ – are unit vectors for $\vec{k_1}$ and $\vec{k_2}$, $\chi^{(r_1)}$ and $\chi^{(r_2)}$ – are spinors of the initial and final nucleons, ω – frequency radiation.

Conclusion

On the basis of the relativistic field-theory Lagrangian, the spin properties and polarizabilities of the nucleon taking into account, the relativistic Compton scattering amplitude which is agreed with the low-energy theorem of this process has been obtained.

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