

ON CONNECTIONS BETWEEN THE BOCHNER–PHILLIPS AND  
HILLE–PHILLIPS FUNCTIONAL CALCULI

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The classical Hille–Phillips functional calculus of generators of  $C_0$ -semigroups, presented in the well-known monograph of these authors, uses Laplace transforms of measures as symbols and leads to bounded operators. Another important functional calculus of semigroup generators — the Bochner–Phillips calculus — uses Bernstein functions as symbols. In this work, an extension of the Hille–Phillips functional calculus is considered that leads to unbounded operators. Connections of this calculus with the Bochner–Phillips functional calculus are indicated. In particular, for generators of uniformly stable semigroups, the multiplication rule and the composition theorem are proved for functions understood in different senses. Conditions for the invertibility of operators that arise in the Bochner–Phillips calculus are obtained. Several examples are given and unsolved problems are formulated.

Keywords: Hille–Phillips functional calculus, Bochner–Phillips functional calculus, fractional powers of operators, subordinate semigroup.

MSC: 47A60, 47D03

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