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К ОПРЕДЕЛЕНИЮ ЕДИНИЦЫ ЭЛЕКТРИЧЕСКОГО ТОКА В СИСТЕМЕ СИ

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TO THE DETERMINATION OF THE UNIT OF ELECTRIC CURRENT IN SI SYSTEM

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Аннотация. В международной системе единиц «СИ» одной из основных является ампер. За время существования единица силы электрического тока – ампер – несколько раз переопределялась. Согласно определению 1948 года ампер определялся через силу взаимодействия двух параллельных прямолинейных проводников бесконечной длины и ничтожно малой площади кругового поперечного сечения. С 2019 года вступили в силу изменения в СИ, которые переопределили ампер на основе фиксации численного значения элементарного электрического заряда, принятые в Резолюции 26-ой ГКМВ в 2018 году. Следствием этого стало, в частности, то, что коэффициенты для перевода между единицами СИ и единицами системы СГС перестали быть точными. В данной работе рассмотрена возможность модификации определения 1948 года путем изменения конфигурации взаимодействующих токов. Бесконечные проводники при этом заменяются круговыми токами конечной длины.

Ключевые слова: *система единиц «СИ», ампер, электрический ток, взаимодействие токов.*

Abstract. In the international system of units "SI", one of the main ones is the ampere. During its existence, the unit of electric current strength, the ampere, was redefined several times. According to the definition of 1948, the ampere was determined through the interaction force of two parallel straight conductors of infinite length and a negligibly small circular cross-sectional area. Since 2019, changes in the SI have come into force, which redefined the ampere based on fixing the numerical value of the elementary electric charge, adopted in Resolution 26 of the CGPM in 2018. The consequence of this was, in particular, that the coefficients for converting between SI units and units of the CGS system were no longer accurate. In this paper, we consider the possibility of modifying the 1948 definition by changing the configuration of interacting currents. In this case, infinite conductors are replaced by circular currents of finite length.

Keywords: *system of units "SI", ampere, electric current, interaction of currents.*

Introduction

In 1960, the XI General Conference on Weights and Measures (CGPM) adopted the standard, which for the first time was called the "International System of Units", and established the international abbreviation for this system "SI". The main units in it were the meter, kilogram, second, ampere, degree Kelvin and candela.

Within the SI, these units are considered to have an independent dimension, that is, none of the basic units can be obtained from others. On January 1, 1963, GOST 9867-61 "International System of Units" SI was introduced in the USSR as the preferred one in all areas of science, technology and the national economy, as well as in teaching.

At present, the SI is adopted as the main system of units by most countries of the world and is almost always used in the field of technology, even in those countries in which traditional units are used in everyday life.

Since 2019, changes to the SI have come into force. As a result of these changes, there are no specific material standards of units in the new version of the SI system. The basic SI units began to be defined through fixed values of fundamental physical constants [1].

1 Ampere is a unit of electric current

One of the basic units of the SI system, the unit of electric current is the ampere. It was adopted at the 1st International Congress of Electricians in Paris in 1881 and named after the French physicist André-Marie Ampère. It was originally defined as one tenth of the current of the CGSM system and defined a current that produces a force of 2 dynes per centimeter of length between two thin conductors 1 cm apart.

In 1893, the unit of current was defined as the current required to electrochemically deposit 1,118 milligrams of silver per second from a solution of silver nitrate. It was assumed that the unit value would not change, but it turned out that it changed by 0,015%. This unit became known as the international ampere.

According to the definition adopted by the IX CGPM in 1948, "The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length" Figure 1.1 [2]. Thus, in fact, the original definition of ampere was returned.
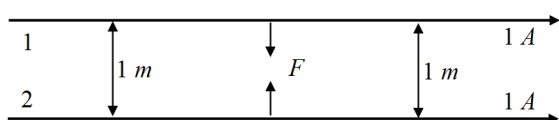

Figure 1.1

Since 2019, changes to the SI have come into force, including the redefinition of ampere based on fixing the numerical value of the elementary charge, adopted in the Resolution of the 26th CGPM in 2018 [1].

The ampere remains the unit of electric current strength, but its value will be set by fixing the numerical value of the elementary electric charge in the SI unit A s, which is equivalent to *Cl*. The wording, effective May 20, 2019, reads: "The ampere, symbol *A*, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge, *e*, to be $1,602176634 \cdot 10^{-19}$ when expressed in the unit *C*, which is equal to *A s*, where the second is defined in terms of $\Delta\nu_{Cs}$." $\Delta\nu_{Cs}$ is the frequency of radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium atom 133.

The value of $\Delta\nu_{Cs}$ at 0 *K* is numerically equal to 9 192 631 770 when it is expressed in the SI unit s^{-1}, which is equivalent to *Hz*. It can be noted that since the 1980s, quantum devices began to be used as a practical implementation of the ampere standard, which tied the ampere to the volt and ohm (1 *A* = 1 *V* / 1 *Ω*) using Ohm's law.

However, the change in the definition of 2019 led to the fact that the expressions for the electrical permeability of vacuum or the electrical constant ε_0 and the magnetic permeability of vacuum or the magnetic constant μ_0 ceased to be exact (in particular, the exact equality $\mu_0 = 4\pi \ 10^{-7} \ H / m$). They began to be performed only numerically (but with great accuracy) and are subject to experimental measurement.

The relative standard uncertainty μ_0 and ε_0 is equal to the relative standard uncertainty of the fine structure constant α. From this, in particular, it follows that the coefficients for converting between SI units and units of the CGS system have ceased to be exact, fixed values, since they are expressed in terms of a magnetic constant.

One of the reasons that made it difficult to implement the 1948 definition of the ampere was the presence of the concept of "infinity" in the definition. In this work, the possibility of eliminating the concept of "infinity" by changing the configuration of interacting currents is considered.

2 Interaction of two circular currents

Consider the interaction of two coaxial turns of the same radius (Figure 2.1).

At currents of the same magnitude and one direction, the forces of mutual attraction arise in the turns. Let the left turn create a magnetic field at the localization points of the right turn, then the Ampère force will act on the right turn. This force will determine the magnitude of the interaction of currents.
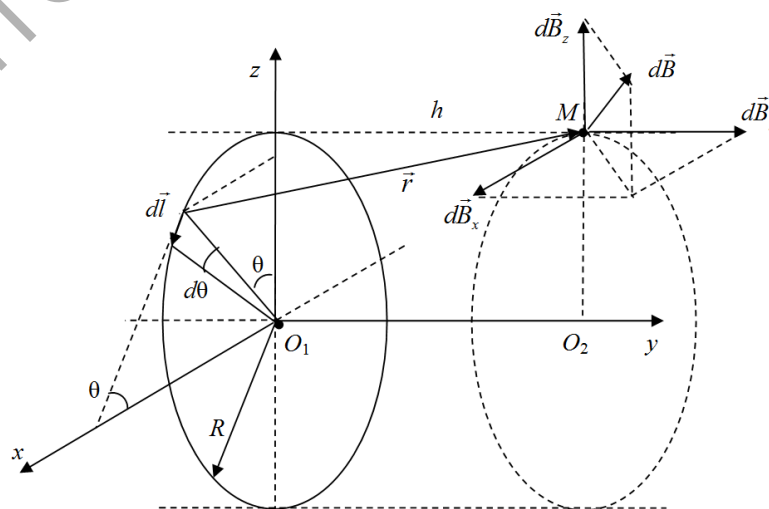

Figure 2.1

Consider a circular current of radius R and a current element $Id\vec{l}$ (left coil, Figure 2.1). The current element $Id\vec{l}$ creates a magnetic field induction at point M. The vector $d\vec{B}$ can be represented in terms of components along the coordinate axes OX, OY, OZ – $\vec{i}dB_x$ $\left(d\vec{B}_x\right)$, $\vec{j}dB_y$ $\left(d\vec{B}_y\right)$, $\vec{k}dB_z$ $\left(d\vec{B}_z\right)$ respectively, where $\vec{i}$, $\vec{j}$, $\vec{k}$ are unit vectors along the corresponding coordinate axes.

The force that attracts the second coil to the first will be affected only by the vector component $\vec{k}dB_z$.

To determine the component $\vec{k}dB_z$ of the magnetic field induction vector $d\vec{B}$ of the circular current at point M (Figure 2.1), we use the Biot-Savart law and the principle of superposition of magnetic fields [3]–[7]. Magnetic induction created by a current element

$$d\vec{B} = \frac{\mu_0 I\left[d\vec{l}\times\vec{r}\right]}{4\pi r^3}, \qquad (2.1)$$

where μ_0 is the magnetic constant, I is the magnitude of the current, the vector $\vec{r}$ is directed from the current element to the point M.

In order to write down a vector $\vec{k}dB_z$, it is necessary to select a component along the OZ axis in the vector product $\left[d\vec{l}\times\vec{r}\right]$ in (2.1). This component, taking into account the properties of the cross product, is equal to $\left(dl_x r_y - dl_y r_x\right)\vec{k}$, where dl_x, r_x, dl_y and r_y are the projections of the vectors $d\vec{l}$ and $\vec{r}$ on the OX, OY axes, $\vec{k}$ is the unit vector along the OZ axis.

Then the vector component $d\vec{B}$ along the OZ axis

$$\vec{k}dB_z = \frac{\mu_0 I}{4\pi r^3}\left(dl_x r_y - dl_y r_x\right)\vec{k}. \qquad (2.2)$$

Vector projections $d\vec{l}$ on the coordinate axes OX, OY are written as: $dl_x = dl\cos\theta$ and $dl_y = 0$. Vector projections $\vec{r}$ on the coordinate axes OX, OY are represented as: $r_x = -R\sin\theta$ and $r_y = y$.

Then for dB_z from expression (2.2) we obtain

$$dB_z = \frac{\mu_0 I}{4\pi r^3} y\,dl\cos\theta. \qquad (2.3)$$

Taking into account that the coil length element $dl = Rd\theta$, from (2.3) we obtain

$$dB_z = \frac{\mu_0 I}{4\pi r^3} yR\cos\theta\,d\theta. \qquad (2.4)$$

Distance from coil length element dl to point M

$$r = \sqrt{r_x^2 + r_y^2 + r_z^2} =$$
$$= \sqrt{\left(R\sin\theta\right)^2 + y^2 + \left(R - R\cos\theta\right)^2} = \qquad (2.5)$$
$$= \sqrt{2R^2 + y^2 - 2R^2\cos\theta}.$$

Then, substituting expression (2.5) into (2.4), we obtain

$$dB_z = \frac{\mu_0 I}{4\pi\left(2R^2 + y^2 - 2R^2\cos\theta\right)^{3/2}} yR\cos\theta\,d\theta. \qquad (2.6)$$

Having integrated over the entire length of the turn, from expression (2.6) we obtain

$$B_z = \int_0^{2\pi} \frac{\mu_0 I}{4\pi\left(2R^2 + y^2 - 2R^2\cos\theta\right)^{3/2}} yR\cos\theta\,d\theta. \qquad (2.7)$$

Knowing the induction of the magnetic field of the left turn at point M, it is possible to determine the force acting on the right turn. Taking into account the Ampère's law for the same currents in the turns (the length of the right turn is $2\pi R$), we obtain

$$F = B_z I 2\pi R. \qquad (2.8)$$

Then, substituting expression (2.7) into (2.8), we obtain

$$F = \int_0^{2\pi} \frac{\mu_0 I^2 R^2}{2\left(2R^2 + y^2 - 2R^2\cos\theta\right)^{3/2}} y\cos\theta\,d\theta. \qquad (2.9)$$

Let us introduce the notation $y/R = k$, then (2.9) will be rewritten in the form

$$F = \frac{\mu_0 k I^2}{2}\int_0^{2\pi} \frac{\cos\theta\,d\theta}{\left(2 + k^2 - 2\cos\theta\right)^{3/2}} = \frac{\mu_0 k I^2}{2} J(k), \quad (2.10)$$

where the integral $J(k)$

$$J(k) = \int_0^{2\pi} \frac{\cos\theta\,d\theta}{\left(2 + k^2 - 2\cos\theta\right)^{3/2}}. \qquad (2.11)$$

The values of the integral $J(k)$ in (2.11) depending on the parameter k are presented in Table 2.1.

Table 2.1

k	$J(k)$	$F(k)$, 10^{-7} N
0,2	47,8695	60,1546
0,4	10,9093	27,4181
0,6	4,28916	16,1697
0,8	2,09071	10,5091
1	1,14331	8,88009
1,2	0,67233	5,06925
1,4	0,416173	3,66085
1,6	0,267981	2,69344
1,8	0,178053	2,01373
2	0,121545	1,52738

Taking into account the data in Table 2.1 ($J(k)$), the dependence of the force F on the parameter k at a current of 1 A and a magnetic constant $\mu_0 = 4\pi\,10^{-7}$ H/m is presented in Table 2.1 ($F(k)$).

3 Possible definition of the unit of electric current

From Table 2.1 it follows that at $k = 1,8$ the interaction force $F = 2,01373\cdot 10^{-7}$ N, and at $k = 2$ the force $F = 1,52738\cdot 10^{-7}$ N.

That is, one can choose such a value of k (slightly more than $k = 1,8$) at which the force F is exactly equal to $2 \cdot 10^{-7}$ N.

At current $I = 1$ A, interaction force $F = 2 \cdot 10^{-7}$ N and magnetic constant $\mu_0 = 4\pi \, 10^{-7} \, H / m$, we obtain an equation for k

$$\int_0^{2\pi} \frac{k \cos\theta \, d\theta}{\left(2 + k^2 - 2\cos\theta\right)^{3/2}} = \frac{1}{\pi}. \qquad (3.1)$$

The value $k = k_0$, which is a solution to Equation (3.1), can be written as $y / R = k_0$, whence $R = y / k_0$. Since the diameter of the coil is equal to twice the radius, then $D = 2y / k_0$.

Equation (3.1) can be solved with a given accuracy.

With $k_0 = 1,8048\ldots$, $y = 1$ m, we get that the diameter of the turns should be equal to $D = 1,1081\ldots$ m.

In view of the foregoing, the definition of the unit of electric current can be written as: "ampere is the strength of an unchanging current, which, when passing through two coaxial circular conductors of the same diameter $D = 1,1081 \ldots m$ one from the other, would cause an interaction force equal to $2 \cdot 10^{-7}$ Newtons" Figure 3.1. At currents in one direction, the coils attract; at currents in opposite directions, the coils repel.
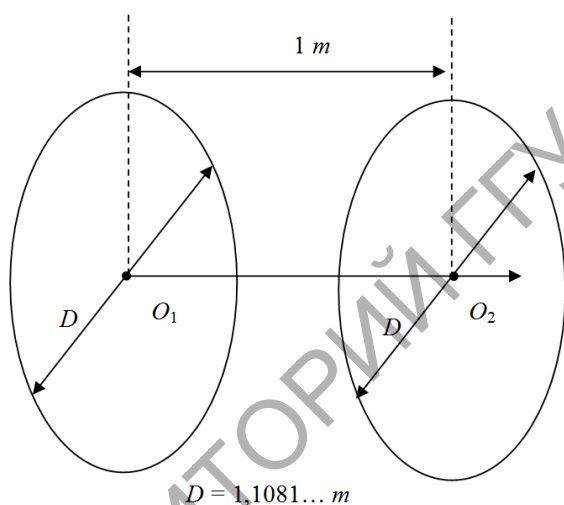

$$D = 1,1081\ldots m$$

Figure 3.1

Conclusion
Thus, the definition of 1948 for the ampere (through the force of interaction of two parallel rectilinear conductors of infinite length and a negligible small circular cross-sectional area) can be modified. For this, the configuration of the interacting currents is changed. Infinite conductors are replaced by circular currents, i.e. conductors of finite length.

The possibility of implementing the definition increases, since there is no concept of "infinity" in the definition. The expressions for the electrical permeability of the vacuum or the electrical constant ε_0 and the magnetic permeability of the vacuum or the magnetic constant μ_0 do not change. The coefficients for conversion between SI units and units of the CGS system remain accurate and are expressed in terms of the magnetic constant.

REFERENCES
1. *Le Système international d'unités* (SI) / The International System of Units (SI), edition 9, – Bureau International des Poids et Mesures (BIPM), 2019.
2. *Le Système international d'unités* (SI) / The International System of Units (SI), edition 7, – Bureau International des Poids et Mesures (BIPM), 1998.
3. *Raymond*, *A*. Serway. Physics for Scientists and Engineers (with PhysicsNOW and info Trac) / A. Serway Raymond, W. Jewett John: 6 th ed., Thomson Brooks / Cole, 2004. – 1296 p.
4. *Detlaf*, *A.A*. Physics course: textbook. allowance for higher educational institutions / A.A. Detlaf, B.M. Yavorsky – 4th ed., corrected. – Moscow: Higher School, 2002. – 718 p.
5. *Matveev*, *A.N*. Electricity and magnetism / A.N. Matveev. – Moscow: Higher School, 1983. – 463 p.
6. *Savelyev*, *I.V*. Course of general physics: in 3 volumes – Vol. 2: Electricity and magnetism. Waves. Optics / I.V. Saveliev. – Moscow: Nauka, 1988. – 480 p.
7. *Shilyaeva*, *K.P*. Physics. Brief theory and tasks: manual / K.P. Shilyaeva, I.O. Delikatnaya, N.A. Akhramenko. – Gomel: BelSUT, 2021. – 211 p.

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