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RESONANCE STATES SPECTRUM AND QUANTUM FIELD THEORY EQUATIONS FOR THE TWO-PARTICLE SYSTEMS

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A method for resonance finding of relativistic two-particle systems, described by covariant three dimensional integral equations [1,2] in the relativistic configurational representation (RCR) [2] is proposed. The method is applied for a model potential and analysis of the relativistic cross sections behaviour is carried out. Partial equations for the scattering states in the RCR for the s-waves have the form [3]

$$\psi^{(j)}(\chi_q, r) = \sin \chi_q mr + \int_0^\infty dr' g^{(j)}(\chi_q, r, r') V(r') \psi^{(j)}(\chi_q, r'), \tag{1}$$

where $j = 1,2,3,4$ corresponds to the kind of the quasipotential type equation. The value χ_q is the rapidity, connected with the energy of the two-particle system by $2E_q = 2m\cosh \chi_q$. The RCR partial Green functions are $g^{(j)}(\chi_q, r, r') = g^{(j)}(\chi_q, r - r') - g^{(j)}(\chi_q, r + r')$, where for example at $j = 2$ [3]

$$g^{(2)}(\chi_q, r) = \frac{(4m\cosh \chi_q)^{-1}}{\cosh[\pi mr/2]} - \frac{i}{m\sinh 2\chi_q} \frac{\sinh\left[\left(\pi + i\chi_q\right) mr\right]}{\sinh[\pi mr]}. \tag{2}$$

Relativistic integral equations for the resonance states have to be homogeneous. In this equation Green functions (GF) for the states with complex energy ($\chi_q = \xi_q + iw_q$) have to be used. It is expected that solutions of such equations will exist only for the discreet complex energy values. One is able to solve such integral equations numerically only for the sufficiently fast decreasing analytical potentials. Numerical solution of these homogeneous equations is possible only in the band $w_{\min} \le w_q \le w_{\max}$, which is dependent on the properties of the potential. However, resonant rapidities may be found beyond this band. In order to solve equation in other domain of complex χ_q, we will make use of the well-known in the non-relativistic theory complex scaling method [4]. After transformation of the real variables r, r' to the complex variables $z = r\exp(i\theta)$, $z' = r'\exp(i\theta)$, $0 \le \theta \le \theta_{\max}$ equations (1) are modified to the resonance states equations:

$$\psi_\theta^{(j)}(\xi_q + iw_q, r) = \int_0^\infty dr' g_\theta^{(j)}(\xi_q + iw_q, r, r') V_\theta(r') \psi_\theta^{(j)}(\xi_q + iw_q, r'). \tag{3}$$

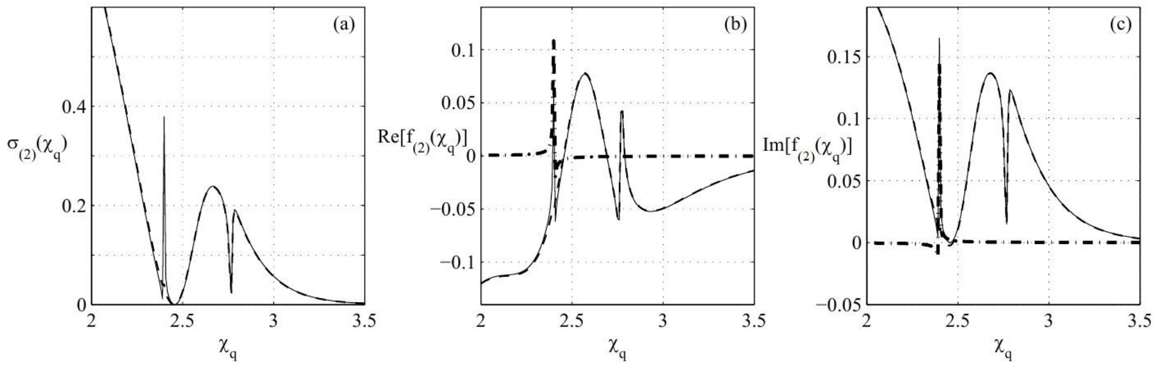
Partial s-wave cross section is defined as $\sigma_{(j)}(\chi_q) = 4\pi |f_{(j)}(\chi_q)|^2$, where scattering amplitude $f(\chi_q)$ is the coefficient in front of the scattered wave, when asymptotic form of the wave function at $r \to \infty$ is considered. In order to study the influence of the resonances on the scattering amplitude (or cross section) let us define the contribution of the R-th resonance to the scattering amplitude by analogy with the non-relativistic case [5] through the residue:

$$\mathrm{Res}[f_{(j)}(\chi_q^R)] / (\chi_q - \chi_q^R).$$

The residue of the scattering amplitude can be found using the Cauchy's theorem. Defining the reduced scattering amplitude and reduced cross section as follows

$$\tilde{f}_{(j)}(\chi_q,\chi_q^R) = f_{(j)}(\chi_q) - \mathrm{Res}[f_{(j)}(\chi_q^R)]\big/(\chi_q - \chi_q^R), \quad \tilde{\sigma}_{(j)}(\chi_q,\chi_q^R) = 4\pi\left|\tilde{f}_{(j)}(\chi_q,\chi_q^R)\right|^2,$$

and then computing and comparing $\sigma_{(j)}$ and $\tilde{\sigma}_{(j)}$, it is possible to identify a feature in the cross section and investigate the influence of the desired resonance.



To illustrate the method presented above we consider equation (3) for the model potential $V(r) = 30r^2\cosh(\pi-\beta)mr\big/\cosh\pi mr,\ \ \beta = \pi/4$. Figure (a) displays cross section (full line) and reduced cross section (dashed line), (b,c) - scattering amplitude (full line), contributions of the first resonance into the scattering amplitude (dash-dotted line) and reduced amplitudes, when the contribution of the first resonance is omitted (dashed line) for this potential. For $j = 2$ first and second resonances lie very close to the real axis. In figure for $\sigma_{(2)}$ and for $f_{(2)}$ one can see narrow peak and narrow trough at the corresponding rapidity values. For the reduced cross sections and amplitudes these structures completely disappear. Third resonance has lager imaginary part and has influence on the wider area of the cross section. For the all four GF we can clearly see that the closer is the resonance to the real axis the narrower is peak in the corresponding area of the cross section. For wider resonances contribution to the cross section is more delocalized but can still be assigned to this resonance.

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