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COVARIANT THREE-DIMENSIONAL EQUATIONS FOR BOUND STATES OF QUARKS AND THE STRUCTURE FUNCTIONS OF HADRONS

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ABSTRACT

The structure functions of hadrons in deep inelastic scattering are calculated in the framework of the single time formulation of QFT. The initial hadron is considered as a bound state of the quark-antiquark pair and the structure functions are expressed through the relativistic wave function of the bound state which obeys the covariant quasipotential equation. A new scaling variable is introduced and explicit expressions for the scaling and prescaling parts of the structure functions are derived. It is shown that the prescaling and exclusive threshold asymptotics of the structure functions in the leading approximation contain only logarithmic terms. The exact solution of the quasipotential equation for the relativistic potential with QCD large Q^2 behaviour is used in order to calculate the structure functions.

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1. INTRODUCTION

Recent theoretical investigations of deep inelastic lepton-hadron scattering (see, for example, the reviews[1)-4)]) show that the wave function of the hadron turns out to be one of the main objects which determine scattering cross-sections. In order to find the behaviour of the hadron wave function in QCD calculations, different authors apply the light-cone operator product expansion which, however, does not allow the construction of the exact expression for the wave function.

In this paper we continue the series of works[5)-12)] devoted to the description of elastic scattering, inclusive reactions, form factors, decays and other processes with the participation of composite particles in the framework of the single time formulation of QFT[13)-17)] where the explicit form of the wave function can be found as a solution of the dynamical equation for bound states. Here we shall consider the structure functions of hadrons in deep inelastic scattering.

For simplicity we assume that the initial hadron (meson) consists of the bound scalar quark and antiquark. In the covariant formulation of the two-body relativistic problem, the single time wave function of the bound state is defined as follows[17)]:

$$\frac{\psi_{BM}(\vec{\Delta}_k)}{2\Delta_k^0} = \int dx\, e^{-ik_1x}\,\delta(\lambda_p x)\langle 0|T(\varphi_q(x)\varphi_{\bar{q}}(0))|P\rangle, \tag{1}$$

where k_1 is the quark momentum, P and M are the momentum and the mass of the meson, $\lambda_P = P/M$ and $\Delta_k = L_P^{-1}k_1$ is the covariantly defined momentum of the quark in the c.m.s. of the quark-antiquark pair[18)]. Figure 1 represents this wave function graphically. Here both the quark and the antiquark are on the mass shell $(k_1^2 = k_2^2 = m^2)$ and since in the meson rest system we have to suppose that the antiquark momentum $\vec{\tilde{\Delta}}_k \equiv \overrightarrow{L_P^{-1}k_2} = -\vec{\Delta}_k$ $(\tilde{\Delta}_k^0 = \Delta_k^0)$ we come to the relationships:

$$\begin{aligned} Pk_1 &= Pk_2\,; \\ k_1+k_2 &= P\cdot 2(Pk_1)/M^2. \end{aligned} \tag{2}$$

Hence we conclude that there is no energy momentum conservation in the vertex presented in Fig. 1 $(k_1 + k_2 \neq P)$.

It was shown in Ref. 17) that the single time wave function (1) satisfies the three-dimensional covariant quasipotential equation of Kadyshevsky type[19]:

$$2\Delta_k^0(M-2\Delta_k^0)\psi_{BM}(\vec{\Delta}_k)=\frac{(2m)^2}{(2\pi)^3}\int\frac{d\vec{\Delta}_p}{2\Delta_p^0}\;V(\vec{\Delta}_k;\vec{\Delta}_p/M)\,\psi_{BM}(\vec{\Delta}_p), \qquad (3)$$

where the kernel V is the so-called quasipotential. We can rewrite this equation in the symbolical form:

$$\psi_{BM} = G_0 V \psi_{BM}, \qquad (4)$$

where the following free Green's function corresponds to the operator G_0:

$$G_0(\vec{\Delta}_k;\vec{\Delta}_p/M) = \frac{2\Delta_k^0\,\delta(\vec{\Delta}_k-\vec{\Delta}_p)}{2\Delta_k^0(M-2\Delta_k^0+i0)}, \qquad (5)$$

and the inverse Green's function is

$$G_0^{-1}(\vec{\Delta}_k;\vec{\Delta}_p/M) = 2\Delta_k^0(M-2\Delta_k^0)2\Delta_k^0\,\delta(\vec{\Delta}_k-\vec{\Delta}_p). \qquad (6)$$

In Fig. 2 we show the diagrams corresponding to Eq. (3) where $\Delta_p = L_P^{-1}p_1$.

Now following Ref. 15) we expand the wave function over the unitary representation of the group of motions in Lobashevsky space[20] in order to transform Eq. (3) into relativistic configurational space:

$$\psi_{BM}(\vec{\Delta}_k) = \int d\vec{r}\,\xi(\vec{\Delta}_k,\vec{r})\,\phi_{BM}(\vec{r}). \qquad (7)$$

In the case of the spherical symmetry, when $\psi_{BM}(\vec{\Delta}_k) = \psi_{BM}(|\vec{\Delta}_k|)$ (S state), we obtain:

$$\psi_{BM}(|\vec{\Delta}_k|) = \frac{4\pi}{m \sinh y} \phi(y), \tag{8}$$

where

$$\phi(y) = \int_0^\infty dr\, r\, \phi_{BM}(r) \sin mry \tag{9}$$

and $y = \ln(\Delta_k^0 + |\vec{\Delta}_k|)/m$ is the rapidity of the quark. In this relativistic co-ordinate $\vec{r}$ space Eq. (3) takes the following form:

$$\hat{H}_0 (M - \hat{H}_0)\, \phi_{BM}(r) = 2m\, V(r) \phi_{BM}(r) \tag{10}$$

for the local quasipotential $V(r)$ and $\hat{H}_0$ is the finite difference operator:

$$\hat{H}_0 = 2m \cosh \frac{i}{m}\frac{\partial}{\partial r} + \frac{2i}{r} \sinh \frac{i}{m}\frac{\partial}{\partial r}\,. \tag{11}$$

Thus, solving Eq. (10) for a given quasipotential $V(r)$ we are able to find the hadron wave function $\phi_{BM}(r)$.

In particular, in Refs 21) and 9) an exact solution of Eq. (10) has been found for the "Coulomb" quasipotential

$$V(r) = -g^2/r \tag{12}$$

in the relativistic $\vec{r}$ space. On the other hand, in Ref. 11) it was shown that if we identify the quasipotential (12) with one gluon exchange contribution, then in momentum space it will have the same asymptotics at very large Q^2 as that in QCD. It was emphasized that this is a result of using the proper relativistic transformation (7) and has nothing to do with the exotic dependence of the coupling constant g^2 on Q^2 (asymptotic freedom).

In the present paper we shall try to express the structure functions of deep inelastic lepton-hadron scattering through the hadron wave function and investigate scaling properties using solutions of the quasipotential equation (10).

2. STRUCTURE FUNCTIONS IN TWO-PARTICLE APPROXIMATION

We start with the usual expression[1)] for the structure tensor describing the hadronic interactions with the virtual photon in deep inelastic electron-hadron scattering:

$$W_{\mu\nu}(P,q) = \frac{1}{4\pi}\sum_N (2\pi)^4\,\delta(P+q-P_N)\times$$
$$\times \langle P|\, \overset{+}{J_\mu}(0)|N\rangle\langle N|\, J_\nu(0)|P\rangle, \tag{13}$$

where the "summation" goes over all intermediate states, P_N is the momentum of the N'th intermediate state, q is the momentum of the virtual photon ($q^2 = -Q^2$) and $J_\mu(0)$ is the hadron current operator.

In this paper we restrict ourselves to the consideration of a two-particle intermediate state (the quark-antiquark pair). It means that instead of Eq. (13) we have:

$$W_{\mu\nu}(P,q) = \frac{1}{2(2\pi)^3}\int dk_1\,\theta(k_1^0)\,\delta(k_1^2-m^2)\int dk_2\,\theta(k_2^0)\,\delta(k_2^2-m^2)\times$$
$$\times\,\delta(P+q-k_1-k_2)\langle k_1,k_2|\,J_\mu(0)|P\rangle^*\langle k_1,k_2|\,J_\nu(0)|P\rangle, \tag{14}$$

hence

$$W_{\mu\nu}(P,q) = \frac{1}{2(2\pi)^3}\int\frac{d\vec{k}_1}{2k_1^0}\,\delta(W^2-2Pk_1-2qk_1)\times$$
$$\times\langle k_1,k_2|\,J_\mu(0)|P\rangle^*\langle k_1,k_2|\,J_\nu(0)|P\rangle, \tag{15}$$

where the usual invariant mass of the final hadron system $W^2 = (P + q)^2$ is introduced. Graphically the matrix element $\langle k_1, k_2 | J_\mu(0) | P\rangle$ is shown in Fig. 3. Now we shall make a change of variables $k_1 \to \Delta_k \equiv L_P^{-1} k_1$ under the integral (15) which is equivalent to the transition to the hadron rest system ($\vec{P} = 0$):

$$W'_{\mu\nu}(M, q') = \frac{1}{2(2\pi)^3} \int \frac{d\vec{\Delta}_k}{2\Delta_k^0}\, \delta\left(W^2 - 2M\Delta_k^0 - 2q'\Delta_k\right) \times$$
$$\times \langle \Delta_k, \tilde{\Delta}_k | \mathcal{J}_\mu(0) | M\rangle^* \langle \Delta_k, \tilde{\Delta}_k | \mathcal{J}_\nu(0) | M\rangle, \tag{16}$$

where

$$\tilde{\Delta}_k \equiv L_P^{-1} k_2\,, \quad q' = L_P^{-1} q$$

and

$$W'_{\mu\nu}(M, q') = \left[L_P^{-1} W(P, q)\right]_{\mu\nu}.$$

It is not difficult to see that the Lorentz transformation gives us

$$q'^0 = Pq/M \equiv \nu; \tag{17}$$

$$|\vec{q}'| = \sqrt{(q'^0)^2 - q^2} = \sqrt{\nu^2 + Q^2}. \tag{18}$$

In order to express the hadronic current matrix element $\langle \Delta_k, \tilde{\Delta}_k | J_\mu(0) | M\rangle$ through the single time quark-antiquark wave function introduced in Eq. (1), we shall construct in analogy with Eq. (1) following Ref. 5) the five-point function in momentum space:

$$\frac{R_\mu(\vec{\Delta}_k ; \vec{\Delta}_p | P^0)}{2\Delta_p^0} = \int dy_1\, dy_2\, e^{ip_1 y_1 + ip_2 y_2}\, \delta\left[\lambda_P (y_1 - y_2)\right] \times$$
$$\times \langle k_1, k_2 | T\left(\mathcal{J}'_\mu(0) \overset{*}{\varphi}_q(y_1) \overset{*}{\varphi}_{\bar{q}}(y_2)\right) | 0\rangle, \tag{19}$$

where

$$P^0 = p_1^0 + p_2^0$$

and

$$\mathcal{J}'_\mu(0) = \left[u^{-1}(L_P^{-1})\,\mathcal{J}(0)\,u(L_P^{-1}) \right]_\mu .$$

From the spectral representation for the function R_μ near the bound state pole $P^0 \cong M$ using Eq. (1) we obtain the following formula:

$$R_\mu(\vec{\Delta}_k; \vec{\Delta}_p | P^0) \underset{P^0 \to M}{=} \frac{\langle \Delta_k, \tilde{\Delta}_k | \mathcal{J}_\mu(0) | M \rangle \overset{*}{\psi}_{BM}(\vec{\Delta}_p)}{2M(P^0 - M - i0)} . \tag{20}$$

In accordance with Ref. 5), the generalized vertex function $\Gamma_\mu(\vec{\Delta}_k; \vec{\Delta}_p \mid P^0)$ is introduced by the relationship:

$$R_\mu = \Gamma_\mu \cdot G , \tag{21}$$

where the operator product should be understood like that in Eq. (4). Now multiplying Eq. (20) by $G^{-1}\psi_{BM}$ and using the normalization condition for the wave function:

$$\left(\psi_{BM} , \frac{\partial}{\partial P^0} G^{-1} \Big|_{P^0 = M} \psi_{BM} \right) = 2M \tag{22}$$

we get in the limit $P^0 \to M$ the following expression for the current matrix element:

$$\langle \Delta_k, \tilde{\Delta}_k | \mathcal{J}_\mu(0) | M \rangle = \int \frac{d\vec{\Delta}_p}{2\Delta_p^0} \Gamma_\mu(\vec{\Delta}_k; \vec{\Delta}_p | M) \psi_{BM}(\vec{\Delta}_p) . \tag{23}$$

The vertex function Γ_μ generally has a very complicated structure which is illustrated in Fig. 3. However, we shall neglect the interaction between the quark and antiquark in the final state, that is, we shall consider here the impulse approximation for the virtual photon hadronic interaction. In this case, the two diagrams of Fig. 4 will define the vertex Γ_μ and, for example, the contribution of the diagram (a) equals

$$\Gamma_\mu^{(a)}(\vec{\Delta}_k\,;\vec{\Delta}_p\,|M) = 2\Delta_k^0\,\delta(\vec{\Delta}_k-\vec{\Delta}_p)\,\tilde{j}^\mu(\tilde{\Delta}_k\,;\tilde{\Delta}_p), \tag{24}$$

where $\tilde{j}_\mu$ is the antiquark current. Substituting Eq. (24) into Eq. (23) we get

$$\langle \Delta_k, \tilde{\Delta}_k\,|\,\mathcal{J}_\mu(0)\,|M\rangle^{(a)} = \tilde{j}^\mu(\tilde{\Delta}_k\,;\tilde{\Delta}_p)\,\varphi_{BM}(\vec{\Delta}_k), \tag{25}$$

where $\tilde{\Delta}_p = (\Delta_k^0, -\vec{\Delta}_k)$.

Then we notice that if we neglect the quark-antiquark interaction in the final state, the two diagrams in Fig. 4 will contribute to the structure tensor (16) incoherently and it is easy to show that their contributions will be equal. Thus, substituting the expression (25) into Eq. (16) we have

$$\begin{aligned} W'_{\mu\nu}(M,q') = \frac{1}{(2\pi)^3}\int\frac{d\vec{\Delta}_k}{2\Delta_k^0}\,\delta\left[W^2-2(\nu+M)\Delta_k^0+2\vec{q}\,'\vec{\Delta}_k\right]\times \\ \times\,\tilde{j}^{*}_\mu(\tilde{\Delta}_k\,;\tilde{\Delta}_p)\,\tilde{j}_\nu(\tilde{\Delta}_k\,;\tilde{\Delta}_p)\,|\varphi_{BM}(\vec{\Delta}_k)|^2. \end{aligned} \tag{26}$$

In the parton model, the special reference frame $|\vec{P}| \to \infty$ is usually introduced, which allows the separation of the parton momentum $\vec{k}_1$ into longitudinal and transverse parts with respect to the momentum of the initial hadron $\vec{P}$. However, this decomposition is not invariant. In our scheme we can introduce the covariant projections of the quark momentum $\vec{\Delta}_k$:

$$\begin{aligned} \Delta_{k\parallel} &= \frac{\vec{q}\,'\vec{\Delta}_k}{|\vec{q}\,'|}\,; \\ \Delta_{k\perp} &= \sqrt{\Delta_k^{0\,2}-m^2-\Delta_{k\parallel}^2}\,. \end{aligned} \tag{27}$$

The covariance of these projections follows from the covariance of q'^0 and $|\vec{q}'|$ [see Eqs (17) and (18)] and $\Delta_k^0 = Pk_1/M$. Finally, from Eq. (26) we obtain:

$$W'_{\mu\nu}(M,q') = \frac{1}{(2\pi)^3}\int \frac{d\vec{\Delta}_k}{2\Delta_k^0}\ \delta\left[W^2 - 2(\nu+M)\Delta_k^0 + 2\sqrt{\nu^2+Q^2}\,\Delta_{k\parallel}\right]\times$$
$$\times\ \tilde{j}^*_\mu(\tilde{\Delta}_k;\tilde{\Delta}_p)\,\tilde{j}_\nu(\tilde{\Delta}_k;\tilde{\Delta}_p)\,|\psi_{BM}(\vec{\Delta}_k)|^2. \tag{28}$$

3. NEW SCALING PROPERTIES OF THE STRUCTURE FUNCTIONS

To begin with we shall consider in this section the case of a scalar hadronic current and take the antiquark current in Eq. (28) in the following simplest form:

$$\tilde{j}(\tilde{\Delta}_k;\tilde{\Delta}_p) = 2M. \tag{29}$$

Then only one structure function will be equal to

$$W(Q^2,\nu) = \frac{2M}{(2\pi)^3}\int\frac{d\vec{\Delta}}{2\Delta^0}\,\delta\left[W^2 - 2(\nu+M)\Delta^0 + 2\sqrt{\nu^2+Q^2}\,\Delta_\parallel\right]|\psi_{BM}(|\vec{\Delta}|)|^2, \tag{30}$$

where $\Delta \equiv \Delta_k$ and we have taken into account the spherical symmetry of ψ_{BM} (S state).

Performing the integration over angles due to the δ function in Eq. (30) we come to the structure function

$$\sqrt{\nu^2+Q^2}\,W(Q^2,\nu) = \frac{2M}{(4\pi)^2}\int_{\Delta_-^0}^{\Delta_+^0} d\Delta^0\,|\psi_{BM}(|\vec{\Delta}|)|^2, \tag{31}$$

where

$$2\Delta^0_{\pm} = \nu + M \pm \varepsilon\sqrt{\nu^2+Q^2}\;; \tag{32}$$

$$\varepsilon = \sqrt{1-4m^2/W^2}\,. \tag{33}$$

Now it is convenient to introduce the rapidity of the quark $y = \ln(\Delta^0 + |\vec{\Delta}|)/m$ under the integral (31) $(\Delta^0 = m\cosh y,\ |\vec{\Delta}| = m\sinh y)$:

$$\sqrt{\nu^2+Q^2}\,W(Q^2,\nu) = \frac{2mM}{(4\pi)^2}\int\limits_{|y_-|}^{y_+} dy\,\sinh y\,|\varphi_{BM}(m\sinh y)|^2, \tag{34}$$

where

$$y_{\pm} = \pm\ln\frac{2m}{(1\mp\varepsilon)(\nu+M-\sqrt{\nu^2+Q^2})}\,. \tag{35}$$

Using the function $\phi(y)$ introduced in Eqs (8) and (9), we get the following expression:

$$\sqrt{\nu^2+Q^2}\,W(Q^2,\nu) = \frac{2M}{m}\int\limits_{|y_-|}^{y_+}\frac{dy}{\sinh y}\,|\phi(y)|^2. \tag{36}$$

Now we shall introduce the new scaling variable ζ in the following way

$$M\zeta = \nu + M - \sqrt{\nu^2+Q^2} = \nu + M - \sqrt{(\nu+M)^2 - W^2}. \tag{37}$$

Then the structure function will take the form:

$$F(\zeta, W^2) \equiv \sqrt{\nu^2+Q^2}\,W(Q^2,\nu) = \frac{2M}{m}\int\limits_{\left|\ln\frac{(1+\varepsilon)M\zeta}{2m}\right|}^{\ln\frac{(1+\varepsilon)W^2}{2mM\zeta}}\frac{dy}{\sinh y}\,|\phi(y)|^2. \tag{38}$$

Thus, in our approach we naturally come to the variables ζ and W^2 instead of $x = Q^2/2M\nu$ and Q^2 which are usually introduced in the Bjorken limit. In the deep inelastic region where $W^2 \to \infty$ and ζ is fixed, we get the following expression $(\varepsilon = 1)$:

$$F(\zeta, W^2) = \frac{2M}{m} \int_{\left|\ln \frac{M\zeta}{m}\right|}^{\ln \frac{W^2}{mM\zeta}} \frac{dy}{\sinh y} \, |\phi(y)|^2 \tag{39}$$

and

$$2M\nu \cong \frac{W^2}{\zeta} \longrightarrow \infty ;$$
$$Q^2 \cong \frac{(1-\zeta)W^2}{\zeta} \longrightarrow \infty , \tag{40}$$

so that $x \cong 1 - \zeta$ is fixed and we are actually in the usual deep inelastic region, but in terms of the other variables ζ and W^2.

In Refs 22) and 23), in the framework of the parton model, a scaling variable ξ taking into account non-zero quark masses was introduced in the following way:

$$2M\xi = (\beta+1)\left(\sqrt{\nu^2+Q^2} - \nu\right), \tag{41}$$

where $\beta = \sqrt{1+4m^2/Q^2}$. It is easy to find the connection between the two variables (37) and (41):

$$2\xi = (\beta+1)(1-\zeta). \tag{42}$$

Hence we see that for massless quarks $(\beta = 1)$ we have $\xi = 1 - \zeta$.

The difference between the two results probably arises from the different treatment of the initial hadronic state. In particular, unlike the parton model, all the constituents in the quasipotential approach always lie on the mass shell and we have to come out from the energy momentum shell according to Eq. (2) in the vertex drawn in Fig. 1. In the parton model only the struck parton is laid up on the mass shell which is achieved by the transition to the infinite momentum reference frame. What is most important is that in contrast to Ref. 23), where the structure functions always depend only on ξ (except for kinematical factors), we get the scaling violation due to the explicit dependence of the integral (39) on W^2 as well.

We can now represent the structure function (38) in the following form:

$$F(\zeta, W^2) = F^S(\zeta) - F^P(\zeta, W^2), \tag{43}$$

where $F^S(\zeta)$ is the scaling part of the structure function depending only on ζ:

$$F^S(\zeta) = \frac{2M}{m} \int_{\left|\ln \frac{M\zeta}{m}\right|}^{\infty} \frac{dy}{\sinh y} \, |\phi(y)|^2 \tag{44}$$

and $F^P(\zeta, W^2)$ is the prescaling part:

$$F^P(\zeta, W^2) = \frac{2M}{m} \int_{\left|\ln \frac{M\zeta}{m}\right|}^{\left|\ln \frac{(1+\varepsilon)M\zeta}{2m}\right|} \frac{dy}{\sinh y} |\phi(y)|^2 + \frac{2M}{m} \int_{\ln \frac{(1+\varepsilon)W^2}{2mM\zeta}}^{\infty} \frac{dy}{\sinh y} \, |\phi(y)|^2 \tag{45}$$

describing the approach of the structure function to the scaling behaviour. Using the representation

$$\int dy \, e^{\pm y} \, |\phi(y)|^2 = e^{\pm y} \sum_{n=0}^{\infty} (\mp)^n \frac{d^n}{dy^n} \, |\phi(y)|^2 \tag{46}$$

we can calculate the asymptotics of the expression (45) in the limit $W^2 \to \infty$ in the leading power approximation:

$$F^P(\zeta, W^2) \cong \frac{2mM}{W^2} \cdot \frac{\ln \frac{m}{M\zeta}}{\left|\ln \frac{m}{M\zeta}\right|} \cdot \frac{\left|\phi\left(\left|\ln \frac{M\zeta}{m}\right|\right)\right|^2}{\sinh \left|\ln \frac{M\zeta}{m}\right|} + \\ + \frac{4M^2\zeta}{W^2} \sum_{n=0}^{\infty} \frac{d^n}{\left(d \ln \frac{W^2}{mM\zeta}\right)^n} \left|\phi\left(\ln \frac{W^2}{mM\zeta}\right)\right|^2. \tag{47}$$

Thus, we have obtained the explicit powerlike prescaling behaviour of the structure function and all logarithmic corrections to it, which are determined by the asymptotics of the wave function (9) and its derivatives at large y. In QCD it was shown that in the lowest orders, the corrections to the structure functions contain only logarithmic terms. In the following sections we shall consider the real case of vector hadronic currents (of scalar quarks), and we shall obtain a similar result but in terms of the explicit form of the initial hadron wave function without using any perturbation theory.

4. LEADING LOG'S APPROXIMATION FOR THE STRUCTURE FUNCTIONS

For the case of the scalar quarks we shall choose the vector antiquark current in Eq. (28) in the following form:

$$\tilde{j}_\nu (\hat{\Delta}_k ; \hat{\Delta}_p) = -(\hat{\Delta}_p + \hat{\Delta}_k)_\nu \tag{48}$$

and

$$\tilde{j}^*_\mu (\hat{\Delta}_k ; \hat{\Delta}_p) = (\hat{\Delta}_p + \hat{\Delta}_k)_\mu . \tag{49}$$

Therefore, from Eq. (28) we get

$$W'_{\mu\nu}(M, q') = -\frac{1}{(2\pi)^3} \int \frac{d\vec{\Delta}_k}{2\Delta^0_k} \delta\left[W^2 - 2(\nu+M)\Delta^0_k + 2\sqrt{\nu^2+Q^2}\,\Delta_{k\parallel}\right] \times$$
$$\times (\hat{\Delta}_p + \hat{\Delta}_k)_\mu (\hat{\Delta}_p + \hat{\Delta}_k)_\nu \,|\psi_{BM}(\vec{\Delta}_k)|^2 . \tag{50}$$

We introduce two invariant functions:

$$V_1(Q^2, \nu) = g^{\mu\nu} W_{\mu\nu}(P, q) = g^{\mu\nu} W'_{\mu\nu}(M, q') \tag{51}$$

and

$$V_2(Q^2, \nu) = P^\mu P^\nu W_{\mu\nu}(P, q)/M^2 = W'_{00}(M, q'). \tag{52}$$

Since $\tilde{\Delta}_p = (\Delta_k^0, -\vec{\Delta}_k)$ and $\tilde{\Delta}_k = (\nu + M - \Delta_k^0, \vec{q}\,' - \vec{\Delta}_k)$, it is not difficult to calculate that

$$V_1(Q^2,\nu) = -\frac{1}{(2\pi)^3}\int \frac{d\vec{\Delta}}{2\Delta^0}\,\delta\left[W^2 - 2(\nu+M)\Delta^0 + 2\sqrt{\nu^2+Q^2}\,\Delta_{\parallel}\right]\times \tag{53}$$

$$\times\left[4(\nu+M)\Delta^0 - 4\vec{\Delta}^2 - W^2\right]\left|\psi_{BM}(|\vec{\Delta}|)\right|^2;$$

$$V_2(Q^2,\nu) = -\frac{(\nu+M)^2}{(2\pi)^3}\int \frac{d\vec{\Delta}}{2\Delta^0}\,\delta\left[W^2 - 2(\nu+M)\Delta^0 + 2\sqrt{\nu^2+Q^2}\,\Delta_{\parallel}\right]\times \tag{54}$$

$$\times\left|\psi_{BM}(|\vec{\Delta}|)\right|^2,$$

where $\Delta \equiv \Delta_k$.

As in the previous section's calculations, by performing the integration over angles we obtain the following expressions:

$$\sqrt{\nu^2+Q^2}\,V_1(Q^2,\nu) = -\frac{1}{(4\pi)^2}\int_{\Delta_-^0}^{\Delta_+^0} d\Delta^0\left[4(\nu+M)\Delta^0 - 4\vec{\Delta}^2 - W^2\right]\left|\psi_{BM}(|\vec{\Delta}|)\right|^2; \tag{55}$$

$$\sqrt{\nu^2+Q^2}\,V_2(Q^2,\nu) = -\frac{(\nu+M)^2}{(4\pi)^2}\int_{\Delta_-^0}^{\Delta_+^0} d\Delta^0\left|\psi_{BM}(|\vec{\Delta}|)\right|^2, \tag{56}$$

where $\Delta_{\pm}^0$ were defined in Eqs (32) and (33). After introducing the rapidity $y = \ln(\Delta^0 + |\vec{\Delta}|)/m$, the wave function (9) and the new scaling variable (37) we find that

$$\sqrt{\nu^2+Q^2}\,V_1(Q^2,\nu) = -\frac{1}{m}\int_{\left|\ln\frac{(1+\varepsilon)M\xi}{2m}\right|}^{\ln\frac{(1+\varepsilon)W^2}{2mM\xi}} \frac{dy}{\sinh y}\left|\phi(y)\right|^2\times \tag{57}$$

$$\times\left[4m(\nu+M)\cosh y - 4m^2\sinh^2 y - W^2\right];$$

$$\sqrt{\nu^2+Q^2}\,V_2(Q^2,\nu) = -\frac{(\nu+M)^2}{m}\int_{\left|\ln\frac{(1+\epsilon)MC_1}{2m}\right|}^{\ln\frac{(1+\epsilon)W^2}{2mMC_1}} \frac{dy}{\sinh y}\,|\Phi(y)|^2. \tag{58}$$

From the usual decomposition of the structure tensor

$$2MW_{\mu\nu}(P,q) = 4M^2\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)W_1(Q^2,\nu) + + 4\left(P_\mu - q_\mu\frac{Pq}{q^2}\right)\left(P_\nu - q_\nu\frac{Pq}{q^2}\right)W_2(Q^2,\nu) \tag{59}$$

we derive the following expressions for the structure functions W_1 and W_2:

$$4MW_1(Q^2,\nu) = -V_1(Q^2,\nu) + \frac{Q^2}{\nu^2+Q^2}\,V_2(Q^2,\nu); \tag{60}$$

$$\frac{4M(\nu^2+Q^2)}{Q^2}\,W_2(Q^2,\nu) = -V_1(Q^2,\nu) + \frac{3Q^2}{\nu^2+Q^2}\,V_2(Q^2,\nu). \tag{61}$$

Hence from Eqs (57) and (58) we find

$$2MW_1(Q^2,\nu) = \frac{1}{2m\sqrt{\nu^2+Q^2}}\int_{\left|\ln\frac{(1+\epsilon)MC_1}{2m}\right|}^{\ln\frac{(1+\epsilon)W^2}{2mMC_1}} \frac{dy}{\sinh y}\,|\Phi(y)|^2 \times \left[4m(\nu+M)\cosh y - 4m^2\sinh^2 y - W^2 - \frac{(\nu+M)^2Q^2}{\nu^2+Q^2}\right]; \tag{62}$$

$$\sqrt{\nu^2+Q^2}\,W_2(Q^2,\nu) = \frac{Q^2}{4mM(\nu^2+Q^2)}\int_{\left|\ln\frac{(1+\epsilon)MC_1}{2m}\right|}^{\ln\frac{(1+\epsilon)W^2}{2mMC_1}} \frac{dy}{\sinh y}\,|\Phi(y)|^2 \times$$

$$\times\left[4m(\nu+M)\cosh y-4m^2\sinh^2 y-W^2-\frac{3(\nu+M)^2Q^2}{\nu^2+Q^2}\right]. \qquad (63)$$

In the limit $W^2 \to \infty$ using Eq. (40) we find

$$F_1(\xi,W^2)\equiv 2MW_1(Q^2,\nu)=\frac{1}{m}\int\limits_{|\ln\frac{M\xi}{m}|}^{\ln\frac{W^2}{mM\xi}} dy\,\frac{2m\cosh y-M}{\sinh y}\,|\phi(y)|^2-$$

$$-\frac{4Mm\xi}{W^2}\int\limits_{|\ln\frac{M\xi}{m}|}^{\ln\frac{W^2}{mM\xi}} dy\,\sinh y\,|\phi(y)|^2; \qquad (64)$$

$$F_2(\xi,W^2)\equiv\sqrt{\nu^2+Q^2}\,W_2(Q^2,\nu)=\frac{1-\xi}{m}\int\limits_{|\ln\frac{M\xi}{m}|}^{\ln\frac{W^2}{mM\xi}} dy\,\frac{2m\cosh y-M(3-2\xi)}{\sinh y}\,|\phi(y)|^2-$$

$$-\frac{4mM\xi(1-\xi)}{W^2}\int\limits_{|\ln\frac{M\xi}{m}|}^{\ln\frac{W^2}{mM\xi}} dy\,\sinh y\,|\phi(y)|^2. \qquad (65)$$

The asymptotics of the second set of integral when $W^2 \to \infty$ in these formulae can be estimated with the help of the representation (46). Leaving only leading power terms we get

$$2\int\limits_{|\ln\frac{M\xi}{m}|}^{\ln\frac{W^2}{mM\xi}} dy\,\sinh y\,|\phi(y)|^2 \cong \frac{W^2}{mM\xi}\sum_{n=0}^{\infty}(-1)^n\frac{d^n}{\left(d\ln\frac{W^2}{mM\xi}\right)^n}\left|\phi\left(\ln\frac{W^2}{mM\xi}\right)\right|^2. \qquad (66)$$

We can now separate the scaling and prescaling parts in the structure functions as was done in the previous section and write down, for example, $F_1(\zeta,W^2)$ in the form:

$$F_1(\zeta,W^2)=F_1^{\,S}(\zeta)-F_1^{\,P}(\zeta,W^2), \tag{67}$$

where

$$F_1^{\,S}(\zeta)=\frac{1}{m}\int\limits_{|\ln\frac{M\zeta}{m}|}^{\infty}dy\;\frac{2m\cosh y-M}{\sinh y}\,|\phi(y)|^2 \tag{68}$$

and in the leading log's approximation

$$F_1^{\,P}(\zeta,W^2)\cong 2\sum_{n=0}^{\infty}(-1)^n\frac{d^n}{\left(d\ln\frac{W^2}{mM\zeta}\right)^n}\int\limits_{\ln\frac{W^2}{mM\zeta}}^{\infty}dy\,|\phi(y)|^2. \tag{69}$$

Thus we conclude that the leading corrections to the scaling part of the structure functions contain only logarithmic terms and we are able to calculate all of them if we know the asymptotics of the wave function $\phi(y)$ as $y\to\infty$.

From Eqs (64) and (65) we derive the following relationship between the structure functions F_1 and F_2:

$$F_2(\zeta,W^2)=(1-\zeta)F_1(\zeta,W^2)-\frac{2M(1-\zeta)^2}{m}\int\limits_{|\ln\frac{M\zeta}{m}|}^{\ln\frac{W^2}{mM\zeta}}\frac{dy}{\sinh y}\,|\phi(y)|^2. \tag{70}$$

Hence we get

$$F_2^{\,S}(\zeta)=(1-\zeta)F_1^{\,S}(\zeta)-\frac{2M(1-\zeta)^2}{m}\int\limits_{|\ln\frac{M\zeta}{m}|}^{\infty}\frac{dy}{\sinh y}\,|\phi(y)|^2 \tag{71}$$

and

$$F_2^{\,p}(\zeta, W^2) = (1-\zeta)\, F_1^{\,p}(\zeta, W^2) \tag{72}$$

in the leading log's approximation.

5. BEHAVIOUR OF THE STRUCTURE FUNCTIONS ON THE EXCLUSIVE THRESHOLD

In this section we shall consider the behaviour of the structure functions in the case when $W^2 = W_0^2$ is fixed and $\zeta \to 0$ (the exclusive threshold). In this region, instead of Eqs (40) we have

$$\begin{aligned} 2M\nu &\cong \frac{W_0^2}{\zeta} \to \infty ; \\ Q^2 &\cong \frac{W_0^2}{\zeta} \to \infty , \end{aligned} \tag{73}$$

and $\varepsilon_0 = \sqrt{1-4m^2/W_0^2}$. From Eqs (62) and (63) in the first non-vanishing approximation in $\zeta \to 0$ we derive the following expression for the structure functions:

$$\begin{aligned} F_1(\zeta, W_0^2) \cong \int_{\ln \frac{2m}{(1+\varepsilon_0)M\zeta}}^{\ln \frac{2m}{(1+\varepsilon_0)M\zeta} + \ln\frac{1+\varepsilon_0}{1-\varepsilon_0}} dy\, |\Phi(y)|^2 \,\times \\ \times \left(2 - \frac{2mM\zeta}{W_0^2} e^{y} \right), \end{aligned} \tag{74}$$

where $\ln\,(2m/(1+\varepsilon_0)M\zeta) \to \infty$.

By replacing the variable $y = \ln(2m/(1+\varepsilon_0)M\zeta) + y'$ we can rewrite Eq. (74) in the form:

$$F_1(\zeta, W_0^2) \cong F_2(\zeta, W_0^2) \cong \int_0^{\ln\frac{1+\varepsilon_0}{1-\varepsilon_0}} dy' \left|\phi\left(\ln\frac{2m}{(1+\varepsilon_0)M\zeta} + y'\right)\right|^2 \times$$
$$\times \left[2-(1-\varepsilon_0)e^{y'}\right]. \tag{75}$$

Hence we obtain

$$F_1(\zeta, W_0^2) \cong F_2(\zeta, W_0^2) \cong$$
$$\cong \sum_{n=0}^{\infty} \frac{a_n(\varepsilon_0)}{n!} \frac{d^n}{\left(d\ln\frac{2m}{(1+\varepsilon_0)M\zeta}\right)^n} \left|\phi\left(\ln\frac{2m}{(1+\varepsilon_0)M\zeta}\right)\right|^2, \tag{76}$$

where

$$a_n(\varepsilon_0) = \int_0^{\ln\frac{1+\varepsilon_0}{1-\varepsilon_0}} dy'\, y'^n \left[2-(1-\varepsilon_0)e^{y'}\right]. \tag{77}$$

Thus, the threshold behaviour of the structure functions is determined by the asymptotics of the wave function $\phi(y)$ as $y \to \infty$ and contains only logarithmic terms. Nevertheless, the structure functions can vanish rather quickly when $\zeta \to 0$ in accordance with experimental data because of a fast powerlike decrease of the wave function.

Concluding this section we notice that the kinematic region of the changing of the new scaling variable ζ at fixed W^2 is defined as follows:

$$\zeta_{max} = 1\,;$$
$$\zeta_{min} \cong \frac{W^2}{2M(\nu+M)}, \tag{78}$$

so that we can reach the exclusive threshold in the limit $(\nu + M) \to \infty$.

6. RELATIVISTIC POTENTIAL WITH QCD, LARGE Q^2 BEHAVIOUR AND THE STRUCTURE FUNCTIONS

In this section, we shall calculate the structure functions using the concrete relativistic quasipotential (12) which corresponds to the QCD one-gluon exchange at large Q^2 [11]. The non-relativistic problem was discussed in Refs 4) and 24) for some various choices of the wave function. In our approach, we have the exact solution of equation (10) with the quasipotential (12) for the relativistic wave function of the initial hadron[9]:

$$\phi_{BM}(r) = c_0 e^{-mry_0}, \tag{79}$$

where the parameter y_0 is defined by the equation $2m \cos y_0 = M$, and c_0 is fixed by the normalization condition (22). It is easy to obtain from Eq. (9) that

$$\phi(y) = \frac{c_0 y_0 y}{m^2 (y^2 + y_0^2)^2}. \tag{80}$$

Hence we derive the scaling parts of the structure functions (68) and (71):

$$F_1^s(\xi) = \frac{2c_0^2 y_0^2}{m^4} \int\limits_{|\ln 2\xi \cos y_0|}^{\infty} dy \, \frac{(\cosh y - \cos y_0)^2}{\sinh y \, (y^2+y_0^2)^4}; \tag{81}$$

$$F_2^s(\xi) = \frac{2c_0^2 y_0^2 (1-\xi)}{m^4} \int\limits_{|\ln 2\xi \cos y_0|}^{\infty} dy \, \frac{[\cosh y - (3-2\xi)\cos y_0]\, y^2}{\sinh y \, (y^2+y_0^2)^4}. \tag{82}$$

From Eqs (69) and (72) we can find the prescaling behaviour of the structure functions in the leading log's approximation. For example, the main terms of these asymptotics equal

$$F_1^P(\zeta, W^2) \cong \frac{2C_0^2 y_0^2}{m^4}\int\limits_{\ln\frac{W^2}{mM\zeta}}^{\infty} dy\, \frac{y^2}{(y^2+y_0^2)^4} \cong \tag{83}$$

$$\cong \frac{2C_0^2 y_0^2}{5m^4}\left(\ln\frac{W^2}{mM\zeta}\right)^{-5}$$

and

$$F_2^P(\zeta, W^2) \cong \frac{2C_0^2 y_0^2(1-\zeta)}{5m^4}\left(\ln\frac{W^2}{mM\zeta}\right)^{-5}. \tag{84}$$

We shall now calculate the main contribution to the threshold behaviour of the structure functions (76):

$$F_1(\zeta, W_0^2) \cong F_2(\zeta, W_0^2) \underset{\zeta\to 0}{\cong} \frac{2C_0^2 y_0^2}{m^4}\left(\ln\frac{1+\varepsilon_0}{1-\varepsilon_0}-\varepsilon_0\right)\left(\ln\frac{2m}{(1+\varepsilon_0)M\zeta}\right)^{-6}. \tag{85}$$

Coming back to the general case, it is not difficult to see that the asymptotics of the wave function $\phi(y)$ at large y will always be defined by the behaviour of $\phi_{BM}(r)$ in Eq. (9) at the origin because at fast oscillation of $\sin rmy$. Therefore, in order to calculate the asymptotics of $\phi(y)$, it is sufficient to know the behaviour of the wave function in co-ordinate space as $r \to 0$. If we assume that near the point $r = 0$

$$\phi_{BM}(r) \sim r^{\alpha-2}, \tag{86}$$

then

$$\phi(y) \underset{y\to\infty}{\sim} \int\limits_0^\infty dr\, r^{\alpha-1} e^{-mr\delta} \sin mry = \tag{87}$$

$$= \frac{\Gamma(\alpha)\sin(\alpha\, \mathrm{arctg}\, y/\delta)}{m^\alpha (y^2+\delta^2)^{\alpha/2}},$$

where the exponential $\exp\{-mr\delta\}$ is introduced for the regularization of the integral. In the limit $y \to \infty$ we get

$$\phi(y) \sim \frac{\Gamma(\alpha)\sin(\pi\alpha/2)}{(my)^{\alpha}}, \quad \alpha \neq 2n, \tag{88}$$

and

$$\phi(y) \sim \frac{(-1)^{n-1}\Gamma(2n)m\delta}{(my)^{2n+1}}, \quad \alpha = 2n. \tag{89}$$

Thus we obtain the powerlike decrease of the wave function and hence the prescaling and threshold behaviour of the structure functions will contain only logarithmic terms in the leading approximation. In the above discussion we see the connection between our approach and the light cone expansion widely used in recent years.

CONCLUSIONS

In the present paper we have managed to express the hadron structure functions of deep inelastic scattering in the two intermediate particle approximation through the relativistic wave function of hadrons in the framework of the single time formulation of QFT. In this approach the hadron is considered as a bound state of quark-antiquark pairs and its single time wave function is found as a solution of the covariant three-dimensional equation with a given quasipotential. This solution could be obtained by applying the expansion over the unitary representation of the group of motions in Lobachevsky space and introducing the relativistic configurational $\vec{r}$ space.

In our approach, a new scaling variable appears which differs from the ones previously introduced. It is shown that in the leading approximation the prescaling corrections to the structure functions contain only logarithmic terms like QCD calculations. However, such behaviour has nothing to do with the exotic dependence of the coupling constant on Q^2, and this is a result of using the proper three-dimensional relativistic configurational space. The results obtained are illustrated with the help of the simplest quasipotential which corresponds to the one-gluon exchange in QCD. In this case, the exact wave function is known and the structure functions can be calculated explicitly.

In our subsequent works we are going to study deep inelastic scattering, taking account of many particle intermediate states, consider the spinor quark case and carry out a more detailed comparison of our approach with QCD and operator product expansion results.

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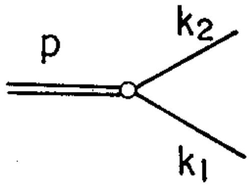

Fig. 1

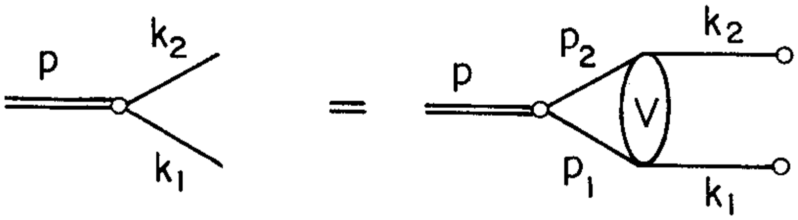

Fig. 2

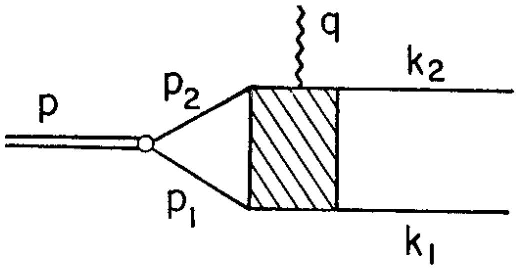

Fig. 3

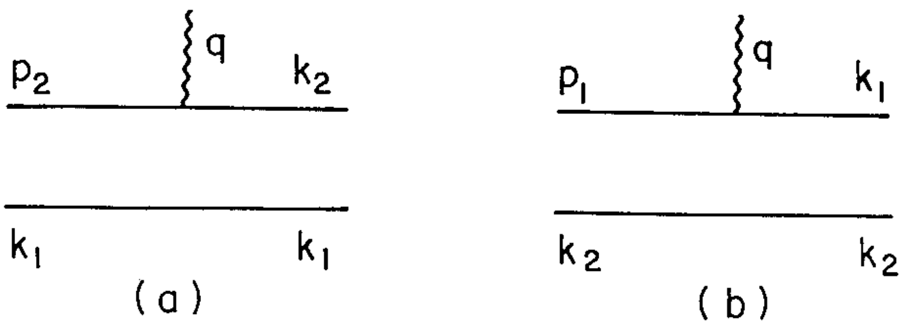

Fig. 4