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МАССИВНОЕ ГРАВИТАЦИОННОЕ ПОЛЕ В ПЛОСКОМ ПРОСТРАНСТВЕ-ВРЕМЕНИ. I. КАЛИБРОВОЧНАЯ ИНВАРИАНТНОСТЬ И ПОЛЕВЫЕ УРАВНЕНИЯ

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A MASSIVE GRAVITATIONAL FIELD IN FLAT SPACETIME. I. GAUGE INVARIANCE AND FIELD EQUATIONS

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Каноническое линейное массивное бесспиновое поле представлено в качестве калибровочно-инвариантной модели гравитации с квадратичным самодействием в рамках специальной теории относительности.

***Ключевые слова**: скалярная гравитация, массивный гравитон, масштабная инвариантность, гравитационно-зависимая масса.*

The canonical linear theory of a massive spinless field is adapted as special-relativistic gauge-invariant model of gravity with a quadratic self-action.

***Keywords**: scalar gravity, massive graviton, scale invariance, gravitationally dependent mass.*

Introduction

Critically rethinking the mathematical methods and physical content of the general theory of relativity, Anatoly Logunov recalled in [1] a long-standing problem of breaking the strong equivalence principle arising at the intersection of gravity and electromagnetism. The essence of this problem of principle raised by Bondi and Gold in [2] is that, unlike a massive electrically neutral point particle, a very similar but charged particle, freely falling in an arbitrary non-uniform gravitational field, emits electromagnetic radiation and appears therefore to be slowly accelerated in a coordinate frame falling with it, revealing thereby the presence of a local gravitational field to a freely falling observer. The conclusion regarding the emergence of electromagnetic self-action causing a non-geodesic free motion of a charged particle in a static gravitational field is supported by a number of theoretical calculations performed within the gravitationally modified electrodynamics inspired by general relativity (see, for example, the papers by Parrott [3] or Poisson et al. [4]). This indisputable violation by the electromagnetic interaction of one of the conceptual conclusions of Einstein's theory of gravitation indicates the approximate nature of the strong principle of equivalence laying in its basis.

On the other hand, as argued Logunov in [1] starting from the Emmy Noether theorem [6] (see §6), the general theory of relativity with its Riemannian geometry of space-time admits no the formulation of energy conservation law (this fact was established by David Hilbert immediately after the advent of this theory [5, pp. 16–17]), as well as of all other fundamental special relativistic conservation laws. This fact not only exposes the limited validity of Einstein's theory of gravitation but also makes doubtful its suitability as a fundamental physical theory of one of the fourth existing fundamental interactions. It should be recalled that for this reason, Logunov and his co-authors back in the 1980s proposed to abandon the general theory of relativity replacing it with a suitable special-relativistic theory of force field (see [1], [7]–[10]).

Looking back in time at several decades of unsuccessful attempts to reconcile the general theory of relativity and quantum theory, we do not find gravity in the short list of fundamental interactions unified within the modern Standard Model of particles and fields. Recall in this connection that this was the heroic time of the last century when quantum electrodynamics, electro-weak unification, quantum chromodynamics, and finally the Standard Model of particles that joined together known fundamental interactions except gravitational, were successfully developed. Steven Weinberg is undoubtedly right in his asserting that "the geometrical approach has driven a wedge between general relativity and the theory of elementary particles" (quoted from the Preface in [11]).

It should be added that the absence in general relativity of an unambiguous positive-definite expression of the energy density of the gravitational field significantly limits the effectiveness of this theory in specific physical applications and makes it unsuitable for solving mass-energy problems presently accumulated in the modern cosmology. Today this circumstance is manifested in the disappointing fact of the absolute helplessness of general relativity in explaining the role of the gravitational interaction in the appearance of so-called "Pioneer anomaly", in solving the problem of "missing mass", in clarification of the nature of "dark energy" which astronomers and cosmologists have encountered more than twenty years ago (a brief remark on the last subject see also in [12, pp. 83–84]).

Physically ridiculous conclusions regarding the concept of mass-energy that arise within the framework of general relativity but beyond the domain of its applicability, are fatal to the theory itself. A very striking example of this is a well-known speculation of Misner, Thorne, and Wheeler around the law of energy conservation in the presence of gravity. In their comprehensive book [13], hiding the failures of Einstein's theory with the laws of energy-momentum conservation under the invented for this case imaginary nonlocalizability of the energy of the gravitational field, we find the conclusion that, if spacetime is not flat at infinity, then, according to general relativity, "one must completely abandon <...> the total mass-energy of the gravitating source", which in this case "is a limited concept" (see page 463).

But authors of [13] themselves do not notice that if the energy of gravitational field is illusive in reality, then the general theory of relativity itself must be considered as a limited concept. Indeed, the hypothesis of equivalence of two masses, inertial and gravitational, is the starting point of any theoretical model of gravitation. Of course, both concepts of these masses should have a clearly defined physical meaning, whereas in the stipulated case we "must completely abandon" [13] at least one of them. In above-mentioned work [7], Denisov and Logunov speak directly about this fact as an internal contradiction of Einstein's theory ofgravitation. They noted the physically meaningless dependence of the result of calculating the inertial mass of the gravitating physical system on the choice of curvilinear three-dimensional coordinates, demonstrated, for example, in [13]–[17]. This fact surely excludes any reasoning about the relationship between the gravitational mass of any body and its inertial mass whose definition is indistinct or do not appear at all. The peak of this absurdity is a non-zero "energy density" of flat empty space resulting from the use of the metric tensor corresponding to the polar coordinates (see original calculations in [18] that give an infinite total "energy"; their comments can be found in [14, Section 61]).

The experience of the application of Einstein's theory of gravitation to the universe led the modern cosmology to the unphysical domain of mythical concepts, such as exploding universe, inflaton, dark mass, dark energy, quintessence, and the creation of universes from nothing. It teaches us that the nature cannot be held hostage to any physical theory, even if it, similar to the general theory of relativity, possesses "the beauty and elegance", according to Dirac [19], and their equations are considered "the greatest achievements of human genius" in words of Fock [20]. As Feynman reminded us in a different context that, "If we find that certain mathematical assumptions lead to a logically inconsistent description of Nature, we change the assumptions, not Nature" [21, Section 13.3].

The unconventional skeptical opinion of Steven Weinberg on the role of geometric ideas in gravity, expressed by him in [11] (see Preface, pp. vii–viii and Section 6.9), stimulated our resolve, completely abandoning Einstein interpretation of the equivalence principle, to try to construct a model of gravitation capable of solving the dark energy problem in the universe and will allow us to take a fresh look at the other problems accumulated today in cosmology.

In this paper, we propose one of the possible dynamical extensions of Newtonian static gravity developed within the constraints followed from the well-known fundamental principles underlying the majority of modern physical theories. This special-relativistic gauge-invariant generalization of the theory of static gravity coincides with the canonical linear theory of a spinless massive field, but it is characterized by a number of features emerged beyond the standard model of particles and fields, specific only of this, fourth type of fundamental interactions. Fully trusting and relying in this way on the fundamental principles of modern theoretical physics, we will try in this paper again to answer the problem questioned 75 years ago by Hermann Weil: "*How far can one get with a linear field theory of gravitation in flat space-time*?" [22]. In order to do this without the risk of "throwing the baby out with the bath water", we had first to overcome the doubts associated with very known experimental tests, and, above all, with the displacement of the Mercury's perihelion and the gravitational deflection of the light beam as it was until now with respect to the scalar and other models of gravitation. We postpone the solution and interpretation of these tests until better times, when we can take into account the cosmological effects associated with the background gravitational field.

In the subsequent parts of this work, relying solely on the proposed gauge-invariant model of a massive scalar gravitational field in combination with the cosmological principle, we arrive at an alternative scenario of the universe evolution, which is in perfect agreement with the old and new astronomical observations. We will also show that the proposed scalar model of gravity rejects the cosmological expansion and clears the science of the universe from the hundred-year layering of *ad hoc* hypotheses that are far from real physics.

1 Nordstroem's mechanics

In our hope to understand the real physics of evolutionary processes in the universe we are faced with a lack of necessary for this a consistent dynamic theory of a coupled to mass attractive field, which, similar to Maxwell's electrodynamics, would be compatible with a clear formulation of the law of conservation of energy *with a positive-definite energy density*. As was explained in detail by Misner, Thorne, and Wheeler [13], the strong equivalence principle, or ultimately the Riemannian geometry of pseudo-Euclidean spacetime, prevents the existence in general relativity of a fully functional energy conservation law in any dynamical system where a gravitational field is taken into account. The disappointing conclusion on the "nonlocalizability" of the energy of the gravitational field tell us that this theory is quite useless to investigate the participation of all the gravitating matter in the universe in the formation of the background energy associated with the collective gravitational field. For this reason, in order to exclude the adjective "dark" from the terms "dark energy" and "dark matter" firmly entrenched in modern cosmology, the replacement of general relativity in future cosmological applications with a field theoretical model of gravity, which would contain a clearly formulated law of energy conservation, and necessarily with positive-definite energy density of the gravitational field, seems to be the only productive recommendation. It turns out that the only way to understand the energetic evolutionary processes of the universe is to develop the theory of the attractive field with a positive definite energy density is to use a scalar field.

In order to prevent in what follows the troubles with the disappearance of the laws of conservation associated with the Poincaré group symmetry, we emphatically assume that the physical space-time is special-relativistic, that is *a priori* flat, both locally and globally. We will use in this case the Minkowski metric tensor $\eta^{\mu\nu}$ with signature $(-,+,+,+)$. Therefore the line element ds will be determined in terms of the four-dimensional coordinate differentials

$$dx^{\mu} = (cdt, dx^{i}) = (cdt, d\mathbf{r}), \qquad (1.1)$$

corresponding to an infinitesimal displacement in space-time, by the expression

$$ds^2 = -\eta_{\mu\nu} dx^{\mu} dx^{\nu} = c^2 dt^2 - \delta_{ij} dx^{i} dx^{j}. \qquad (1.2)$$

We next proceed to derive the equation of motion of classical particles in the presence of a scalar field coupled to their mass. For this purpose we use the Poincare-invariant spatial density of the joined Lagrangian $\mathscr{L}_{\text{mattr}} = \mathscr{L}_0 + \mathscr{L}_{\text{int.}}$ of gravitationally interacting particles (dustlike matter) given by the expression

$$\mathscr{L}_{\text{mattr}} = -c^2 \sum_a m_a \sqrt{1 - \frac{v_a^2}{c^2}} \phi^2 \delta^{(3)}(\mathbf{r} - \mathbf{r}_a), \qquad (1.3)$$

where the "nominal" mass m_a of individual particle (its gravitational charge) and the primary field variable ϕ are "non-minimally" coupled. For a single particle in the presence of a gravitational field from the density (1.3), it follows the joined Lagrangian, which we shall write down in the form [23]:

$$L_{\text{p.}} = -c^2 m \phi^2 \sqrt{1 - \frac{v^2}{c^2}}. \qquad (1.4)$$

It is to be noted here that the use by Einstein and Fokker in [24] of the conformally flat metric ($g_{\mu\nu} = \phi^4 \eta_{\mu\nu}$ – in our notation) is not quite equivalent to the Lagrangian formalism used below. This is because the metric approach fails when $\phi \to 0$, so that the Riemannian manifold of spacetime degenerates into a point, if we are talking, for example, about a background metric.

The practice in manipulating a scalar field coupled to the mass each time gave rise different authors to a Lagrangian very similar to (1.4) with multiplicative inclusion of the interaction in the Lagrangian of a free particle. Naturally, in all these cases, the same form of the equation of motion of the test particle in the external gravitational field was reproduced. Thus, independently of the specific definition of the basic field variable and the form of the field equations (usually non-linear) proposed occasionally by different authors, which tried to develop the theory of gravity within the scalar approach, the motion of a test particle in the scalar gravitational field obeys the equation that was obtained for the first time by Nordström in [25]–[27].

Now we can derive Nordström's equation of motion of every individual particle from the principle of least action by varying the appropriate action. Thus, as a result of the using of Lagrangian (1.4), we get a covariant four-dimensional form of the equation of motion of a particle in an arbitrary external gravitational field g^{μ}:

$$c^2 \frac{d(m\phi^2 u^{\mu})}{ds} = m\phi^2 g^{\mu}. \qquad (1.5)$$

In this equation, $u^{\mu} = dx^{\mu}/ds$ represents the dimensionless four-vector of the velocity of a particle. We have also introduced here the strength four-vector g_{μ} of the gravitational field, defined in terms of logarithmic derivatives of ϕ with respect to the space-time coordinates by the connection

$$g_{\mu} = -2c^2 \frac{1}{\phi} \partial_{\mu} \phi. \qquad (1.6)$$

As a reminder of this circumstance, we will sometimes call the field variable ϕ the *logarithmic potential*.

Along with the initial field variable ϕ, it is convenient to introduce the another dynamical characteristic of the field by means of the equality

$$\phi^2 = e^{\Phi/c^2}. \qquad (1.7)$$

Then the strength four-vector g_{μ}, in agreement with its previous representation in form (1.6), may be expressed as an ordinary gradient of this new scalar field variable:

$$g_\mu = -\partial_\mu \Phi. \quad (1.8)$$

So, this relation defines Φ as the *usual potential* of the vector force field g_μ, which can be used instead of the logarithmic potential ϕ of this field, determined by (1.6).

The four-dimensional equation (1.5) covers both the Lagrange's equation for momentum **p** canonically conjugated to the radius vector **r** and the equation for the energy $\mathscr{E}$ of a particle:

$$\frac{d\mathbf{p}}{dt} = \frac{\partial L_p}{\partial \mathbf{r}}, \; \frac{d\mathscr{E}}{dt} = -\frac{\partial L_p}{\partial t}, \quad (1.9)$$

where as usual,

$$\mathbf{p} = \frac{\partial L_p}{\partial \mathbf{v}} \text{ and } \mathscr{E} = \mathbf{v}\frac{\partial L_p}{\partial \mathbf{v}} - L_p. \quad (1.10)$$

From (1.5), it follows that we deal here with the field-dependent physical quantity

$$\tilde{m} = m\phi^2, \quad (1.11)$$

which appears in this equation simultaneously in two physical meanings: as the inertial mass of the particle (on the left) and as its passive gravitational mass (on the right). Thus, the *actual* inertial mass $\tilde{m}$, which, in accordance with (1.4) and the second equation in (1.10), determines the gravitationally dependent content of energy

$$\mathscr{E} = c^2\tilde{m}$$

stored in a particle at rest, is a dynamical variable. Whereas its *nominal* mass m remains the constant which is the measure of as much as possible amount of energy that a given particle or body can accommodate. On the other hand, in the case of elementary particles, the constant parameter m represents the gravitational analogue of charge, which characterizes the individual susceptibility of a certain sort of particles to the influence of the "mass-coupled" field ϕ.

The Lorentz-covariant equation of motion (1.5), expressed for the space and time components separately in the form

$$\frac{d}{dt}\left(\frac{\tilde{m}\mathbf{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \tilde{m}\sqrt{1-\frac{v^2}{c^2}} \cdot \mathbf{g}, \quad (1.12)$$

$$\frac{d}{dt}\left(\frac{\tilde{m}c^2}{\sqrt{1-\frac{v^2}{c^2}}}\right) = c\tilde{m}\sqrt{1-\frac{v^2}{c^2}} \cdot Q, \quad (1.13)$$

was first postulated by Nordström in [25]. In these formulas we have introduced the notation

$$\mathbf{g} = -2c^2\frac{1}{\phi}\nabla\phi, \quad (1.14)$$

$$Q = 2c\frac{1}{\phi}\frac{\partial\phi}{\partial t}, \quad (1.15)$$

for the two three-dimensional force characteristics of the gravitational field, its strengthes: vector **g** and scalar Q. These two field observables constitute the four-vector (1.6) of a field strength:

$$g^\mu = (Q, \mathbf{g}). \quad (1.16)$$

We note in passing that, in connection with the discussion of the second Nordström's theory of gravitation, equation (1.12) was also obtained by Einstein in [28] from the principle of least action.

After some simple transformations and elimination of potential by dividing by ϕ^2, the Nordström's equation (1.5) can be rewritten as the equation, containing the constant nominal mass m only. Indeed, having in mind (1.6), we get in this way the equation

$$c^2 m\frac{du^\mu}{ds} = m(\delta^\mu_\nu + u^\mu u_\nu)g^\nu, \quad (1.17)$$

where constant nominal mass m can be canceled. Thus, we arrive at an equation of motion that does not depend on the nominal mass of the particle, as it should.

It must be emphasized that, since the transition from (1.5) to (1.17) is accompanied by the division of the original equation by ϕ^2, these equations are equivalent with the exception of the cases when ϕ passes through zero. It is clear that if $\phi \to 0$, then the particle completely loses its inertial mass m^* and accelerates to the speed of light. In this case, similar to photon, the description of its motion by classical equations (1.5) or (1.17) loses the physical meaning.

From (1.17) we see that the necessary condition of orthogonality

$$u_\mu\frac{du^\mu}{ds} = 0, \quad (1.18)$$

which arises by virtue of the equality

$$u^\mu u_\mu = -1, \quad (1.19)$$

is satisfied identically.

The spatial part of (1.17) reduces to the equation of motion in the form [23]

$$\frac{d}{dt}\left(\frac{m\mathbf{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}\left(\mathbf{g} + \frac{1}{c^2}\mathbf{v}\times\mathbf{v}\times\mathbf{g} - \frac{1}{c}\mathbf{v}Q\right), \quad (1.20)$$

which can also be obtained directly from (1.12) by using (1.6) and (1.11). Inspection of this equation shows that, in favor of the principle of equivalence of gravitation and inertia, both the inertia of a moving particle and its susceptibility to the influence of gravitational field are determined by the same factor $m/\sqrt{1-v^2/c^2}$.

By means of some obvious transformations, the relativistic equation of motion (1.20) can be rewritten in the simpler equivalent form:

$$\frac{d\mathbf{v}}{dt} = \left(1 - \frac{v^2}{c^2}\right)\left(\mathbf{g} - \frac{1}{c}\mathbf{v}Q\right), \quad (1.21)$$

where the first factor in parentheses on the right-hand side restricts the increase of the velocity of a particle making the light velocity insurmountable.

For velocities small compared with the velocity of light, both equations (1.20) and (1.21) go over into the simple Newton's equation of motion of a particle in the external gravitational field, which looks like the equation of an ordinary mass-independent free fall with the acceleration $\mathbf{g}$:

$$\frac{d\mathbf{v}}{dt} = \mathbf{g}.$$

Thus in the non-relativistic approximation, the three-dimensional vector $\mathbf{g}$ formed by the spatial components of the four-vector (1.16) corresponds to the usual gravitational acceleration of free fall.

It is useful to compare (1.21) with an analogous equation of motion of a test particle in the vector model of gravitation, which reads

$$\frac{d\mathbf{v}}{dt} = \sqrt{1 - \frac{v^2}{c^2}}\left(\mathbf{g} - \frac{1}{c^2}\mathbf{v}(\mathbf{v}\cdot\mathbf{g}) + \frac{1}{c}\mathbf{v}\times\mathbf{h}\right). \quad (1.22)$$

It can be obtained from the known equation of motion for a charged particle in electromagnetic field resolved with respect to acceleration (see, for example, the problem in the end of §17 in [29]) after replacing electric charge e of a particle by its "gravitational charge" m and electromagnetic "six-vector" of strengthes $(\mathbf{E}, \mathbf{H})$ by its gravitational analog $(\mathbf{g}, \mathbf{h})$. The difference in relativistic free motion of a test particle in the static gravitational field $\mathbf{g}$ in a scalar model of gravitation and in the Maxwellized gravity is striking. To see this, it suffices to compare two equations: the first, appeared in Nordström mechanics,

$$\frac{d\mathbf{v}}{dt} = \left(1 - \frac{v^2}{c^2}\right)\mathbf{g},$$

and the second, associated with the supposed Maxwell – Lorentz-like gravitational force,

$$\frac{d\mathbf{v}}{dt} = \sqrt{1 - \frac{v^2}{c^2}}\left(\mathbf{g} - \frac{1}{c^2}\mathbf{v}(\mathbf{v}\cdot\mathbf{g})\right)$$

that follow from (1.21) and (1.22) respectively, if we set $Q = 0$ and $\mathbf{h} = 0$. The similar characteristic difference between the conclusions of the scalar and vector models of gravity was noted earlier by Norton [30].

2 Gauge invariance of the massive scalar field

The equation of the motion of a test particle in an external variable gravitational field, especially expressed in the simple form (1.21), clearly shows that all the components of the four-dimensional strength vector $g^{\mu} = (\mathbf{g}, Q)$ are the physical characteristics of the field that are accessible to direct measurement. Ignoring almost insurmountable difficulties of certain measurements due to the extreme weakness of the corresponding effects, we shall content ourselves here with the understanding that these measurements can be performed, at least in principle, by measuring the velocity and acceleration of the test particle. This means that both the force characteristics of the field, its strengths $\mathbf{g}$ and Q, which directly affect the motion of massive particles, are the conditionally measurable, that is, uniquely determined physical quantities.

Unlike the field observables g^{μ}, the scalar function ϕ, which appeared in Lagrange formalism in (1.4) as original field variable, is not completely unique. This can be easily seen from (1.6), or, equivalently, from (1.14) and (1.15). Indeed, for a given gravitational field comprehensively determined by set of measurable quantities $\mathbf{g}$ and Q, their logarithmic potential ϕ is determined with the help of these relations to within an arbitrary non-zero coefficient. From (1.14) and (1.15) it follows that the generalized scale transformation

$$\phi \to \phi' = \kappa\phi \quad (2.1)$$

with an arbitrary non-zero, both *positive* or *negative*, constant κ does not change both field observables $\mathbf{g}$ and Q and leaves unchanged the equation of particles motion. This important fact allows us to consider the admissible transformation (2.1) of the field variable ϕ as a *special form of gauge transformations* for the mass-coupled field of zero spin.

Here it is useful to pay special attention to the fact that, in addition to the non-uniqueness of ϕ, the transformation (2.1) also corresponds to a non-unique choice of the usual field potential Φ defined for a given strength four-vector g_{μ} by (1.8). Indeed, using (1.7), it is easy to see that the multiplicative transformation (2.1) of the field variable ϕ corresponds to the additive transformation of the potential,

$$\Phi \to \Phi' = \Phi + \xi, \quad (2.2)$$

with an arbitrary finite constant ξ, as it should in view of equation (1.8). Here it is assumed that by virtue of the relation (1.7) two arbitrary constants, ξ and κ, appearing in (2.1) and (2.2) are related to each other by the equality $\xi = c^2 \ln \kappa^2$.

We recall that a scale transformation of the field variable very closely analogous to (2.1) was established earlier as a special form of the gauge transformation of a scalar field in the refined *second* Nordström's theory of gravitation [26], [27] by Dicke [31], and then by Wellner and Sandri [32]. But much early, this problem was treated in more detail by Bergmann [33], who also operated with a scalar potential, logarithmic in the sense of definition (1.6). The Lorentz-invariant equation for this field variable in [33] is not invariant with respect to its a simple, like (2.1), scale transformation. Bergmann eliminated this non-invariance by a suitable additional re-scaling of the four-dimensional coordinates. Applied together, these two types of scale transformations mutually compensate each other leaving the field equation unchanged. As shown in [33], such joined transformation forms the gauge group, with respect to which Nordström's second theory of the gravitational field remains invariant. This fact means ultimately that, in accordance with such a combined solution of the gauge problem in Nordström's theory, we need to stretch or shrink the time and distance scales appropriately to the shift in the origin of the ordinary gravitational potential Φ similar to (2.2).

In contrast to the foregoing approach to the problem, we will seek the field equations which are themselves invariant with respect to the scale transformation (2.1) of field function ϕ. Consequently, they will be automatically invariant with respect to the admissible shift transformation (2.2) of the potential Φ. In this case, one would like to hope that there is a way to construct a much more preferable theory of gravity, which would be insensitive to our freedom to redefine the unit of mass. Of course, this possibility should also include changing the units of energy, momentum, force and other related physical quantities along with gravitational and Planck constants. At the same time, unlike the method used by Bergmann in [33], we would not like to associate such transformations with the freedom to choose units of time and distance trying to avoid the geometric interpretation of gravity leading to troubles with conservation laws.

The case of a similar special relativistic scalar field theory, gauge-invariant in the stated sense, but only massless, was considered fairly in detail in [23]. It was established that the implementation of the intention to develop a viable minimal dynamical extension of Newton's static gravity within the restrictions dictated by the standard principles and requirements of the classical field theory is indeed possible. Among them are the principle of least action, the principle of simplicity, the Lorentz covariance and the gauge invariance of the theory, the existence of a clear formulation of the special-relativistic conservation laws of energy, momentum, angular momentum, and center-of-mass motion together with the Maxwell's principle of positive definiteness of field energy density, and finally the Newton's principle of proportionality of the inertial and gravitational masses. Under the conditions, when direct crucial experiments are ineffective, the use of these principles, accumulated by modern theoretical physics beyond the limits of gravitational phenomena themselves, is of decisive importance for constructing a self-consistent theoretical model of gravitational interaction acceptable from the viewpoint of the existing physics of particles and fields.

A characteristic feature of the variational principle in the derivation of the equation of motion of a particle which mass is coupled to a spin-zero field ϕ is the scale transformation $\mathcal{S}_{\text{mattr}} \to \mathcal{S}'_{\text{mattr}} = \kappa^2 \mathcal{S}_{\text{mattr}}$ of the action (1.3) under the gauge transformation (2.1) of the field. We recall that such transformation of an action is admissible in the general case as one of the manifestations of the non-uniqueness of the Lagrangian (see, e.g., [34, p. 291], or [35, Section 6.3]). The Euler – Lagrange equations, being linear and uniform with respect to the Lagrangian, are, of course, invariant under this transformation. We also recall that such a non-uniqueness of the definition of the Lagrangian of a free particle corresponds to the natural freedom of choice of a unit of mass and, as a consequence, units of other physical quantities related to the mass (for more details, see [29, §27] and [36, pp. 4–8]).

According to the principle of gauge invariance, the desired equations describing a total system of interacting matter and field must also be invariant with respect to the scale transformation (2.1). In order to satisfy this condition, we must require the fulfilment of the scale transformation of a *total action*,

$$\mathcal{S} \to \mathcal{S}' = \kappa^2 \mathcal{S}, \tag{2.3}$$

under the admissible gauge transformation (2.1) of the logarithmic potential of the field.

The gauge invariance of the action itself is of fundamental importance in the formulation of modern physical theories. Its fulfillment, for example, is the starting point in the proof of Noether's theorem. The absence of this invariance, as shown by equation (2.3), should not be of particular concern, since in the present case this defect is easily eliminated. To do this, it suffices to rewrite the action in absolute units as a dimensionless quantity, dividing it by the Planck constant. (It is interesting that the very existence of a fundamental physical constant with the dimension of action guarantees the invariance of Lagrangian theories with respect to the scale transformations of all physical quantities, thereby ensuring the freedom of choice of their units.)

This possibility becomes obvious if we note that the transformation (2.1) in addition to the change in the unit of measurement of the inertial mass $\tilde{m} = m\phi^2$ entails a change in the numerical values of other physical quantities, including energy, momentum, Lagrangian, and the Planck constant. In all these cases, because of (2.1), the square of the gauge parameter κ plays the role of the conversion factor, so that for the transformation of the numerical value of Planck constat associated with gauge transformation (2.1), we can write

$$\hbar \to \hbar' = \kappa^2 \hbar. \tag{2.4}$$

Nevertheless, the Planck's quantum of action should be considered as a fundamental true physical constant. But because this constant has a dimension of action, the substitution (2.4) should be considered no more than recalculation of its numerical value. Such a recalculation indeed is necessary, since, according to (1.11), the gauge transformation (2.1) causes the replacement

$$\tilde{m} \to \tilde{m}' = \kappa^2 \tilde{m},$$

that is, the transition to a new system of units of physical quantities with a new unit of the inertial mass.

3 Gauge-invariant model of a massive spinless gravitational field

It is easy to see that the required property (2.3) of total action $\mathcal{S} = \mathcal{S}_{\text{mattr}} + \mathcal{S}_{\text{field}}$ will be satisfied if, obeying the principle of simplicity, we get the quadratic in the field total Lagrangian density for a system of interacting classical particles and a massive scalar field:

$$\mathscr{L} = \mathscr{L}_{\text{mattr}} - \frac{c^4}{2\pi G_N}(\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \varkappa^2\phi^2). \quad (3.1)$$

Here the spatial density of the Lagrangian of gravitationally interacting classical particles (dustlike matter) is given by the expression (1.3).

We have introduced in (3.1) a coefficient containing usual Newton's gravitational coupling constant G_N, so that the field variable ϕ remains dimensionless and the theory would lead to the Newtonian static limit in conventional notation. These expressions are identical to those presented in [23], except that (3.1) contains a standard term associated with mass $m_g = \hbar\varkappa / c$ of a spinless graviton.

The field equation obtained by varying the action, corresponding to the expressions (3.1) and (1.3), with respect to the field variable $\phi(x)$ is linear and uniform:

$$\left(\Box - \varkappa^2 - \frac{2\pi G_N}{c^2}\Theta\right)\phi = 0. \quad (3.2)$$

Here $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is the D'Alembertian operator. The source density of the field, denoted in this equation by Θ, is connected with the mass of gravitating particles by the relation

$$\Theta = \sum_a m_a\sqrt{1 - \frac{v_a^2}{c^2}}\,\delta^{(3)}(\mathbf{r} - \mathbf{r}_a). \quad (3.3)$$

This expression is a four-dimensional scalar, as it should. It is known as the trace (divided by c^2) of the energy-momentum tensor of a non-gravitating dustlike system of free particles (see, e.g., [29, §34]).

As was to be expected, the field equation (3.2) remains unchanged with respect to the gauge (scale) transformation (2.1) of the field variable with constant non-zero parameter κ. It should also be noted that the equation (3.2), being uniform and linear, satisfies the requirement of simplicity.

Using (1.6), equation (3.2) can also be written directly for the field strength vector g_μ. Expressing [in accordance with (1.6)] the four-dimensional gradient of logarithmic potential ϕ in (3.2) in terms of the field g_μ as $\partial_\mu\phi = -(1/2c^2)\phi g_\mu$, and then dividing the resulting equality by ϕ, we obtain

$$\partial_\mu g^\mu = \frac{1}{2c^2}g_\mu g^\mu - 2c^2\varkappa^2 - 4\pi G_N\Theta. \quad (3.4)$$

Recall also that the vector field g_μ satisfies the potentiality condition (1.8). In view of this fact, we have additionally the homogeneous linear tensor equation

$$\partial_\mu g_\nu - \partial_\nu g_\mu = 0. \quad (3.5)$$

This equation, like the first pair of Maxwell equations in electrodynamics, is satisfied regardless of the nature of the field source, filtering out physically unacceptable solutions to the basic equation (3.4).

The last two field equations together with equation (1.17) of the motion of a massive particle in a gravitational field constitute a complete system of Lorentz-covariant and gauge-invariant equations that are compatible with the laws of conservation of energy and momentum with a positive gravitational energy density [37]. We will use them in what follows to describe the gravitational field and its interaction with matter in the framework of the special theory of relativity, primarily for cosmological applications. Of course, the field equation (3.2) retains its practical importance: to solve the nonlinear equation (3.4) satisfying the condition (3.5), it is convenient to first obtain a solution of a simpler linear equation (3.2), and then use the relation (1.6), or two relations (1.14) and (1.15) separately.

In the case of resting gravitating masses distributed in space with density $\varrho(\mathbf{r})$, the equation (3.4) reduces to the non-linear relativistic generalization of the equation of Newtonian static gravity,

$$\nabla\cdot\mathbf{g} = \frac{1}{2c^2}\mathbf{g}^2 - 2c^2\varkappa^2 - 4\pi G_N\varrho, \quad (3.6)$$

which differs from the equation obtained early in [23] by having the term $-2c^2\varkappa^2$ associated with the mass m_g of graviton. This equation is accompanied by a supplementary condition, which follows from (3.5), that $\mathbf{g}$ is a potential field, that is, "curl-free":

$$\nabla\times\mathbf{g} = 0. \quad (3.7)$$

We note, incidentally, that in the case of massless field, the same as (3.6) nonlinear equation for the vector field strength $\mathbf{g}$ outside the region where the masses producing this field are located, that is, in space where $\varrho = 0$, was manufactured by Brillouin [38]. In four-dimensional form, like (3.4) but without two last terms, similar equation with quadratic self-action was constructed by Deser and Halpern [39] (see Appendix in their paper).

Somewhat later, almost this equation appeared also in Hooft's lectures [40] (see page 13). (The Hooft's equation is transformed into the Brillouin's equation by a simple replacing $\mathbf{g} \to -\mathbf{g}$.)

Returning to the equation (3.4) it should be stressed that under the conditions of the evolving universe the restriction $\varkappa = 0$ is not necessary at all, in order to obtain the Newtonian behavior of static field at large distances from a gravitating body. Although the search for solutions of the field equations presented here is beyond the scope of this paper, we note that in a homogeneous non-expanding universe filled with gravitating dust matter and the background gravitational field created by it, the local post-Newtonian static field is described by equation (3.6) *but without the mass term quadratic in* $\varkappa$, even when the mass of gravition does not vanish in the basic field equation (3.2) and (3.4), from which we start. In this case, a spherically symmetric solution of equation (3.6), (3.7) for a static field outside a central body, as shown in [38] and [23], for large distances has an asymptotic behavior corresponding to the Newtonian inverse square law. As for the role of the mass parameter $\varkappa$ in general equtions (3.2) and (3.4), it participates in the formation of a slowly evolving spatially homogeneous background field Q.

Finally, we also note that, in contrast to the Lagrangian formalism of scalar-tensor theories, where, along with the tensor gravitational field, a scalar field is often used to parameterize dark energy, as, for example, in [41]–[43], our total Lagrangian (3.1) is quadratic and homogeneous with respect to the dynamic variable ϕ of the field and its four-dimensional gradient $\partial_\mu\phi$. For this reason, our field equation (3.2) is linear and gauge-invariant in the sense of scale transformation (2.1), as it should. (This circumstance, by the way, makes the usual procedure for quantizing the gravitational field an almost trivial task.) Thus, Lagrangian (3.1) generates an initially linear field equation that clearly does not contain signs of any self-action, the fact that is usually considered self-evident for a gravitational field. Nevertheless, as seen from our considerations, the nonlinear terms quadratic in the components of the field strength g^μ, which can be interpreted as a necessary gravitational "self-action", appear in the transformed post-Newtonian equation (3.4) (or (3.6) in the static limit), represented in terms of the field strengthes $g^\mu = (Q, \mathbf{g})$.

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