On the $\mathfrak{F}$-hypercenter of hereditary $\check{S}$-formations

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All groups considered here will be finite. Recall that a formation $\mathfrak{F}$ is called a formation with the Shemetkov property if all s-critical for $\mathfrak{F}$-groups are Schmidt groups or groups of prime order. A. N. Skiba [**1**] showed that every hereditary formation with the Shemetkov property in the universe of all soluble groups is saturated. There are examples of hereditary non-saturated formations with the Shemetkov property (see [**2**]). According to [**3**] every hereditary formation with the Shemetkov property is solubly saturated (composition, Baer-local).

Recall that a subgroup U of G is called $\mathfrak{X}$-*maximal* in G provided that (a) $U \in \mathfrak{X}$, and (b) if $U \leq V \leq G$ and $V \in \mathfrak{X}$, then $U = V$. The symbol $\mathrm{Int}_{\mathfrak{X}}(G)$ denotes the intersection of all $\mathfrak{X}$-maximal subgroups of G. A a chief factor H/K of G is called $\mathfrak{X}$-*central* in G provided $H/K \rtimes G/C_G(H/K) \in \mathfrak{X}$. The symbol $\mathrm{Z}_{\mathfrak{X}}(G)$ denotes the $\mathfrak{X}$-*hypercenter* of G, that is, the largest normal subgroup of G such that every chief factor H/K of G below it is $\mathfrak{X}$-central.

L. A. Shemetkov possed the following question on Gomel Algebraic seminar in 1995 "For what non-empty normally hereditary solubly saturated formations $\mathfrak{X}$ do the equality $\mathrm{Int}_{\mathfrak{X}}(G) = \mathrm{Z}_{\mathfrak{X}}(G)$ hold for every group G?" The solution to this question for hereditary saturated formations was obtained by A. N. Skiba in [**4**] (for the soluble case, see also [**5**]) and for the class of all quasi-$\mathfrak{F}$-groups, where $\mathfrak{F}$ is a hereditary saturated formation, was given in [**6**]. In particular, the intersection of maximal quasinilpotent subgroups is the quasinilpotent hypercenter. Note that methods of [**4, 5**] don't very useful in cases of non-saturated or non-hereditary formations. Let $\sigma = \{\pi_i \,|\, i \in I\}$ be a partition of the set of all primes $\mathbb{P}$ into mutually disjoint subsets. Recall that $\times_{i\in I}\mathfrak{G}_{\pi_i} = (G \,|\, \mathrm{O}_{\pi_i}(G)$ is a Hall π_i-subgroup of G for all $i \in I)$.

Theorem. *Let $\mathfrak{F} \neq (1)$ be a hereditary formation with the Shemetkov property. Then $\mathrm{Z}_{\mathfrak{F}}(G) = \mathrm{Int}_{\mathfrak{F}}(G)$ holds for every group G if and only if there is a partition $\sigma = \{\pi_i \,|\, i \in I\}$ of $\mathbb{P}$ into mutually disjoint subsets such that $\mathfrak{F} = \times_{i\in I}\mathfrak{G}_{\pi_i}$.*

References

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