On hereditary Baer–Shemetkov formations of finite groups

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All considered groups are finite. L.A. Shemetkov possed the following problem on Gomel Algebraic seminar in 1995: *For what non-empty* (*normally*) *hereditary* (*local*, *Baer-local*) *formations* $\mathfrak{F}$ *do the intersection of* $\mathfrak{F}$*-maximal subgroups coincide with the* $\mathfrak{F}$*-hypercenter in any group ?*

The origin of this problem may be traced back to the R. Baer's result [**1**] which states that the intersection of maximal nilpotent subgroups coincides with the hypercenter in every group. Different cases of this problem were considered in [**2, 3, 4**].

We shall say that a formation $\mathfrak{F}$ is a Baer-Shemetkov formation in a class $\mathfrak{X}$ if for every $\mathfrak{X}$-group the intersection of all its $\mathfrak{F}$-maximal subgroups coincides with its $\mathfrak{F}$-hypercenter. If $\mathfrak{X}$ is the class of all groups, then we shall say that $\mathfrak{F}$ is a Baer-Shemetkov formation.

Using the notions of the N-critical $\Gamma_{Nc}(\mathfrak{X})$ graph of a class $\mathfrak{X}$ from [**5**] and of a Z-saturated formation from [**6**] we obtain the following.

Theorem. *Let* $\mathfrak{F} \neq (1)$ *be a hereditary formation. The following statements are equivalent.*

(1) $\mathfrak{F}$ *is the Baer-Shemetkov formation.*
(2) *The following conditions hold:*
  (2.1) $\mathfrak{F}$ *is a* Z*-saturated formation.*
  (2.2) *There exists a partition* $\sigma = \{\pi_i \mid i \in I\}$ *of the set of all primes* $\mathbb{P}$ *such that* $\Gamma_{Nc}(\mathfrak{F})$ *is the union of complete directed graphs on the vertex sets* $\pi_i, i \in I$*.*
  (2.3) $\mathfrak{F}$ *is a Baer-Shemetkov formation in the class of all* π_i*-groups for every* $i \in I$*.*

References

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