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МАССИВНОЕ ГРАВИТАЦИОННОЕ ПОЛЕ В ПЛОСКОМ ПРОСТРАНСТВЕ-ВРЕМЕНИ. II. ЗАКОНЫ СОХРАНЕНИЯ И ГРАВИТАЦИОННАЯ ИЗМЕНЧИВОСТЬ ИНЕРТНОЙ МАССЫ

М.А. Сердюкова, А.Н. Сердюков

Гомельский государственный университет им. Ф. Скорины

A MASSIVE GRAVITATIONAL FIELD IN FLAT SPACETIME. II. CONSERVATION LAWS AND GRAVITATIONAL VARIABILITY OF THE INERTIAL MASS

M.A. Serdyukova, A.N. Serdyukov

F. Scorina Gomel State University

Построен канонический тензор энергии-импульса и сформулированы законы сохранения энергии и импульса линейного массивного беспинового поля, адаптированного в качестве калибровочно-инвариантной модели гравитации в рамках специальной теории относительности. Показано, что общее требование положительной определенности плотности энергии любой физической реальности в случае гравитационного поля предопределяет его скалярную природу и далеко идущую гравитационную изменчивость инертной массы частиц материи.

Ключевые слова: *массивная скалярная гравитация*, *энергия гравитационного поля*, *гравитационная изменчивость массы.*

For a linear massive spinless field adapted as a gauge-invariant model of gravity in the framework of the special theory of relativity, a canonical energy-momentum tensor is constructed and the laws of conservation of energy and momentum are formulated. It is shown that the general requirement of a positive definiteness of the energy density of any physical reality in the case of an attractive field determines its scalar nature and far-reaching gravitational variability of the inertial mass of matter particles.

Keywords: *spinless massive gravity*, *gravitational energy*, *variable mass.*

Introduction

By all the criteria of the modern Standard Model of particle physics, the depressing fact of the absence of ten special relativistic integrals of motion in the general theory of relativity is the worst thing that should have happened with a physical theory in order to abandon it without hesitation as a model of one of the four existing fundamental interactions. Each time, referring to general relativity and fitting Riemannian geometry to physical reality, we must not forget that on the other side of the scale there is the outstanding achievement of a higher physical and philosophical resonance, obtained almost simultaneously with the advent of the general theory of relativity. This is 100-year-old Emmy Noether's brilliant theorem on the nature of conservation laws. This theorem is the exciting result, which explains the origin of conservation of energy, momentum, angular momentum, and center-of-mass velocity for the matter and physical fields from the undistorted symmetry of the Minkowski space-time. The principles of conservation that nature obeys, which, thanks to Emmy Noether, have become a theorem, are an outstanding achievement of physical science, and they cannot be sacrificed even for the fascinating Einstein's idea of a strong equiavalence, which creates the Riemannian curvature of spacetime and makes it one of the physical fields capable of spectacularly interpreting the four well-known tests.

On the other hand, although the approximation of a weak gravitational field within general relativity allows us to distinguish it as a tensor field against the background of the flat Minkowski metric and to formulate the law of conservation of energy, nevertheless this well-known trick does not resolve all the energy problems associated with gravitation. Postulating gravity in such an approach as a classical tensor field of the second rank, we immediately encounter the insoluble problem of the lack of positive definiteness of the energy density of this field.

Our consideration of the law of energy conservation has shown that among the force fields of various tensor dimensions, which could have attractive properties, only one, simplest of them, the field of zero rank, can satisfy the necessary general physical requirement for the field energy density to be positively defined. This means that in the classical field-theoretical approach to the problem of constructing a dynamic theory of gravity, in which all ten special-relativistic integrals of motion are present, definitely only a scalar field can be considered as a candidate to represent the gravitational field.

1 The law of energy-momentum conservation of massive spin-zero gravitational field

Continuing, begun in [1] , the adaptation of the canonical scalar massive field as a possible model of gravity for its further use in solving existing problems of dark mass and dark energy in cosmology, in this article we formulate the law of energy-momentum conservation, which is a stumbling block for the general theory of relativity.

For a system of interacting scalar field and massive particles, we form the action invariant with respect to the Lorentz group. Obeying the principle of simplicity we use the expression quadratic in the field as the Lagrangian density for a system of interacting classical particles and a massive scalar field,

$$\mathscr{L}_{field} = -\frac{c^4}{2\pi G_N}\left(\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{c^2 m_g^2}{\hbar^2}\phi^2\right), \quad (1.1)$$

and the spatial density of the Lagrangian of gravitationally interacting classical particles (dustlike matter) coupled to this field, is given by the expression

$$\mathscr{L}_p = -c^2\sum_a m_a\sqrt{1-\frac{v_a^2}{c^2}\phi^2}\,\delta^{(3)}(\mathbf{r}-\mathbf{r}_a). \quad (1.2)$$

In the case of the real classical scalar field ϕ described by the Lagrangian density (1.1), the canonical energy-momentum tensor

$$T^\mu_\nu = \mathscr{L}\delta^\mu_\nu - \frac{\partial\mathscr{L}}{\partial_\mu\phi}\partial_\nu\phi \quad (1.3)$$

leads immediately to the expression

$$T_{\mu\nu} = \frac{c^4}{\pi G_N}\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(\partial_\sigma\phi\partial^\sigma\varphi + \varkappa^2\varphi^2)\eta_{\mu\nu}\right). \quad (1.4)$$

As we see, this tensor is symmetric, as it should be for the spin-zero field. Using the definition

$$g_\mu = -2c^2\frac{1}{\phi}\partial_\mu\phi \quad (1.5)$$

of the strength vector in terms of the field variable $\phi(x)$ and its derivatives with respect to four-dimensional coordinates, we can represent the energy-momentum tensor (1.4) in the form [2]

$$T_{\mu\nu} = \frac{\phi^2}{4\pi G_N}\left(g_\mu g_\nu - \frac{1}{2}(g_\sigma g^\sigma + 4c^4\varkappa^2)\eta_{\mu\nu}\right). \quad (1.6)$$

The densities of energy, energy flux, momentum, and momentum flux of the field appear in this formalism in the usual way as the following components of the energy-momentum tensor:

$$W = T_{00} = T^{00},\quad S_i = -cT_{0i} = cT^{0i},$$

$$G_i = -c^{-1}T_{i0} = c^{-1}T^{i0},$$

and $\sigma_{ij} = T_{ij} = T^{ij}$ respectively.

Thus, from (1.6), using the notation $g^\mu = (Q,\mathbf{g})$, we obtain all four dynamical characteristics of the field listed above:

$$W = \frac{\phi^2}{8\pi G_N}(\mathbf{g}^2 + Q^2 + 4c^4\varkappa^2), \quad (1.7)$$

$$\mathbf{S} = \frac{c\phi^2}{4\pi G_N}Q\mathbf{g}, \quad (1.8)$$

$$\mathbf{G} = \frac{\phi^2}{4\pi G_N c}Q\mathbf{g}, \quad (1.9)$$

$$\sigma_{ij} = \frac{\phi^2}{4\pi G_N}(g_i g_j + Q^2\delta_{ij}) - W\delta_{ij}. \quad (1.10)$$

Now we return to the equation of motion of a test particle in the field, expressed in the four-dimensional form (equation (1.5) in the previous paper [1]):

$$\frac{dp^\mu}{d\tau} = m\phi^2 g^\mu. \quad (1.11)$$

We recall that in this formula $u^\mu = c^{-1}dx^\mu/d\tau$ is the four-velocity of a particle, $d\tau = c^{-1}ds = \sqrt{1-(v/c)^2}\,dt$ is its proper time. The four-vector $p^\mu = cm\phi^2 u^\mu$, represented in components as

$$p^\mu = \left(\frac{1}{c}\mathscr{E}, \mathbf{p}\right), \quad (1.12)$$

consists of the canonical expressions $\mathbf{p} = \partial L_p/\partial\mathbf{v}$ and $\mathscr{E} = \mathbf{v}\partial L_p/\partial\mathbf{v} - L_p$ for momentum and energy of a separate particle, if we keep in mind its Lagrange function

$$L_p = \int\mathscr{L}_p dV = -c^2 m\phi^2\sqrt{1-\frac{v^2}{c^2}}. \quad (1.13)$$

As can be seen from the equation (1.11) represented in unpacked three-dimensional form, these are the same field-dependent expressions that were first introduced by Nordström in [3] :

$$\mathscr{E} = \frac{c^2 m\phi^2}{\sqrt{1-(v/c)^2}}, \quad (1.14)$$

$$\mathbf{p} = \frac{m\phi^2\mathbf{v}}{\sqrt{1-(v/c)^2}}. \quad (1.15)$$

Presented in these formulas the energy content factor ϕ^2 is the same that has already appeared in formula

$$\tilde{m} = m\phi^2 \quad (1.16)$$

in [1] for the inertial and gravitational masses of a particle. It is important to note as a significant achievement of a theory that this factor presents also in (1.7)–(1.10) for the densities and fluxes of the corresponding physical quantities of the field itself. Thereby the numerical values of all these physical quantities are multiplied by the same conversion factor κ^2 under the gauge transformation $\phi \to \phi' = \kappa\phi$, just as in the case where the adopted unit of mass is replaced by another one.

We emphasize the fact that in other numerous scalar relativistic models of the gravitational field, motivated by different reasons, this is not the case. Particularly, in Nordström's theory presented in [4], the mass of a particle, its energy and momentum depend on the gravitational field, as in (1.14)–(1.16), by the factor ϕ^2 (denoted by Nordström as $e^{g\Phi/c^2}$), whereas the densities of energy and energy flux of the field itself are proportional to ϕ^4 (for details of analysis, see in [5]).

From the exact expression (1.14), it is useful to get an approximate formula for estimating the small changes in the energy content of nonrelativistic particles moving in a weak gravitational field. Setting in (1.14) $\phi^2 = e^{\Phi/c^2}$ and taking the traditional convenient gauge condition $\Phi = 0$ for the gravitational potential at infinity, we arrive at the known expression (see formula (87.10) in [6])

$$\mathscr{E} \approx mc^2 + \frac{mv^2}{2} + m\Phi \qquad (1.17)$$

for the energy of particle (1.14) in the nonrelativistic and weak-field approximation, linear in small quantities v^2/c^2 and Φ/c^2.

It is straightforward to verify that in empty space in the absence of gravitating masses, the expression (1.6) obtained as the energy-momentum tensor of the field satisfies the continuity equation

$$\partial^{\nu} T_{\mu\nu} = 0. \qquad (1.18)$$

This equation expresses in a differential form the conservation of energy and momentum of a free gravitational field, which in the case under consideration is a closed physical system.

In the presence of massive particles, the total four-divergence of the energy-momentum tensor of the field is no longer equal to zero due to the energy and momentum exchange between the field and matter. Using (1.5) and the field equations (3.4) and (3.5) of our previous paper [1], it can be shown immediately from (1.6) that, instead of (1.18), in this case

$$\partial^{\nu} T_{\mu\nu} = -\sum_a m_a \phi^2 \sqrt{1 - \frac{v_a^2}{c^2}} \delta^{(3)}(\mathbf{r} - \mathbf{r}_a) g_{\mu}. \qquad (1.19)$$

Of course, for the closed system consisting of the gravitational field and the massive particles gravitationally interacting with each other, the full energy and momentum are strictly conserved. In fact, integrating equation (1.19) over a certain volume V bounded by a closed smooth surface Σ with the aid of equation (1.11) for the four-vector (1.12) and using the Gauss theorem, we obtain the balanced equation for the energy and momentum in the integral form

$$\frac{d}{dt}\left(\sum_{(V)} p^{\mu} + \frac{1}{c} \iiint_V T^{\mu 0} dV \right) = -\oint\!\!\oint_{\Sigma} T^{\mu i} df_i, \qquad (1.20)$$

where $d\mathbf{f} = (df_i)$ is an infinitesimal normal vector of the the closed surface Σ directed outward of the chosen volume V. The summation in (1.20) extends over all the particles contained in this volume. This equation expresses the law of conservation of total energy and momentum of the field and matter in the integral form.

Setting in (1.20) the index μ equal to zero and using the formulas (1.14), (1.7), and (1.8), we find the equation expressed the conservation law of total energy of the particles and field:

$$\frac{d}{dt}\left[\sum_{(V)} \frac{c^2 m \phi^2}{\sqrt{1 - \frac{v^2}{c^2}}} + \iiint_V \frac{\phi^2}{8\pi G_N} (\mathbf{g}^2 + Q^2 + 4c^4 \varkappa^2) dV \right] = -\oint\!\!\oint_{\Sigma} \frac{c\phi^2}{4\pi G_N} Q \mathbf{g} \cdot d\mathbf{f}. \qquad (1.21)$$

This equation asserts that the rate of change of the total energy of particles given by formula (1.14) and of the field with the positive density defined by (1.7) in a certain volume V is exactly equal to the amount of energy passing per unit time with the field energy flux of density (1.8) into the surrounding space or back through the closed surface Σ bounding this volume.

Along with (1.21), choosing the spatial components of (1.20) and omitting the indexes, we obtain the separate three-dimensional vector equation

$$\frac{d}{dt}\left(\sum_{(V)} \frac{m\phi^2 \mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}} + \iiint_V \frac{\phi^2 Q \mathbf{g}}{4\pi G_N c} dV \right) = -\oint\!\!\oint_{\Sigma} \sigma \cdot d\mathbf{f}, \qquad (1.22)$$

which expresses the momentum conservation of particles and field. The three-dimensional tensor of second rank σ on the right-hand side of this equation represents the momentum flux density of the field; its components are defined by (1.10).

Analysis of the integral relation (1.22) reveals that, if we use the perturbed Lagrangian (1.13), the generalized momentum $\partial L / \partial \mathbf{v}$, canonically conjugate to $\mathbf{r}$ coincides with (1.15) and participates in the conservation law as a momentum of a particle in a gravitational field. Whereas it is well known that in classical mechanics of a charged particle in an electromagnetic field, the unperturbed Lagrangian of a free particle should be used for this purpose. For this reason, after obtaining formulas (1.14) and (1.15) for the energy and momentum of a particle using in their canonical definition (1.3) the Lagrange function (1.13), some doubts can still arise against the actual physical meaning of these quantities.

The proved theorems of conservation of energy and momentum in the form of equations (1.21) and (1.22), containing expressions (1.14) and (1.15), demonstrate the groundlessness of such doubts. Precisely the covariant form (1.20) of these equations enables us to associate $T^{\mu 0}$ with p^{μ} and gives the final arguments to identify with confidence the corresponding components of tensor (1.6) as densities of energy, energy flux, momentum, and momentum flux of the field listed in (1.7)–(1.10).

Let us turn for the moment to the post-Newtonian equation of a static gravitational field (equation (3.6) in [1]). As we have already noted in [1], this equation in the case of massless field, that is in the form

$$\nabla \cdot \mathbf{g} = \frac{1}{2c^2}\mathbf{g}^2 - 4\pi G_N \varrho, \qquad (1.23)$$

was proposed by Brillouin [7] and Hooft [8]. In this connection we note that the appearance in it of the nonlinear term quadratic in field **g** was motivated by these authors by the necessity to include the field energy into the source of the field itself. This was done in [7], [8] "by hand", using the negative *pseudo*-energy density

$$W^{\text{pseudo}} = -\frac{g^2}{8\pi G_N} \qquad (1.24)$$

principally different of positive expression (1.7). In different dynamical theories of gravitation, including general relativity, this expression is indeed widely known and currently accepted as the energy density of static gravitational field in Newtonian approximation. Indeed, formula (1.24) is derived by many authors. It can be found, for example, in books [7], [8], which were already quoted, and also in [6], [9], [10], [11].

In this connection we note that, although the post-Newtonian equation (1.23) (similarly to the covariant four-dimensional equation (3.4) in [1]) represents the field **g** as a self-acting one, as it should, our starting equation (3.2) in [1] for the main scalar field variable ϕ has the canonical form; that is, it is essentially linear and homogeneous, as required by gauge symmetry.

Returning now to the equation (1.13) in [1], we see that if a particle moves in static external field $\phi(\mathbf{r})$, which is characterized by the vector strength $\mathbf{g} \neq 0$, while its scalar strength $Q = 0$, then the energy (1.14) of this particle is conserved.

In turn, as can be seen from (1.12) in [1], if the field variable ϕ depends only on time, so that $\mathbf{g} = 0$, the particle momentum (1.15) is conserved despite the presence of field Q in this case. Although field strength Q causes, according to (1.21) in [1] , a change in the velocity of a moving particle, its momentum in the presence of this field is conserved. From formula (1.15) of the present paper, it follows that this is possible simultaneously with a suitable balanced change in the variable inertial mass $\tilde{m} = m\phi^2$ of a particle.

We make now one remark relating to the expressions for the densities of the field energy and momentum and of their fluxes in formulas (1.7)–(1.10) and in (1.21), (1.22). From these formulas, it may seem at first sight that if ϕ vanishes at some instant of time or within a finite region of space, then the total energy and momentum of particles and fields together with fluxes of the field energy and momentum vanish simultaneously. However, this is not the case, as can be seen by comparing the formulas (1.4) and (1.6). We recall that the factor ϕ^2 arises in energy-momentum tensor (1.6) and naturally in expressions (1.7) and (1.8) as a result of the replacement in (1.4) of the ordinary gradient $\partial_\mu \phi$ by logarithmic derivatives of ϕ that are hidden in the observables g_μ.

As for the expressions (1.14) and (1.15) of the energy and momentum of a moving particle, these quantities do not vanish also when $\phi \to 0$. This can be easily proved by writing down the Hamiltonian of the particle

$$\mathscr{H} = v_i \frac{\partial L_p}{\partial v_i} - L_p = c\sqrt{p^2 + c^2 m^2 \phi^4}, \quad (1.25)$$

which corresponds to the Lagrangian (1.13) and reproduces the equation of motion (1.12) in [1] as the canonical Hamilton equation

$$\frac{d\mathbf{p}}{dt} = -\frac{\partial H}{\partial \mathbf{r}}.$$

From (1.25) it follows that if $\phi \to 0$, then the relation between the energy and momentum of the particle takes the form $\mathscr{E} = cp$, that is, as in the case of a massless particle moving with the light velocity.

2 Gravitational variability of the inertial mass

Our consideration of the law of energy conservation in the previous section shows that the logic of the scalar field approach to the problem of gravity within the framework of the special theory of relativity that we follow here allows us to specify the initial concepts of inertial mass and mass, understood as a gravitational charge, which were introduced in the construction of the Lagrangian (1.2). The results already obtained in previous sections make it possible to establish a clear connection between these two fundamental physical concepts and, therefore, to better understand the physical essence hidden in them.

In order to clearly trace the connection between the parameter m identified by (1.13) as the gravitational charge of a particle and its inertial properties, resulting from the adopted Lagrangian (1.1) [accounting (1.2)], we first turn to the general definition of the inertial mass as such in the case of a moving particle. Very reasonable arguments in favor of the special-relativistic definition of this physical quantity as the magnitude (in sense of pseudo-Euclidean Minkowski metric) of momentum-energy four-vector are given by Taylor and Wheeler in [12] (Section 12). But even earlier, such a Poincaré-invariant definition of inertial mass as a measure of the rest energy of a relativistic particle, given by the expression

$$\frac{1}{c}\sqrt{-p_\mu p^\mu} = \frac{1}{c^2}\sqrt{\mathscr{E}^2 - c^2 p^2},$$

was used by Herman Weil [13] (pp. 407–408), and then was especially convincingly and persistently advocated by Lev Okun' [14], [15] .

Two previously obtained formulas (1.14) and (1.15) for the energy and momentum enable us to calculate this quantity. The expression found in this way is already known to us: it coincides with (1.16), so that

$$\tilde{m} = \frac{1}{c}\sqrt{-p_\mu p^\mu},$$

as it should. Using, as before, the definition $\phi^2 = e^{\Phi/c^2}$ of potential Φ, we write it again in the original form

$$\tilde{m} = me^{\Phi/c^2} \qquad (2.1)$$

obtained for the first time by Hunnar Nordström in his "first" theory of gravitation (see [3], formula (1.5)) and also in the "second" one ([4], formula (53)).

The orthodox (non-metric) field-theoretical approach to gravity, presented here, shows that the gravitational variability of the inertial mass with necessity is predetermined by the very scalar nature of the field coupled to it, independently of the specific type and form of field equations. This fact is caused already by the special form of the gravitational analogue of the Poynting vector. It is easy to see that energy flow vector, as the only possible three-dimensional vector quadratic form in the components of the strength four-vector

$$g^\mu = (Q, \mathbf{g}) = \left(\frac{1}{c}\frac{\partial \Phi}{\partial t}, -\nabla\Phi\right), \qquad (2.2)$$

must be proportional, as in (1.8), to the product $Q\mathbf{g}$. We write it again in the form

$$\mathbf{S} = \frac{ce^{\Phi/c^2}}{4\pi G_N} Q\mathbf{g}. \qquad (2.3)$$

This means that the flow of gravitational energy arises only in the presence of two fields of strengths, $\mathbf{g}$ and Q, simultaneously. But in all cases, the field energy is transferred exclusively along or against the strength field $\mathbf{g}$.

Consequently, in space where a gravitating substance is present, an external gravitational field with a time-varying potential Φ_{ext} and, therefore, with non-zero strength Q_{ext} initiates the redistribution of field energy in space. The transfer of the energy around each individual body occurs along or opposite the direction of its proper convergent field $\mathbf{g}_{prop}$, that is, into or out the body, depending on the sign of the scalar strength Q_{ext} of the external field. It is worth recalling that the described gravitational mechanism of changing the inertial mass of a gravitating body was known still to Nordström over a hundred years ago and was presented in his pioneer work [3].

If the gravitational potential Φ_{ext} increases, that is, if the time component $Q_{ext} = c^{-1}\partial\Phi_{ext}/\partial t$ of the four-vector strength g^μ of the field in (2.2) is positive, then the induced energy flux, according to (1.8), will be convergent, just as the strength $\mathbf{g}_{prop}$ of the field produced by the body. Thus, in this case, because of the energy inflow of the gravitational field from the surrounding space, there is an increase in the energy $\mathscr{E} = \tilde{m}c^2$ stored in the body in the form of its inertial mass $\tilde{m}$. On the contrary, an external field with negative strength Q_{ext} (that is, with a decreasing potential Φ_{ext}) will provoke the process of evacuation of energy accumulated in the body into the surrounding space and its transformation in the form of the energy of gravitational field. The actual inertial mass $\tilde{m}$ of the body, determined by formula (2.1), will decrease in this case. Of course, the same applies to the inertial mass of the atoms that make up the body, and in general to all massive elementary particles.

A similar mechanism of mass change due to energy transfer works in the absence of the field Q_{ext} if the massive gravitating particle moves in an external static field $\mathbf{g}_{ext}$ produced by another massive body at rest. In this case, the nonzero field Q, which provokes, according to (2.3), the transfer of the field energy, appears in the rest frame of the moving particle. This fact can be easily verified by going to this reference frame.

As an example, let us consider the case when a test body (particle) moves uniformly with a certain velocity $\mathbf{v}$ in the external static gravitational field $\mathbf{g}_{ext}$ produced by some other massive body, for example, by the earth. Suppose that in the rest frame of the earth (the daily rotation is neglected), this body at the point x^μ of its own world line has a velocity $\mathbf{v}$ directed upward, opposite the field strength $\mathbf{g}_{ext}$. Now we choose in the reference frame associated with the earth the three-dimensional coordinate system in which the x^3-axis is directed along the velocity of the particle. So, in the frame chosen in this way, $\mathbf{v} = (0, 0, v)$ and the strength four-vector in the indicated world point x^μ is

$$g^\mu_{ext} = (0, \mathbf{g}_{ext}) = (0, 0, 0, -g_{ext}).$$

Then, as a result of the Lorentz transformations, the set of transformed components of this four-vector

$$g'^\mu_{ext} = (Q'_{ext}, \mathbf{g}'_{ext})$$

in the rest frame of the particle at the same world point is

$$g'^\mu_{ext} = \left(\frac{(v/c)g_{ext}}{\sqrt{1-(v/c)^2}}, 0, 0, \frac{-g_{ext}}{\sqrt{1-(v/c)^2}}\right).$$

As expected, the nonvanishing scalar strength of the gravitational field

$$Q'_{ext} = \frac{(v/c)g_{ext}}{\sqrt{1-(v/c)^2}} \qquad (2.4)$$

appeared among them as the time component.

The formula (2.4) thus obtained determines the positive scalar strength Q'_{ext} of the external field, which together with the vector self-field $\mathbf{g}_{proper}$ of the particle forms a convergent flow of gravitational energy (1.8):

$$\mathbf{S} = \frac{1}{4\pi G_N} \frac{v g_{ext} e^{\Phi_{tot}/c^2}}{\sqrt{1-(v/c)^2}} \mathbf{g}_{proper},$$

that increases the particle rest energy and, consequently, its inertial mass as it moves opposite the external field $\mathbf{g}_{ext}$. On the contrary, the motion of a particle along $\mathbf{g}_{ext}$ leads to the appearance of a scalar field strength Q'_{ext} in its rest frame, similar to (2.4), but negative. Accordingly, this field will provoke an outflow of energy accumulated in the particle and a corresponding decrease in its inertial mass.

There exists the essential difference between the geometric patterns of energy transfer arising in two similar cases: around a massive particle in a *scalar* gravitational field, where the density of energy flux $\mathbf{S}$ is proportional to $Q\mathbf{g}$, and around a charged particle in a *vector* electromagnetic field, where $\mathbf{S}$ is proportional to $\mathbf{E}\times\mathbf{H}$. The additional electromagnetic energy flows associated with the proper electric field of a charged particle, which arise when it appears in an external electromagnetic field, are strictly orthogonal to the radial direction of the Coulomb field of this particle. For this reason, there never arise radial convergent or divergent flows of electromagnetic energy that could change the rest energy of a charged particle. As a result, unlike the scalar model of gravitation, in the Maxwell – Lorentz electrodynamics, the proper energies of particles and hence their inertial masses remain inviolable.

The above elementary reasoning shows that the variability of the inertial mass (mass defect) of isolated material particles is absent in electrodynamics and inherent in scalar theories of the gravitational field. It is easy to understand that this difference in the features of mass with respect of these two types of fields is due to the difference in their tensor dimensions (tensor ranks). In other words, this difference is due the vector nature of the electromagnetic field and the scalar nature (adopted in our study) of the gravitational field. [Note, by the way, that the scalar-tensor theory of gravitation proposed in [16], which parametrizes the hypothetical evolution of the gravitational constant by means of an auxiliary scalar field minimally coupled to matter, also leads for the same reason to the correctional variability of the inertial mass on a cosmological time scale.]

Thus, we have found answers to the fundamental questions arising in the scalar model of gravity, *how* and *why* the actual inertial mass of particles (1.16) or, equivalently, (2.1) changes as they are at rest in a gravitational field that varies with time, or if they move in an external gravitational field. We stated also that there is a big difference between the gravitational variability of the rest energy of any massive particles (macroscopic bodies, atoms, nuclei, and even elementary particles), on the one hand, and the usual small defect of mass associated with other fundamental interactions: electromagnetic, weak, and strong, which manifests itself, for example, in the decay of unstable particles, as well as in nuclear and chemical transformations, on the other hand. We see that in the first case the inertial mass of elementary particles can be changed up to zero when the logarithmic potential of external gravitational field varying in time or space goes through zero.

In connection with the unexpectedly discovered progress in understanding the variability of the inertial mass, we recall that Vladimir Fock in [17] (Section 34) formulated the problem: why does the rest energy of elementary particles in all processes except annihilation “behaves passively”, so that even its small part never turns into other types of energy, in particular, in “active” kinetic form? Since the special theory of relativity neither in itself nor with the help of general relativity can explain the special stability of the prevailing part of the rest energy of matter, Fock actually leaves the problem unanswered, stating that the cause of the inviolability of the energy stored in the mass of elementary particles is of a quantum nature.

Leaving aside the Fock's hypothesis about the quantum nature of the inertial mass, in all other respects he is undoubtedly right. The rest energy of particles participating in electromagnetic interaction is inviolable, and the inclusion of Einstein's theory of gravitation does not make the understanding of the problem raised by Fock more profound. Whereas, having overcome Einstein’s “taboo” and turned to the scalar special relativistic theory of gravitation, we can meaningfully discuss the dynamics of the formation of the inertial mass of elementary particles, hidden in the phenomenology of $\mathscr{E} = mc^2$, with the participation of gravitational interaction.

3 Special-relativistic effect of the gravitational shift of atomic spectra

According to the general expression (1.14), the energy of atom in n-th stationary state located at rest in gravitational field with potential Φ is

$$\mathscr{E}_n = \mathscr{E}_n^0 e^{\Phi/c^2}.$$

Here $\mathscr{E}_n^0$ is the rest energy of this or another identical atom in the same quantum state removed far from gravitating matter, where we choose the gauge condition $\Phi = 0$. Therefore, the frequency of the photon emitted by the atom changes with a change of potential as

$$\omega_{nm}(\Phi) = \frac{\mathscr{E}_n^0 - \mathscr{E}_m^0}{\hbar} e^{\Phi/c^2}.$$

From this formula it follows that the wavelength

$$\lambda_{nm}(\Phi) = \frac{2\pi c}{\omega_{nm}(\Phi)} = \frac{2\pi c\hbar}{\mathscr{E}_n^0 - \mathscr{E}_m^0} e^{-\Phi/c^2} \qquad (3.1)$$

increases with decreasing potential. Introducing the usual “redshift”' parameter equal to the fractional increase in the radiated wavelength when the potential decreases by an amount $\Delta\Phi$:

$$z = \frac{\lambda_{nm}(\Phi - \Delta\Phi) - \lambda_{nm}(\Phi)}{\lambda_{nm}(\Phi)},$$

we get from (3.1) the rigorously valid relation regardless of the value of $\Delta\Phi$:

$$z = e^{\Delta\Phi/c^2} - 1. \qquad (3.2)$$

In cases of a weak field or small potential variations, when $|\Delta\Phi|/c^2 \ll 1$, we can easily deduce from (3.2) the well-known approximate formula

$$z \approx \frac{\Delta\Phi}{c^2}. \qquad (3.3)$$

A detailed analysis of the classical phenomenon of a decrease of the characteristic frequencies of atoms and nuclei placed in a static gravitational field on the basis of a formula that coincides with our approximate formula (3.3) is given in a methodological article [18] by Okun', Selivanov, and Telegdi. As in our article, the decrease in the frequency of a photon or gamma-quantum emitted by atom or nucleus in a static gravitational field is considered in [18] as a result of a "blue shift" of these sources, that is, due to increase in their rest energy with height; at the same time, it is assumed that the frequency of the propagating electromagnetic radiation is unaffected by the presence of a gravitational field.

The special-relativistic "blue shift" of mass-energy (2.1) in the gravitational field has been known thanks to Nordström [3] for more than a hundred years. But unfortunately, in the literature there is sometimes an erroneous statement that the gravitational redshift cannot be explained within the special theory of relativity. A characteristic examples of this are the books [19] and [20] by Norbert Straumann. In the first of them a brave verdict "*The Gravitational Red Shift is not Consistent with Special Relativity*" author has made as the title of the relevant section. But we just saw a completely opposite conclusion, therefore, the author's pseudo-arguments given to prove the quoted statement, as well as the pseudo-proof itself, we leave without comment.

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