Table 1 – Comparison of the values of the root-mean-square radii of light mesons

R.m.s. radii	[1]	[5]	This work
$< r_{\pi^\pm} >$, fm	0.659 ± 0.004	–	0.534 ± 0.006
$< r_{\rho^\pm} >$, fm	–	0.748	0.615 ± 0.006

Analysis of table 1 shows that the proposed model gives values comparable with modern experimental data and other models. It should be noted that the contribution of the structure functions of constituent quarks was not investigated in the work: the usage of mean-square radii $\left\langle r_q^2 \right\rangle = a \big/ m_q^2$ [6] can lead to values close to those of experimental data.

Conclusion. The paper presents a calculation of the electromagnetic characteristics of mesons consisting of quarks. It is shown that the use of model parameters obtained from leptonic decays and mesons leads to results on the electromagnetic mean-square radii of mesons that correlate with experimental data and other models.

Literature

1. Review of Particle Physics / S. Navas [et al.] (Particle Data Group) // Phys. Rev. D. – 2024. – Vol. 110 (3). – 030001 pp.

2. Keister, B. D. Relativistic Hamiltonian dynamics in nuclear and particle physics / B. D Keister, W. N. Polyzou // Advances in nuclear physics. – 1991. – Vol. 20. – P. 225–479.

3. Jaus, W. Consistent treatment of spin-1 mesons in the light-front quark model / W. Jaus // Phys. Rev. D. – 2003. – Vol. 67. – 094010 pp.

4. Light quark masses / H. Leutwyler, J. Iliopoulos, R. Gastmans, J. M. Gerard. – NATO Sci. Ser. B. – 1997. – P. 149–164.

5. Krutov, A. F Electroweak properties of ρ-meson in the instant form of relativistic quantum mechanics / A. F. Krutov, R. G. Polezhaev, V. E Troitsky // EPJ Web Conf. – 2017. – Vol. 138. – 02007 pp.

6. Andreev, V. V. Poincare-covariant models of two-particle systems with quantum field potentials / V. V Andreev. – Gomel: Francisk Skorina Gomel State University, 2008. – 294 pp (in Russian).

A. V. Ivashkevich[1], P. O. Sachenok[2], E. M. Ovsiyuk[3], V. V. Kisel[4]

[1]B. I. Stepanov Institute of Physics of National Academy of Sciences of Belarus, Minsk, Republic of Belarus,

[2]Mozyr State Pedagogical University named after I. P. Shamyakin, Mozyr, Republic of Belarus,

[3]Francisk Skorina Gomel State University, Gomel, Republic of Belarus,

[4]Belarusian State Agrarian Technical University, Minsk, Republic of Belarus

SPIN 1/2 PARTICLE WITH ANOMALOUS MAGNETIC MOMENT AND POLARIZABILITY IN THE EXTERNAL MAGNETIC FIELD

In the paper [1], within the general method by Gel'fand–Yaglom [2], starting with the extended set of representations of the Lorentz group, it was constructed a generalized equation for a spin 1/2 particle with two additional characteristics (concerning general formalism see in [3], [4]). After eliminating the accessory variables of the complete wave function, it was derived the generalized Dirac-like equation, the last includes two additional interaction terms which are interpreted as related to anomalous magnetic moment and a second additional characteristics:

$$\left\{ \gamma^c i(\partial_c + ieA_c) + \frac{e\mu}{2M} j^{[ab]} F_{[ab]} + \frac{e\sigma}{2M^2} \gamma^c i(\partial_c + ieA_c) j^{[ab]} F_{[ab]} - M \right\} \Psi = 0; \qquad (1)$$

the parameter μ corresponds to anomalous magnetic moment of a spin 1/2 particle, and the second parameter σ looks as related to a polarizability of the particle. Let us consider this equation in presence of the uniform magnetic field. We will apply the cylindrical coordinates and the tetrad formalism. Let the field be oriented along the axis z, $A_\phi = +eBr^2/2,\ F_{12} = B$. Then the above equation (1) takes on the form

$$\left\{ \left[\gamma^0 i\partial_t + \gamma^1 \left(\partial_r + \frac{1}{2r} \right) + \frac{\gamma^2}{r} + (i\partial_\phi - eBr^2/2 + ij^{12}) \ + \right.\right.$$
$$\left.\left. + \ \gamma^3 i\partial_z \right] \left(1 + \frac{e\sigma}{M^2} j^{12} F_{12} \right) + \frac{e\mu}{M} j^{12} F_{12} - M \right\} \Psi = 0. \qquad (2)$$

We will apply the following substitution for the wave function

$$\Psi = \frac{1}{\sqrt{r}} e^{-i\epsilon t} e^{im\phi} e^{ikz} \begin{vmatrix} f_1(r) \\ f_2(r) \\ f_3(r) \\ f_4(r) \end{vmatrix} = \frac{1}{\sqrt{r}} e^{-i\epsilon t} e^{im\phi} e^{ikz} F(r).$$

Let us simplify the notations $eB \Rightarrow B,\ eF_{12} \Rightarrow +B,\ e\mu \Rightarrow \mu,\ e\sigma \Rightarrow \sigma$, and

$$a_{m+1/2} = \frac{d}{dr} + \frac{m + 1/2 + Br^2/2}{r}, \quad b_{m-1/2} = \frac{d}{dr} - \frac{m - 1/2 + Br^2/2}{r},$$

the equation (2) leads to

$$-a_{m+1/2} f_4(r) \left(\frac{B\sigma}{2M^2} + i \right) + f_3(r)(m+k) \left(1 - \frac{iB\sigma}{2M^2} \right) + f_1(r) \left(-\frac{iB\mu}{2M} - M \right) = 0,$$

$$b_{m-1/2} f_3(r) \left(\frac{B\sigma}{2M^2} - i \right) + f_4(r)(m-k) \left(1 + \frac{iB\sigma}{2M^2} \right) + f_2(r) \left(+\frac{iB\mu}{2M} - M \right) = 0,$$

$$a_{m+1/2} f_2(r) \left(\frac{B\sigma}{2M^2} + i \right) + f_1(r)(m-k) \left(1 - \frac{iB\sigma}{2M^2} \right) + f_3(r) \left(-\frac{iB\mu}{2M} - M \right) = 0,$$

$$-b_{m-1/2} f_1(r) \left(\frac{B\sigma}{2M^2} - i \right) + f_2(r)(m+k) \left(1 + \frac{iB\sigma}{2M^2} \right) + f_4(r) \left(+\frac{iB\mu}{2M} - M \right) = 0.$$

In order to resolve this system, we will apply the method by Fedorov–Gronskiy [5]. It is based on the use of projective operators related to the third spin projection

$$Y = ij^{12} = \begin{vmatrix} 1/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & -1/2 \end{vmatrix}; \quad P_{+} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}, \quad P_{-} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix};$$

according to this approach, each projective constituent is determined through one function:

$$\Psi_{+}(r) = \begin{vmatrix} f_1 \\ 0 \\ f_3 \\ 0 \end{vmatrix} F_1(r), \quad \Psi_{-}(r) = \begin{vmatrix} 0 \\ f_2 \\ 0 \\ f_4 \end{vmatrix} F_2(r).$$

We impose differential constraints that permit us to transform all equations into algebraic ones:

$$a_{m+1/2} F_2(r) = C_1 F_1, \quad b_{m-1/2} F_1(r) = C_2 F_2,$$

taking into account these constraints we get the algebraic system

$$-C_1 \left(\frac{B\sigma}{2M^2} + i \right) f_4 + (m+k) \left(1 - \frac{iB\sigma}{2M^2} \right) f_3 + \left(-\frac{iB\mu}{2M} - M \right) f_1 = 0,$$

$$C_2 \left(\frac{B\sigma}{2M^2} - i \right) f_3 + (m-k) \left(1 + \frac{iB\sigma}{2M^2} \right) f_4 + \left(+\frac{iB\mu}{2M} - M \right) f_2 = 0,$$

$$C_1 \left(\frac{B\sigma}{2M^2} + i \right) f_2 + (m-k) \left(1 - \frac{iB\sigma}{2M^2} \right) f_1 + \left(-\frac{iB\mu}{2M} - M \right) f_3 = 0,$$

$$-C_2 \left(\frac{B\sigma}{2M^2} - i \right) f_1 + (m+k) \left(1 + \frac{iB\sigma}{2M^2} \right) f_2 + \left(+\frac{iB\mu}{2M} - M \right) f_4 = 0.$$

Without loss of generality, we can equate two parameters, $C_2 = C_1 = C$, so obtaining

$$\left(a_{m+1/2}\, b_{m-1/2} - C^2 \right) F_1(r) = 0, \quad \left(b_{m-1/2}\, a_{m+1/2} - C^2 \right) F_2(r) = 0; \tag{3}$$

then the above algebraic system reads simpler

$$-\left(\frac{iB\mu}{2M}+M\right)f_1+0\cdot f_2+(k+m)\left(1-\frac{iB\sigma}{2M^2}\right)f_3-C\left(\frac{B\sigma}{2M^2}+i\right)f_4=0,$$

$$0\cdot f_1+\left(\frac{iB\mu}{2M}-M\right)f_2+C\left(\frac{B\sigma}{2M^2}-i\right)f_3+(m-k)\left(1+\frac{iB\sigma}{2M^2}\right)f_4=0,$$

$$(m-k)\left(1-\frac{iB\sigma}{2M^2}\right)f_1+C\left(\frac{B\sigma}{2M^2}+i\right)f_2-\left(\frac{iB\mu}{2M}+M\right)f_3+0\cdot f_4=0, \qquad (4)$$

$$-C\left(\frac{B\sigma}{2M^2}-i\right)f_1+(m+k)\left(1+\frac{iB\sigma}{2M^2}\right)f_2+0\cdot f_3+\left(\frac{iB\mu}{2M}-M\right)f_4=0.$$

In explicit form the equations (3) read

$$\frac{d^2F_1}{dr^2}+\frac{1}{r}\frac{dF_1}{dr}+\left[-\frac{1}{4}B^2r^2-\frac{1}{2}B-mB-C^2-\frac{(m-1/2)^2}{r^2}\right]F_1=0\,,$$

$$\frac{d^2F_2}{dr^2}+\frac{1}{r}\frac{dF_2}{dr}+\left[-\frac{1}{4}B^2r^2-\frac{1}{2}B-mB-C^2-\frac{(m+1/2)^2}{r^2}\right]F_2=0\,.$$

Let us transform them to the variable, $x=-Br^2/2$. These equations are related by simple symmetry $B\Rightarrow -B,\ m\Rightarrow -m,\ F_1\Rightarrow F_2$; so it suffices to solve the equation for $F_1(x)=x^Ae^{Dx}f_1(x)$:

$$f''+\left(\frac{2A+1}{x}+2D\right)f'+\frac{A^2}{x^2}f-\frac{1}{4}\frac{(1/2-m)^2}{x^2}f+D^2f-$$

$$-\frac{1}{4}f+\frac{(2A+1)D}{x}f+\frac{1}{4}\frac{1+2m+2C^2/B}{x}f=0.$$

In order to have finite solutions, we should use

$$A=+\frac{|m-1/2|}{2},\ D=+1/2\ (\text{let } B>0).$$

In this way, in the variable $y=-x$ we get a confluent hypergeometric equation with parameters

$$c=|m-1/2|+1,\quad a=\frac{|m-1/2|+m+1/2}{2}+\frac{C^2}{2B}+\frac{1}{2}.$$

The polynomial condition $a=-n$ gives the following quantization rule

$$C^2=-2B\left(n+\frac{|m-1/2|+m+1/2}{2}+\frac{1}{2}\right),\quad n=0,1,2,...$$

Let us turn again to the algebraic system (4). It is convenient to apply dimensionless quantities

$$\frac{m}{M} = E, \quad \frac{k}{M} = K, \quad \frac{C}{M} = c,$$

$$b = \frac{B^2}{2M}, \quad \frac{B\sigma}{2M^2} = ib\sigma, \quad \frac{iB\mu}{2M^2} = ib\mu;$$

then the system (4) in matrix form reads

$$\begin{vmatrix} -(ib\mu+1) & 0 & (E+K)(1-ib\sigma) & -c(b\sigma+i) \\ 0 & (ib\mu-1) & c(b\sigma-i) & (E-K)(1+ib\sigma) \\ (E-K)(1-ib\sigma) & c(b\sigma+i) & -(ib\mu+1) & 0 \\ -c(b\sigma-i) & (E+K)(1+ib\sigma) & 0 & (ib\mu-1) \end{vmatrix} \begin{vmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{vmatrix} = 0. \qquad (5)$$

From vanishing its determinant, we derive a bi-quadratic equation

$$\det A = b^4\left[\left(E^2-K^2+c^2\right)\sigma^2-\mu^2\right]^2+\left(E^2-K^2+c^2-1\right)^2+$$

$$+b^2\left\{2\left(E^2-K^2-c^2+1\right)\mu^2+2\left[\left(E^2-K^2+c^2\right)^2+E^2-K^2-c^2\right]\sigma^2+\right.$$

$$\left.+\,8\left(E^2-K^2\right)\sigma\mu\right\}=0.$$

For parameters $E_{1,2} > 0$, we obtain expressions

$$E_{1,2} = E_{\pm} = \frac{1}{1+b^2\sigma^2}\left[\pm 2b\left(\sigma+\mu\right)\sqrt{c^2\left(1+b^2\sigma^2\right)^2-\left(b^2\mu\sigma-1\right)^2}+\left(\left(-c^2+K^2\right)\sigma^4+\sigma^2\mu^2\right)b^4+\right.$$

$$\left.+\left(\left(-1-2c^2+2K^2\right)\sigma^2-4\sigma\mu-\mu^2\right)b^2-c^2+1+K^2\right]^{1/2}.$$

Substituting expression for $E_{1,2}$ in the system (5) we can find two types of the wave functions. The energy spectra depend in a complicated way on additional characteristics; by this reason these spectra may be studied numerically. By physical reason, two additional parameters should are imaginary; only then we get the physically interpretable positive energies.

Literature

1. Spin 1/2 particle with anomalous magnetic moment and polarizability / V. V. Kisel, A. V. Bury, P. O. Sachenok, E. M. Ovsiyuk // Vesnik of Brest University. Series 4. Physics, Mathematics. − 2024. − № 2 (in Russian).

2. Gel'fand, I. M. General relativistically invariant equations and infinite dimensional representations of the Lorentz group / I. M. Gel'fand, A. M. Yaglom // J. Experimental and Theoretical Physics. − 1948. − Vol. 18. − P. 703−733 (in Russian).

3. Pletukhov, V. A. Relativistic wave equations and intrinsic degrees of freedom / V. A. Pletukhov, V. M. Red'kov, V. I. Strazhev. − Minsk: Belarusian Science, 2015. – 327 p. (in Russian).

4. Spin 1/2 particle with anomalous magnetic moment in a uniform magnetic field, exact solutions / E. M. Ovsiyuk, V. V. Kisel, Y. A. Voynova, O. V. Veko, V. M. Red'kov // Nonlinear Phenomena in Complex Systems. − 2016. − Vol. 19, № 2. − P. 153−165.

5. Gronskiy, V. K. Magnetic properties of a particle with spin 3/2 / V. K. Gronskiy, F. I. Fedorov // Doklady of the National Academy of Sciences of Belarus. − 1960. − Vol. 4, № 7. − P. 278−283 (in Russian).