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Mathematical models of contact quasi-static interaction between fibrous composite bodies

Yu.M. Pleskachevsky, V.V. Mozharovsky and Yu.F. Rouba
Metal-Polymer Research Institute of Belarussian Academy of Sciences,
32a Kirov Str., 246652 Gomel, Republic of Belarus, E-Mail: plesk%polymer.gomel.by@brc.minsk.by,
Gomel State University named after F. Skorina

Abstract

Investigations have been carried out in contact quasi-static interaction of cylindrical and rectangular bodies from fibrous composite materials. Mathematical models are proposed taking account of fiber disposition in contact bodies and their effect on contact parameters (impact strength, duration, contact zones, etc.). Computing methods have been developed for the cases of fiber arrangement parallel to impact, fiber arrangement perpendicular to contact surface, fiber arrangement perpendicular to impact.

1 Introduction

The theory of quasi-static impact in elastic medium has received a great deal of attention in literature devoted to anisotropic body. Thus, Hertz's theory reflected, for e.g., in [3] describes some variation regularities in elastic impact parameters, i.e., in duration, maximum contact force and elastic deformation value. This was followed by Shtaerman's [1] contact problem solution for a generalized contact between elastic bodies. Isotropic cylinder impact has been considered by Dinnik [2], while Goldsmith in his book [3] has extended investigations to impact interaction between solids. Analytical study results of composite material impact were reflected elsewhere [4]. Moon [5], in his turn, has described analytical simulation of impact as applied to composite materials. It should be noted that, the known Hertz's quasi-static theory is frequently used for the approximated impact theoretical analysis in the domain of anisotropic materials. It is based on static solution of the contact interaction problem

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between an elastic indenter (striker) and anisotropic half-space (target).

A vast review on modelling deformation processes occurring in laminated composites under impact loading is given in manuscript [9]. In work [10] an experimental formula of contact force dependence on penetration depth is used to develop impact theory.

The theory of quasi-static impact between a spherical indenter and anisotropic half-space has been developed in detail by Greszczuk [6]. Three main approaches to the impact problem has been discriminated in the work. They are the determination of pressure on the target surface, identification of stress state in the composite target and estimation of damage type. The author have developed the theory for stress state computation in a composite using the finite element programs.

Investigation of a cylinder impact against a composite is aimed at assisting machine part design. As for example, modern fibrous composites used in gear or chain transmissions necessitate consideration of metal roller impact on a chain transmission sprocket made of a fiber composite. So, it is urgent to create new models showing interaction and theoretical dependences for the quasi-static theoretical solution.

Similar to the classical theory of elastic bodies impact, the present work is based on the static theory of contact unaccounted for wave effects. Previous to static contact problem solution, let us consider models describing a fibrous composite contact interaction with an indenter.

2 Contact interaction models for fibrous materials

The above review of contemporary investigations in contact behavior of friction joints made of fibrous composite materials [7] visualizes the necessity of new mathematical models taking account for fiber arrangement in contacting bodies.

The following models of a rigid cylindrical body-composite friction joint will be considered here (Fig.1a, Fg.1b, Fig.1c):

1.Fibers in a body are arranged parallel to X-axis (longitudinal disposition);
2.Fibers are perpendicular to contact surface (normal disposition);
3.Fibers are arranged perpendicular to XY plane (transverse disposition).

To develop a mathematical model taking into account the afore-mentioned three cases, the mathematical elasticity theory is used of anisotropic medium and the micromechanical concept. The material characteristics involving fiber volume containment in the matrix, elasticity modulus and Poisson's ratio taken in different directions were applied when estimating contact parameters.

The stress ($\sigma_x,\sigma_y,\tau_{xy}$) to deformation ($\varepsilon_x,\varepsilon_y,\gamma_{xy}$) dependence can be written using the generalized Hook's law:

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix},$$

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where S_{ij} constants for plane deformation are determined by

$$S_{11} = \frac{1-\nu_{13}\nu_{31}}{E_1}, \; S_{12} = -\frac{\nu_{12}+\nu_{13}\nu_{31}}{E_1}, \; S_{22} = \frac{1-\nu_{32}\nu_{23}}{E_2}, \; S_{66} = \frac{1}{G_{12}}. \qquad (1)$$

at the plane-stressed state $\nu_{j3} = \nu_{3j} = 0$, $j=1,2$. A relation exists between elasticity muduli: $\nu_{ij}/E_i = \nu_{ji}/E_j$, $i,j=1,2,3$. Let us consider some models.

2.1 Longitudinal arrangement of fibers

At a longitudinal layout (Fig. 1a) the fibers are parallel to X-axis.

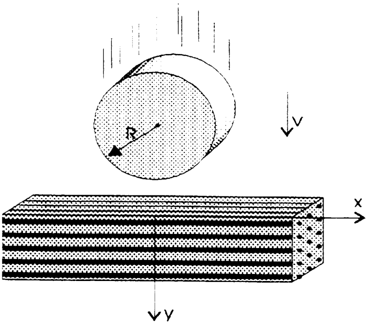

Figure 1a: Cylinder impact against orthotropic base at longitudinal fiber arrangement in the matrix.

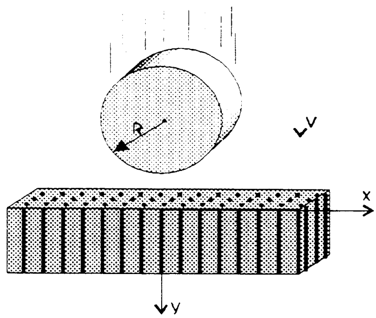

Figure 1b: Cylinder impact against orthotropic base at normal fiber arrangement in the matrix.

For approximate engineering calculations of elastic stable fibrous materials, the dependences (1) and the following relations can be used:

$$E_1 = E_x = V_f E_f + (1-V_f)E_m; \; E_2 = E_y = \frac{E_m \cdot (1+\eta \cdot V_f)}{(1-\eta \cdot V_f)},$$

where $\eta = (E_f - E_m)/(E_f + E_m)$ is chosen according to [11]; V_f - volume fraction of fibre.

$$G_{12} = G_{xy} = G_m \frac{G_f(1+V_f)+G_m(1-V_f)}{G_f(1-V_f)+G_m(1+V_f)}; \; \nu_{12} = V_f \nu_f + (1-V_f)\nu_m,$$

from symmetrical properties it results: $E_3=E_2$; $\nu_{13} = \nu_{12}$; $\nu_{31} = \nu_{21}$; $\nu_{32} = \nu_{23}$; $\nu_{21} = (E_2/E_1)\cdot \nu_{12}$. Indices **f** (**m**) mean fiber (matrix).

2.2 Normal arrangement of fibers

At a normal disposition (Fig. 1b) the fibers are parallel to Y-axis. Elasticity moduli are expressed through the fiber and matrix characteristics and those of their content

$$E_2 = V_f E_f + (1-V_f)E_m; \; E_1 = \frac{E_f E_m}{V_f E_m + (1-V_f)E_f};$$

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$$\nu_{12} = V_f \nu_f + (1 - V_f)\nu_m ;\ \nu_{13} = 1 - \nu_{12} - \frac{E_1}{3K} ; K = \frac{K_f K_m}{V_f K_m + (1 - V_f) K_f},$$

where $K_f = E_f/(3 - 6\nu_f)$; $K_m = E_m/(3 - 6\nu_m)$.

From the symmetry properties: $\nu_{21} = (E_2/E_1)\nu_{12}$; $\nu_{32} = \nu_{12}$; $\nu_{23} = \nu_{21}$; $\nu_{31} = \nu_{13}$; $E_3 = E_1$; $G_{13} = G_{12}$ (is determined as in ch. 2.1).

2.3 Transverse arrangement of fibers

At transverse disposition (Fig.1c) the fibers are perpendicular to XY-axis.

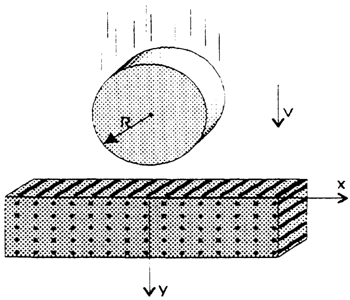

Figure 1c: Rigid cylinder impact against orthotropic base at transverse fiber arrangement in the matrix.

So, elasticity moduli will be

$$E_3 = V_f E_f + (1 - V_f)E_m ;\ E_1 = \frac{E_f E_m}{V_f E_m + (1 - V_f)E_f} ;\ G_{12} = \frac{E_1}{2(1 + \nu_{12})} ;$$

$$\nu_{31} = V_f \nu_f + (1 - V_f)\nu_m ;\ \nu_{32} = (E_3/E_2)\nu_{23} ;\ \nu_{12} = 1 - \nu_{13} - E_1/(3K) ;$$

$$K = K_f K_m / [V_f K_m + (1 - V_f)K_f] \text{, where } \nu_{23} = \nu_{13} ; E_2 = E_1 .$$

It turns out that, from analytical dependences disclosing the material physical parameters, i.e., elasticity modulus through fiber volume content in the composite matrix, the static contact problem can be solved which would be helpful to determine dynamic characteristics of the indenter-target impact interaction.

3 Contact problem on cylindrical indenter-orthotropic half-space interaction

The contact problem solution is based on Green's function that defines vertical displacement of **y=0** boundary for the orthotropic half-space loaded by a concentrated force **P** [8].

$$\upsilon = \frac{P}{2\pi(\beta_1 - \beta_2)} \left[S_{22} \left(\beta_1^2 \ln\left|\frac{x\beta_1}{h}\right| - \beta_2^2 \ln\left|\frac{x\beta_2}{h}\right| \right) + S_{12} \ln\left|\frac{\beta_2}{\beta_1}\right| \right], \quad (2)$$

where $\beta_{1,2} = \left(\sqrt{\left(S_{66} + 2S_{12} \pm \sqrt{(S_{66} + 2S_{12})^2 - 4S_{11}S_{22}} \right) / 2S_{11}} \right)^{-1}$.

Green's function (2) is derived from boundary conditions: vertical displacement equals zero, ($\upsilon=0$ at $x=0$, $y=h$) and the absence of turning, ($d\upsilon/dx = 0$ at $x=0, y=0$). The contact problem solution is reduced to determination of unknown pressure distribution law satisfying equilibrium condition $\int_{-a}^{a} p(t)dt = P$.

Presumably, stresses σ_{yy} and τ_{xy} are scattered on the half-space boundary and are given as:

$$\sigma_{yy} = \begin{cases} p(x), & -a \le x \le a, y = 0; \\ 0, & a < |x|; \end{cases} \qquad \tau_{xy}=0,\ -\infty \le x \le \infty,\ y=0.$$

Using dependence (2) and substituting Green's function, we can find the displacement for the above stated boundary conditions:

$$\upsilon = \left\{ -\frac{S_{22}}{\pi(\beta_1 - \beta_2)} \beta_1^2 \int_{-a}^{a} p(t) \ln\left| \frac{\beta_1}{h}(x-t) \right| dt + \right.$$

$$\left. + \frac{S_{22}}{\pi(\beta_1 - \beta_2)} \beta_2^2 \int_{-a}^{a} p(t) \ln\left| \frac{\beta_2}{h}(x-t) \right| dt - \frac{S_{22}}{\pi(\beta_1 - \beta_2)} \ln\frac{\beta_2}{\beta_1} P \right\}. \qquad (3)$$

Using Green's function, boundary and equilibrium conditions, we obtain an integral equation for pressure determination (analogous to work [8])

$$\frac{1}{\pi}\theta_s \int_{-a}^{a} p(t) \ln|t - x| dt = f(x) + const, \qquad (4)$$

where $\theta_s = (\beta_1 + \beta_2)S_{22}$, $f(x)$ is the equation for the cylinder contour.

By presenting the approximating cylinder equation as $f(x) = x^2/2R$ (R-radius of rigid cylinder), we obtain a known solution of equation for small contact areas

$$p(x) = \sqrt{a^2 - x^2}/R\theta_s, \qquad -a \le x \le a \qquad (5)$$

By substituting the solution into equilibrium condition, we can easily determine half-width of the contact area

$$a = \sqrt{2PR\theta_s/\pi}. \qquad (6)$$

Using the law of pressure distribution in the contact area, the maximum displacement of the orthotropic half-space boundary can be found when a rigid cylindrical punch is impressed. Furthermore, using dependences for displacements (3) at $x=0$, we obtain

$$\upsilon = -\frac{1}{\pi}\theta_s \left[\int_{-a}^{a} p(t) \ln|t| dt - K_0 P \right], \qquad (7)$$

where

$$\mathbf{K}_0 = \left[S_{22}(\beta_1 + \beta_2)\ln h - \frac{S_{22}}{\beta_1 - \beta_2}(\beta_1^2 \ln\beta_1 - \beta_2^2 \ln\beta_2) - \frac{S_{12}}{\beta_1 - \beta_2}\ln\frac{\beta_2}{\beta_1} \right].$$

By substituting $\mathbf{p} = \left(2\mathbf{P}\sqrt{\mathbf{a}^2 - \mathbf{x}^2}\right)/(\pi\mathbf{a})$ into (7) and calculating integral, the orthotropic half-space boundary displacement can be found

$$\upsilon = -\frac{\mathbf{P}}{\pi}\left\{ S_{22}(\beta_1 + \beta_2)\left[\ln\frac{\mathbf{a}}{2\mathbf{h}} - \frac{1}{2}\right] + \frac{S_{22}}{\beta_1 - \beta_2}(\beta_1^2 \ln\beta_1 - \beta_2^2 \ln\beta_2) + \right.$$
$$\left. + S_{12}\ln(\beta_2/\beta_1)/(\beta_1 - \beta_2)\right\}. \quad (8)$$

4 Investigation of a cylindrical indenter collision with an elastic composite base

Let the mass of a rigid cylindrical indenter be $\mathbf{m}_1$, that of orthotropic half-space $\mathbf{m}_2$, $\mathbf{V}$ - the indenter motion velocity, $\mathbf{R}$ - radius of cylindrical indenter, $\mathbf{l}$ - cylinder length. Then, linear force will be $\mathbf{P'=P/l}$ and mass $\mathbf{m'=m/l}$ (the prime is further omitted). The maximum impact strength $\mathbf{P}_{max}$, impact time τ and maximum approach δ_{max} should be determined. Taking $\mathbf{m}_2 \to \infty$, an equation of energy balance is written

$$\frac{1}{2}\mathbf{m}_1\mathbf{V}^2 = \int_0^{\delta} \mathbf{P}d\delta. \quad (9)$$

where δ is the value of approach at a static impression of cylindrical punch into orthotropic half-space determined by formula (8):

$$\delta = \frac{\mathbf{P}}{\pi}\theta_s \ln\frac{\mathbf{a}e^{\Lambda - \frac{1}{2}}}{2\mathbf{R}}. \quad (10)$$

Here, $\Lambda = \frac{1}{S_{22}(\beta_1 + \beta_2)}\left[\frac{S_{22}}{\beta_1 - \beta_2}(\beta_1^2 \ln\beta_1 - \beta_2^2 \ln\beta_2) + \frac{S_{12}}{\beta_1 - \beta_2}\ln\frac{\beta_2}{\beta_1}\right]$.

By substituting half-width of contact area (6) into (10) we obtain

$$d\delta = \frac{\theta_s}{2\pi}\ln\frac{\mathbf{P}\theta_s e^{2\Lambda}}{4\pi\mathbf{R}}d\mathbf{P}.$$

Using (9) and taking account for $\mathbf{a}$, we find

$$\mathbf{V}^2 = \left(\frac{d\delta}{dt}\right)^2 = \frac{\theta_s}{2\mathbf{m}_1\pi}\int_0^{\mathbf{P}_{max}} \mathbf{P}\ln\frac{\mathbf{P}\theta_s e^{2\Lambda}}{4\pi\mathbf{R}}d\mathbf{P}.$$

Since in the moment of maximum approach $d\delta/dt = 0$, the equation for maximum impact force can be written as [8]

$$\mathbf{P}_{max}^2 \ln\left(4\pi\mathbf{R}e^{\frac{1}{2}-2\Lambda}/(\mathbf{P}_{max}\theta_s)\right) = 2\pi\mathbf{V}^2\mathbf{m}_1/\theta_s. \quad (11)$$

From solution of transcendental equation (11) we can find the maximum impact force. For convenience we introduce

$$\mathbf{W} = \mathbf{P}_{max}\theta_s/\left(4\pi\mathbf{R}e^{\frac{1}{2}-2\Lambda}\right); \mathbf{U}^2 = \mathbf{V}^2\theta_s\mathbf{m}_1/\left(8\pi\mathbf{R}^2e^{1-4\Lambda}\right). \quad (12)$$

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Then, equation (11) will have the form

$$\mathbf{W}^2/\ln\mathbf{W} = \mathbf{U}^2. \tag{13}$$

Equation (13) is solved by the iteration method. As the first approximation we take $\mathbf{W_0}=\mathbf{U}$. Upon several iterations we have

$$\mathbf{W} \approx \mathbf{U}\sqrt{\ln\mathbf{U} + \tfrac{1}{2}\ln\ln\mathbf{U}}.$$

By integrating (11) similar to [2], impact time is determined

$$\tau = \frac{\theta_s}{\pi V}\int_0^{P_{max}} \ln\left(P\theta_s e^{2\Lambda} \Big/ \sqrt{1 + \frac{\theta_s}{2\pi V^2}\frac{P^2}{m_1}\ln\frac{P\theta_s}{4\pi\, \mathrm{Re}^{\frac{1}{2}-2\Lambda}}}\right) dP. \tag{14}$$

Further reasoning of integral (14) calculation method is given in [2]. Omitting transformations, the final result is yielded

$$\tau \approx \pi m_1 V/P_{max} + \theta_s P_{max}\ln 2/(4V).$$

Stress in a composite at an impact is computed using programs based on finite element method. We consider that at a cylinder impact against orthotropic surface, pressure distributes following the law: $p(x) = p_0\sqrt{(a^2 - x^2)}$, where $p_0 = 2P_{max}/(\pi a)$.

To determine stresses in an elastic orthotropic body, the contact surface is divided into strips where the mean constant pressure **p** acts. Using dependences for stresses under concentrated forces **p(s)ds** [8], stress in a random point is calculated so

$$\sigma_x = \frac{y}{\pi(\beta_1 - \beta_2)}\int_{x1}^{x2}\left(\left(\beta_1^2(x-s)^2 + y^2\right)^{-1} - \left(\beta_2^2(x-s)^2 + y^2\right)^{-1}\right)p(s)ds;$$

$$\sigma_y = -\frac{y}{\pi(\beta_1 - \beta_2)}\int_{x1}^{x2}\left(\left((x-s)^2 + \frac{y^2}{\beta_1^2}\right)^{-1} - \left((x-s)^2 + \frac{y^2}{\beta_2^2}\right)^{-1}\right)p(s)ds;$$

$$\tau_{xy} = -\frac{1}{\pi(\beta_1 - \beta_2)}\int_{x1}^{x2}(x-s)\left(\left((x-s)^2 + \frac{y^2}{\beta_1^2}\right)^{-1} - \left((x-s)^2 + \frac{y^2}{\beta_2^2}\right)^{-1}\right)p(s)ds;$$

$[x_1,x_2]$ - pressure variation area; $x_1 \le s \le x_2$; **s** - coordinate along x-axis relative to origin of coordinates.

Discrete approximation of pressure distribution on the boundary is solved by dividing into **n** segments (finite elements). Normal stresses on each finite element are taken constant.

The total stress in the body is determined as follows

$$\sigma_x = \sum_{i=1}^{n}(\sigma_x)_i; \quad \sigma_y = \sum_{i=1}^{n}(\sigma_y)_i; \quad \tau_{xy} = \sum_{i=1}^{n}(\tau_{xy})_i;$$

Upon transformations and integral calculation, stresses on the i-th segment are calculated using the formulas:

$$(\sigma_y)_i = -\frac{1}{\pi}p(s_i)L_{0i}; (\tau_{xy})_i = -\frac{1}{\pi}p(s_i)L_{1i}; (\sigma_x)_i = -\frac{1}{\pi}p(s_i)L_{2i};$$

where

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$$L_{0i} = \frac{1}{\beta_1 - \beta_2}\left[\beta_1 \operatorname{arctg}\frac{\beta_1 B}{y} - \beta_1 \operatorname{arctg}\frac{\beta_1 A}{y} + \beta_2 \operatorname{arctg}\frac{\beta_2 A}{y} - \beta_2 \operatorname{arctg}\frac{\beta_2 B}{y}\right];$$

$$L_{1i} = \frac{1}{2(\beta_1 - \beta_2)}\left(\ln(B^2 + \frac{y^2}{\beta_1^2}) - \ln(A^2 + \frac{y^2}{\beta_1^2}) - \ln(B^2 + \frac{y^2}{\beta_2^2}) + \ln(A^2 + \frac{y^2}{\beta_2^2})\right);$$

$$L_{2i} = \frac{1}{\beta_1 - \beta_2}\left[-\frac{1}{\beta_1}(\operatorname{arctg}\frac{\beta_1 B}{y} - \operatorname{arctg}\frac{\beta_1 A}{y}) - \frac{1}{\beta_2}(\operatorname{arctg}\frac{\beta_2 A}{y} - \operatorname{arctg}\frac{\beta_2 B}{y})\right];$$

$A = x - s_i + L; B = x - s_i - L; s_i - L \le s_i \le s_i + L; \Delta s;$ **L** is the finite element half-width.

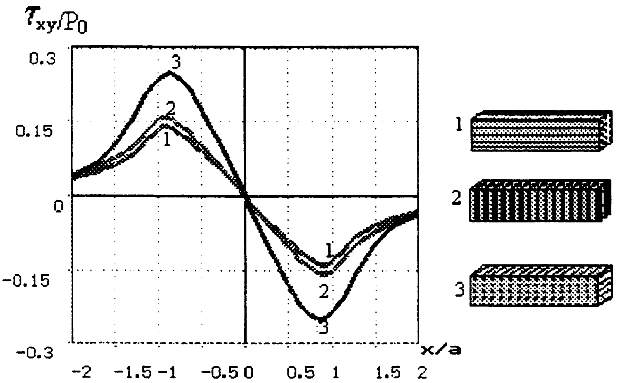

Figure 2: Variation of tangent stress in a composite at different fiber packing.

For e.g., in Fig. 2, fiber arrangement is shown to effect tangent stress in the composite at cylindrical indenter impact. The calculation was made for aluminum reinforced by graphite fibers **Gr/Al** [7] (elastic stable fibers: E_f=380 GPa, G_f=3.7 GPa, ν_f=0.2; that of matrix: E_m=70 GPa, G_m=26 GPa, ν_m=0.34; fiber volume content in the matrix V_f=0.8) at a depth **y**=0.5a.

5 Calculation of composite coating contact strength at impact loading

The performance of impact devices (mechanical, electro-magnetic devices, electromagnetic relay, etc.) can be substantially improved by application of a thin-film polymer-based composite coating on the part surface. In work [8] a method to calculate polymer coating strength at a multicycle impact loading has been developed. The method is based on the analysis of polymer coating stress-strain state and the results of their contact durability investigations.

In the framework of the quasi-static impact theory for the case of plane deformation, the problem is considered of the rigid rectangular indenter collision with an elastic coating on a rigid base (Fig.3). Assuming that the indenter of mass m_1 falls with velocity υ_0 and impact duration surpasses considerably that of direct and reverse wave distribution in the colliding bodies, the equation of indenter motion can be written through penetration δ and impact force **P(t)**:

$$\mathbf{M}\ddot{\delta} = -\mathbf{P(t)}, \text{ where } \mathbf{M} = \mathbf{m_1 m_2/(m_1 + m_2)}. \tag{15}$$

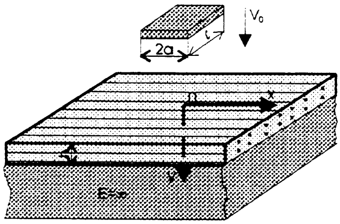

Figure 3: A rectangular punch impact against elastic coating on a rigid base.

Notice that, thin films are characterized by an approximate linear relation of displacement δ to pressure $\mathbf{p_0}$: $\delta(t) \approx \mathbf{p(t)/k_1}$ and even pressure distribution under the punch $\mathbf{p} = \mathbf{P/S}$, where **S=2al**; coefficient $\mathbf{k_1}$ characterizes elastic and geometrical properties of the coating:

$$\mathbf{1/k_1} = \left(\mathbf{S_{22}} - \mathbf{S_{12}^2/S_{11}}\right)\mathbf{h},$$

At initial conditions $\mathbf{t} = \mathbf{0}, \delta = \mathbf{0}, \dot{\delta} = \upsilon = \upsilon_0$ (υ_0 is the initial velocity of indenter) Eq.(15) takes the form

$$\ddot{\delta} + \mathbf{k_1 S}\delta/\mathbf{M} = \mathbf{0}. \tag{16}$$

Taking $\mathbf{k=k_1S}$ and integrating (16) with substitution of symbols upon transformations, contact duration is determined $\tau = \pi\sqrt{\mathbf{M/k}}$, maximum penetration $\delta_{\max} = \sqrt{\mathbf{M/k}}\,\upsilon_0$, maximum impact force $\mathbf{P}_{\max} = \sqrt{\mathbf{kM}}\,\upsilon_0$.

Presuming that normal contact stresses are the main contributor into the coating damage, taking into account that mean stress $\sigma = -\mathbf{P}_{\max}/\mathbf{S}$ and denoting $\left(\mathbf{S_{22}} - \mathbf{S_{12}^2/S_{11}}\right)^{-1} = \mathbf{C}$, a formula for the rated contact compressive stress at impact $\sigma_I \approx \sqrt{\mathbf{M}\upsilon_0^2\mathbf{C}/(\mathbf{hS})}$ is obtained.

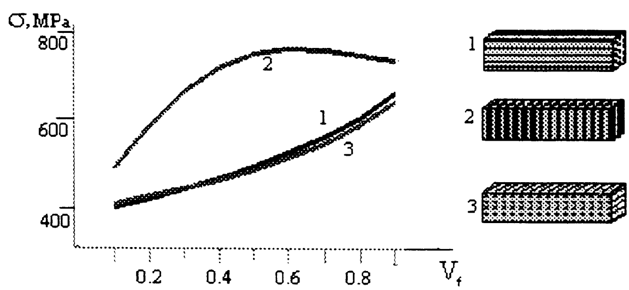

Figure 4: Stress variation at different fiber arrangement.

In Fig. 4, variation of σ is shown depending on fiber disposition at a rectangular punch impact against Gr/Al material (elastic constants see above). Punch mass in this case is **m**=0.8 kg; coating area **S**=$24\cdot10^{-6}$ m^2, coating thickness **h** = 1500 μm, punch falling velocity υ=0.8 m/s.

Experimental investigation [8] has been conducted for composite coatings on urethan thermoplast base with polyester resin addition (the material is taken showing isotropic properties) using a highly durable stand.

To estimate coating durability at a given limiting contact stress and coating thickness $\mathbf{h}_p$, experimental dependences $\sigma_{\lim I}$ under fixed N cycle number of impact loading have been derived [8]. Using the above formula and values we can determine the limiting contact stress, limiting coating thickness.

The described method can be used to design composite coatings prone to damage during operation because of local breakthrough with further propagation of damaged zones.

6 Conclusion

The suggested mathematical models describing the effect of fiber orientation on quasi-static impact parameters are approximated. They do not take into account such processes as fiber pulling, their bonding with matrix, fiber size and shape, etc. Nevertheless, the obtained method can be used to estimate the effect of fiber properties, their arrangement and volume content on the composite stress at impacting. By using the suggested analytical dependences, calculations can be made for different materials aimed at their application in designing gear and chain transmissions, as well as impact devices made of fibrous composites.

Key words: impact, contact, cylinder, composite.

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