УДК 537.2
DOI: https://doi.org/10.54341/20778708_2025_2_63_7
EDN: SLSGBO

НЕКОТОРЫЕ ОСОБЕННОСТИ РЕШЕНИЯ ФИЗИЧЕСКИХ ЗАДАЧ С БЕСКОНЕЧНЫМИ ВЕЛИЧИНАМИ

Н.А. Ахраменко, М.В. Буй

Белорусский государственный университет транспорта, *Гомель*

SOME FEATURES OF SOLVING PHYSICAL PROBLEMS WITH INFINITE QUANTITIES

N.A. Akhramenko, M.V. Bui

Belarusian State University of Transport, *Gomel*

Аннотация. В разнообразных задачах по физике часто используются какие-либо величины, являющиеся бесконечными. Встречаются случаи, когда сразу несколько величин стремятся к бесконечности. Иногда встречаются ситуации, требующие более аккуратного подхода при решении. В данной работе на примере задачи из раздела «Электростатика» рассматриваются некоторые особенности решения при использовании бесконечных величин.

Ключевые слова: *электрическое поле*, *напряженность*, *полуплоскость*, *поверхностная плотность заряда.*

Для цитирования: *Ахраменко*, *Н.А.* Некоторые особенности решения физических задач с бесконечными величинами / Н.А. Ахраменко, М.В. Буй // Проблемы физики, математики и техники. – 2025. – № 2 (63). – С. 7–10. – DOI: https://doi.org/10.54341/20778708_2025_2_63_7. – EDN: SLSGBO

Abstract. In various problems in physics, some quantities that are infinite are often used. There are cases when several quantities simultaneously tend to infinity. Sometimes there are situations that require a more careful approach to solving. In this paper, using the example of a problem from the section "Electrostatics", some features of the solution when using infinite quantities are considered.

Keywords: *electric field*, *intensity*, *half-plane*, *surface charge density.*

For citation: *Akhramenko*, *N.A.* Some features of solving physical problems with infinite quantities / N.A. Akhramenko, M.V. Bui // Problems of Physics, Mathematics and Technics. – 2025. – № 2 (63). – P. 7–10. – DOI: https://doi.org/10.54341/20778708_2025_2_63_7. – EDN: SLSGBO

Introduction

When considering physical problems, we often encounter problems where one of the parameters (or several parameters) is infinite. Sometimes we encounter situations that require a more careful approach when solving problems with infinite quantities. As an example, let us consider the following problem (offered at the Moscow City Physics Olympiad) [1].

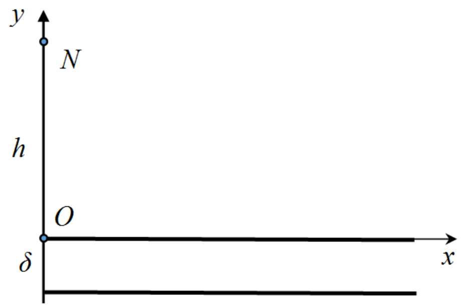

Figure 0.1 – Uniformly charged half-planes (thickened segments), perpendicular to the plane of the drawing

Two parallel half-planes are uniformly charged with charge density + σ on the upper half-plane and − σ on the lower half-plane. Find the magnitude and direction of the electric field strength E at point N, which is located at a height h above the edge of the half-planes (Figure 0.1). The distance between the half-planes δ is small compared to h.

In this problem, the geometric dimensions of the half-planes are infinite parameters. Let us consider possible options for analyzing and solving this problem.

1 Preliminary analysis using the formula for an infinite plane

Let us consider the possibility of using the formula for the electric field strength of an infinite plane $E = \frac{\sigma}{2\varepsilon_0}$ [2]–[6]. The field strength in this case is perpendicular to the plane (i.e. there is only a component perpendicular to the plane, and the component parallel to the plane is zero). Then for half-planes (upper and lower, respectively), for reasons of symmetry, there will be components perpendicular to both planes and equal to

$$E_{1\perp} = \frac{\sigma}{4\varepsilon_0} \text{ and } E_{2\perp} = -\frac{\sigma}{4\varepsilon_0}.$$

The resulting component perpendicular to both planes will be equal to zero due to the fact that the half-planes have different charge signs with the same absolute value. As for the component parallel to both half-planes (along the *OX* axis), the question remains open. Thus, the use of the formula for the field of an infinite plane as applied to this problem shows that the component perpendicular to the plane will be equal to zero ($E_\perp = 0$), which will be used further (we will find only the component of the vector $\vec{E}$ parallel to the half-planes).

2 Solution with consideration of mutually compensating strips

Let us draw two planes through point *N*, perpendicular to the plane of the drawing (Figure 2.1) so that they form a small angle $d\varphi$ with each other and pass through both charged half-planes.

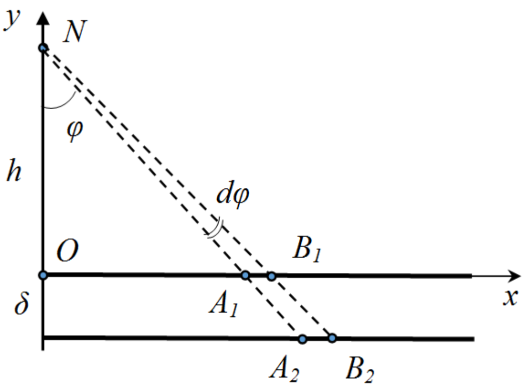

Figure 2.1 – Two planes (dashed lines) perpendicular to the plane of the drawing and intersecting both charged half-planes

These planes will cut out two narrow strips A_1B_1 and A_2B_2 in the charged half-planes, perpendicular to the plane of the drawing (Figure 2.1). It is known that an infinitely long uniformly charged thread creates an electric field of intensity [2]–[6]

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\tau}{r},$$

where τ – is the linear charge density of the thread, r – is the distance from the thread to the point under consideration.

The strips A_1B_1 and A_2B_2 can be considered infinitely narrow, and then they create the same field at point *N* as a uniformly charged thread. That is, the strips A_1B_1 and A_2B_2 will create a field of intensity at point *N*

$$E_1 = \frac{1}{2\pi\varepsilon_0}\frac{\tau_1}{|NA_1|} = \frac{1}{2\pi\varepsilon_0}\frac{\sigma|A_1B_1|}{|NA_1|}$$

и

$$E_2 = \frac{1}{2\pi\varepsilon_0}\frac{\tau_2}{|NA_2|} = \frac{1}{2\pi\varepsilon_0}\frac{\sigma|A_2B_2|}{|NA_2|},$$

where $\tau_1 = \sigma|A_1B_1|$ and $\tau_2 = \sigma|A_2B_2|$.

In this case, the vector $\vec{E}_1$ is directed from the strip A_1B_1, and the vector $\vec{E}_2$ is directed toward the strip A_2B_2. From the similarity of the triangles A_1B_1N and A_2B_2N, we obtain that $\frac{|A_1B_1|}{|NA_1|} = \frac{|A_2B_2|}{|NA_2|}$, therefore, the fields created by both strips at point *N* compensate each other due to the different charges of the half-planes, and the resulting field is zero.

Such reasoning is valid for all pairs of strips cut from the upper and lower half-planes. If the segment A_1B_1 tends to infinity, then the segment A_2B_2 will also tend to infinity, i. e. both half-planes will be covered by these paired strips in this partition. Such partitions into paired strips can cover the upper and lower half-planes.

In this case it may seem that the resulting electric field strength at point *N* is zero. However, this approach is incorrect, since points B_1 and B_2 simultaneously tend to infinity according to different laws (point B_2 of the lower half-plane tends to infinity faster). The correct approach is one in which the points of both half-planes simultaneously tend to infinity according to the same law.

Let us consider the part of the upper and lower half-planes located at the same distance from the *OY* axis (same width $|OB_1| + \Delta l$)

When dividing the upper and lower half-planes into paired strips, it turns out that for the outermost strip of width Δl (Figure 2.2) from the upper half-plane, there is no paired strip on the lower half-plane. Therefore, the sought field strength *E* at point *N* will be equal to the strength created at this point by a strip of width Δl.

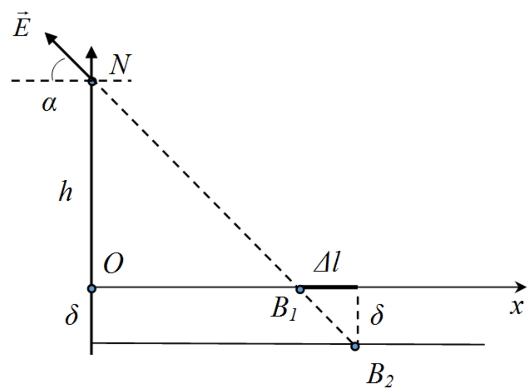

Figure 2.2 – Field strength $\vec{E}$ at point *N* created by a strip of width Δl from the upper half-plane.

The magnitude of the field strength *E* at point *N* will be equal to

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\sigma\Delta l}{|NB_1|}.$$

Projection of a vector $\vec{E}$ onto the OX axis

$$E_x = -\frac{1}{2\pi\varepsilon_0}\frac{\sigma\Delta l}{|NB_1|}\cos\alpha = -\frac{1}{2\pi\varepsilon_0}\frac{\sigma\Delta l}{|NB_1|}\frac{|OB_1|}{|NB_1|} = -\frac{1}{2\pi\varepsilon_0}\frac{\sigma\Delta l|OB_1|}{|NB_1|^2}. \quad (2.1)$$

From the constructions in Figure 2.2 it follows

$$\frac{\delta}{\Delta l} = \frac{h}{|OB_1|} \text{ or } \Delta l = \frac{\delta}{h}|OB_1|.$$

Substituting Δl into expression (2.1) we obtain

$$E_x = -\frac{1}{2\pi\varepsilon_0}\frac{\sigma|OB_1|}{|NB_1|^2}\frac{\delta}{h}|OB_1| = -\frac{1}{2\pi\varepsilon_0}\frac{\sigma|OB_1|^2}{|NB_1|^2}\frac{\delta}{h}. \quad (2.2)$$

As point B_2 tends to infinity, we obtain that $|OB_1| \approx |NB_1|$ and then from expression (2.2) it follows that

$$E_x = -\frac{1}{2\pi\varepsilon_0}\frac{\sigma\delta}{h}. \quad (2.3)$$

Consequently, the vector of the electric field intensity created by the system under consideration at point N is directed parallel to the half-planes and opposite to the OX axis. In absolute value, it is equal to $\frac{1}{2\pi\varepsilon_0}\frac{\sigma\delta}{h}$. It is precisely this type of answer to the problem under consideration that is given in [1].

3 Solution with finding the sum of the intensities of both half-planes

We will find only the components of the vector $\vec{E}$ parallel to both half-planes along the OX axis (in point 1 it was determined that $E_\perp = 0$). Let us consider the upper half-plane (Figure 3.1).

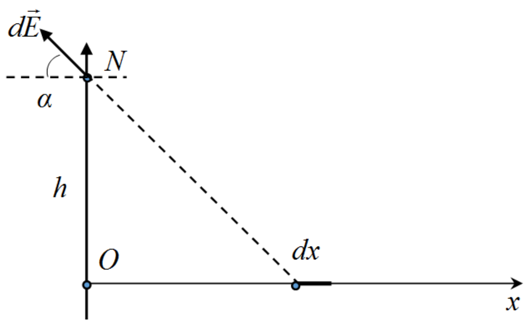

Figure 3.1 – A strip of the upper half-plane of width dx, perpendicular to the plane of the drawing.

A strip of width dx creates a field of intensity at point N

$$dE = \frac{\sigma dx}{2\pi\varepsilon_0\sqrt{h^2+x^2}}.$$

Component E_{1x} parallel to the half-plane along the OX axis (projection of the vector $\vec{E}$ onto the OX axis)

$$dE_{1x} = -\frac{\sigma dx}{2\pi\varepsilon_0\sqrt{h^2+x^2}}\cos\alpha = -\frac{\sigma x dx}{2\pi\varepsilon_0(h^2+x^2)}. \quad (3.1)$$

Integrating expression (3.1) in the range from 0 to x, we obtain

$$E_{1x} = -\frac{\sigma}{2\pi\varepsilon_0}\int_0^x \frac{xdx}{(h^2+x^2)} = -\frac{\sigma}{2\pi\varepsilon_0}\frac{1}{2}\ln(h^2+x^2)\Big|_0^x = -\frac{\sigma}{4\pi\varepsilon_0}\ln\frac{h^2+x^2}{h^2}. \quad (3.2)$$

Similarly, for the lower half-plane, replacing h with $h+\delta$ and σ with $-\sigma$, we obtain

$$E_{2x} = \frac{\sigma}{2\pi\varepsilon_0}\int_0^x \frac{xdx}{(h+\delta)^2+x^2} = \frac{\sigma}{4\pi\varepsilon_0}\ln\frac{(h+\delta)^2+x^2}{(h+\delta)^2}. \quad (3.3)$$

If we set the x coordinate to infinity separately in expressions (3.2) and (3.3), we obtain that $E_{1x} \to -\infty$ and $E_{2x} \to \infty$. Therefore, it turns out that $E_x = E_{1x} + E_{2x}$ and the uncertainty is of the form $\infty - \infty$.

However, if we write the resulting tension in the form $E_x = E_{1x} + E_{2x}$, then using (3.2) and (3.3), we obtain

$$E_x = -\frac{\sigma}{4\pi\varepsilon_0}\ln\frac{h^2+x^2}{h^2} + \frac{\sigma}{4\pi\varepsilon_0}\ln\frac{(h+\delta)^2+x^2}{(h+\delta)^2}. \quad (3.4)$$

From (3.4) after transformations we obtain

$$E_x = \frac{\sigma}{4\pi\varepsilon_0}\ln\left[\frac{(h+\delta)^2+x^2}{(h+\delta)^2}\frac{h^2}{h^2+x^2}\right]. \quad (3.5)$$

In expression (3.5) we can now let the x coordinate tend to infinity.

$$E_x = \lim_{x\to\infty}\left\{\frac{\sigma}{4\pi\varepsilon_0}\ln\left[\frac{(h+\delta)^2+x^2}{(h+\delta)^2}\frac{h^2}{h^2+x^2}\right]\right\} = \frac{\sigma}{4\pi\varepsilon_0}\ln\frac{h^2}{(h+\delta)^2}. \quad (3.6)$$

Let us carry out transformations in expression (3.6)

$$E_x = \frac{\sigma}{4\pi\varepsilon_0}\ln\frac{h^2}{(h+\delta)^2} = \frac{\sigma}{2\pi\varepsilon_0}\ln\frac{h}{h+\delta} = -\frac{\sigma}{2\pi\varepsilon_0}\ln\frac{h+\delta}{h} = -\frac{\sigma}{2\pi\varepsilon_0}\ln\left(1+\frac{\delta}{h}\right). \quad (3.7)$$

Thus, the resulting electric field strength at point N is equal to

$$E_x = -\frac{\sigma}{2\pi\varepsilon_0}\ln\left(1+\frac{\delta}{h}\right). \quad (3.8)$$

The minus sign indicates that the vector $\vec{E}$ is directed against the OX axis.

4 Discussion of solution options

In the solution to the problem in paragraph 2 (this solution is similar to that considered in [1]), the situation is not analyzed and it is not taken into account that the strip of width Δl, as it moves away from the boundary of the half-planes, also increases in width to infinity. It turns out that from the original problem with infinities we come to another problem with infinities (the field strength of an infinitely distant half-plane). And then the question arises about the possibility of using the formula

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\tau}{r}$$

for calculations, since the strip can no longer be considered narrow.

From the constructions in Figure 4.1 we can write

$$\frac{h}{x_1} = \frac{\delta}{x_2 - x_1} \Rightarrow \; x_2 - x_1 = \frac{\delta x_1}{h} = \Delta l \Rightarrow$$

$$x_2 = x_1\left(1 + \frac{\delta}{h}\right). \qquad (4.1)$$

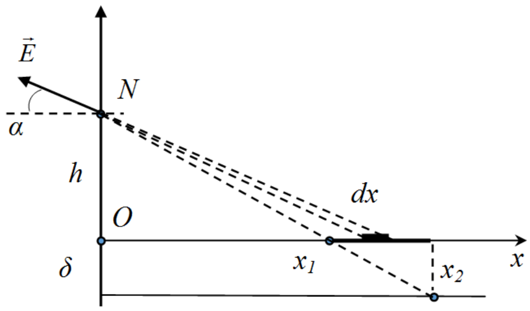

Figure 4.1 – A strip of the upper half-plane of width dx, located between points x_1 and x_2 ($x_2 - x_1 = \Delta l$)

Electric field strength E_x in point N

$$E_x = -\frac{\sigma}{2\pi\varepsilon_0}\int_{x_1}^{x_2}\frac{x\,dx}{h^2 + x^2}. \qquad (4.2)$$

Substituting the coordinate x_2 from (4.1) into the upper limit of the integral (4.2), we obtain

$$E_x = -\frac{\sigma}{2\pi\varepsilon_0}\int_{x_1}^{x_1(1+\delta/h)}\frac{x\,dx}{h^2 + x^2}. \qquad (4.3)$$

Let us calculate the integral (4.3)

$$E_x = -\frac{\sigma}{2\pi\varepsilon_0}\int_{x_1}^{x_1(1+\delta/h)}\frac{x\,dx}{h^2 + x^2} =$$

$$= -\frac{\sigma}{4\pi\varepsilon_0}\ln\left(h^2 + x^2\right)\Big|_{x_1}^{x_1(1+\delta/h)} =$$

$$= -\frac{\sigma}{4\pi\varepsilon_0}\ln\frac{h^2 + x_1^2(1+\delta/h)^2}{h^2 + x_1^2}. \qquad (4.4)$$

Let us tend the coordinate x_1 to infinity in expression (4.4)

$$E_x = \lim_{x_1\to\infty}\left[-\frac{\sigma}{4\pi\varepsilon_0}\ln\frac{h^2 + x_1^2(1+\delta/h)^2}{h^2 + x_1^2}\right] \Rightarrow$$

$$\Rightarrow E_x = -\frac{\sigma}{4\pi\varepsilon_0}\ln\left(1 + \frac{\delta}{h}\right)^2 = -\frac{\sigma}{2\pi\varepsilon_0}\ln\left(1 + \frac{\delta}{h}\right). \quad (4.5)$$

Thus, taking into account that the strip Δl actually tends to infinity in width at $x \to \infty$, we obtain an answer corresponding to (3.8). This means that the answer (2.3) is inaccurate, although at $\delta \ll h$ the differences are insignificant.

From the above it follows that when considering problems with infinite quantities, it is advisable to consider the different possibilities of these quantities tending to infinity and to take into account the nuances that arise.

REFERENCES

1. *Задачи Московских городских олимпиад по физике* 1986–2005 / С.Д. Варламов, В.И. Зинковский, М.В. Семёнов [и др.]; под ред. М.В. Семёнова, А.А. Якуты. – 2-е изд., перераб. и доп. – Москва: МЦНМО, 2006. – 621 с.

2. *Raymond*, *A. Serway. Physics for Scientists and Engineers* (*with PhysicsNOW and info Trac*) / A.S. Raymond, W.J. John: 6th ed., Thomson Brooks / Cole, 2004. – 1296 p.

3. *Детлаф*, *А.А.* Курс физики: учебное пособие для ВТУЗов / А.А. Детлаф, Б.М. Яворский. – 4-е изд., испр. – Москва: Высшая школа, 2002. – 718 с.

4. *Савельев*, *И.В.* Курс общей физики. В 3-х томах. Т. 2: Электричество и магнетизм. Волны. Оптика / И.В. Савельев. – 2-е изд., перераб. – Москва: Наука, 1982. – 496 с.

5. *Трофимова*, *Т.И.* Курс физики: учеб. пособие для вузов / Т.И. Трофимова. – 17-е изд., стер. – Москва: Издательский центр «Академия», 2008. – 560 с.

6. *Шиляева*, *К.П.* Физика. Краткая теория и задачи: пособие / К.П. Шиляева, И.О. Деликатная, Н.А. Ахраменко. – Гомель: БелГУТ, 2021. – 211 с.

Поступила в редакцию 12.03.2025.

Информация об авторах

Ахраменко Николай Арсеньевич – к.т.н., доцент

Буй Михаил Владимирович – к.ф.-м.н., доцент