LEPTON DECAY CONSTANTS OF THE PSEUDOSCALAR MESONS IN THE RELATIVISTIC HAMILTONIAN DYNAMICS

© 2003 V.V. Andreev[1], A.F. Krutov[2], O.I. Shro[3], V.E. Troitsky[4]

Abstract

Calculation of the lepton decay constants of the heavy–light pseudoscalar mesons B , B_s , D , D_s is performed in the framework of instant form of relativistic Hamiltonian dynamics (RHD). Interaction between constituent quarks is given by the smeared potential with linear rise at large distances, Coulomb behavior at small distances and spin-spin interaction. Wave functions in the sense of RHD are calculated approximately by the variational method. Masses of the light quarks are fixed by the fitting of lepton decays constants of pion and kaon. The theoretical uncertainties in the values of lepton constants associated with uncertainties in the quark masses and heavy–light meson masses are estimated. Results are in good agreement with lattice calculations and experiments.

1. Introduction

It is well known that the lepton decays of the heavy mesons are the important source of information about the parameters of Standard Model (SM) (e.g. Cabbibo– Kobayashi–Maskawa matrix elements), and also can serve for searches of physics beyond SM (see e.g. [1]). The retrieval of this information calls for the precise calculations of the lepton decay constants. The values of these constants are determined by the structure of the mesons, and therefore the nonperturbative approaches are necessary for the calculations.

There exist different approaches to this problem, for example, the calculations on lattices [2, 3, 4, 5, 6, 7, 8, 9, 10], QCD sum rules (see e.g. [11, 12, 13]), and constituent quark model (CQM) (see e.g. review [14] and references therein).

Results of computation of the lepton decay constants in the listed approaches are in rather poor agreement with each other and often have a sufficiently large theoretical uncertainties. So, the development of new approaches to this problem and the realization of new calculations are necessary nowadays. In particular, the calculations have to include the relativistic effects by correct way.

CQM is widely and successfully used for the description of hadron properties at low and intermediate energies [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. The reasons for this are well known: first, CQM uses the physically adequate degrees of freedom; second, CQM describes nonperturbative effects. These facts give the possibility to use CQM for the investigation of so-called "soft"structure of hadrons in contrast to QCD (see e.g., [29]).

Among the relativistic approaches the different forms of relativistic CQM hold an important position, for example, quasipotential approach [18], dispersion relations [23],

[1]Andreev Victor Vasil'evich, e-mail: andreev@gsu.unibel.by, Department of Theoretical Physics, Gomel State University, Sovietskaya St., 104, Gomel, 246019, Belarus.

[2]Krutov Alexander Fedorovich, e-mail: krutov@ssu.samara.ru, Department of General and Theoretical Physics, Samara State University, Ac. Pavlov St., 1, Samara, 443011, Russia.

[3]Shro Oleg Ivanovich, e-mail: oshro@ssu.samara.ru, Department of Computational Physics, Samara State University, Ac. Pavlov St., 1, Samara, 443011, Russia.

[4]Troitsky Vadim Eugenievich, e-mail: troitsky@theory.sinp.msu.ru, D.V. Skobeltsyn Institute of Nuclear Physics, Moscow State University, Vorobjovy Gory, Moscow, 119899, Russia.

and different forms of Relativistic Hamiltonian Dynamics (RHD): light-front dynamics [16, 24, 28] and point dynamics [22, 25] (the RHD has been detailed in references [30, 31, 32]).

The main feature of CQM versus QCD is the extraction of a finite number of the most important degrees of freedom needed to the hadron. Dynamical effects of QCD are incorporated in CQM through the effective (constituent) quark mass and internal quark structure in terms of the quark form factors. So, in the framework of CQM, constituent quarks have all the material properties of free particles and interact with each other through the confinement potential. This means that the constituent quark is characterized by an effective mass, a mean-square radius and so on. Let us remark that the concept of extended constituent quarks also appears in some quantum field theory models (see e.g. [33]). In this context one can imagine that CQM is initiated by QCD. However, it is very important for us to remind ourselves that CQM is not a direct consequence of QCD, but a very successful phenomenological model.

In this work for the calculation of the lepton decay constants of the heavy–light pseudoscalar mesons f_B , f_{Bs} , f_D , f_{Ds} we use the instant form of RHD in the version developed by the authors [17, 19, 20, 21, 27].

Contrary to field theory, RHD is dealing with finite number of degrees of freedom from the very beginning. This is certainly a kind of a model approach. The preserving of the Poincaré algebra ensures the relativistic invariance. So, the covariance of the description in the frame of RHD is due to the existence of the unique unitary representation of the inhomogeneous group $SL(2,C)$ on the Hilbert space of composite system states with finite number of degrees of freedom [30]. The mathematics of RHD is similar to that of nonrelativistic quantum mechanics and permits to assimilate the sophisticated methods of phenomenological potentials and can be generalized to describe three or more particles.

The distinctive property of our version of the instant form of RHD is the method of construction of current transition matrix elements [20, 27, 34], which includes the relativistic covariance conditions. Our approach to the construction of the current operator includes the following main points:

1. We extract from the current matrix element of composite system the reduced matrix elements (form factors) containing the dynamical information about the process. In general these form factors are generalized functions.
2. Along with form factors we extract from the matrix element a part which defines the symmetry properties of the current: the transformation properties under Lorentz transformation, discrete symmetries, conservation laws etc.
3. The physical approximations which are used to calculate the current are formulated not in terms of operators but in terms of form factors.

This approach was used successfully for the description of electromagnetic properties of light mesons [17, 20] and semileptonic decays of light and heavy–light mesons [34, 35].

In general the different model of quark–antiquark interaction can be used in RHD. In our work we use the interaction which incorporates wide varieties of such models, that is the smeared potential with linear rise at large distances, Coulomb behavior at small distances and spin-spin interaction. The variation of the smearing parameter transforms the potential behavior significantly.

In the present paper the wave functions in the sense of RHD are calculated approximately by the variational method. Wave function of ground state of harmonic oscillator is used as a trial function.

The lepton decay constants of the present paper are in good agreement with lattice calculations and existing experimental data. The ratios of these lepton decay constants agree well with the existing estimations, too.

The dependence of the lepton decay constants on quarks masses and uncertainties in heavy–light meson masses is investigated in the present paper. The uncertainty in the determination of the lepton decay constants from these dependences is varied through a range from 1.4% to 4.9% for different heavy–light mesons.

In the present paper it is shown that the relativistic effects are important for the lepton decay constants of B–, B_s–, D–, D_s– mesons.

The role of the corrections in the inverse mass of heavy quark $1/m_Q$ in the limit $m_Q \to \infty$ is clarified for different pseudoscalar heavy–light mesons.

The model independent estimation of the possible relativistic corrections in the lepton decay constant calculations in the potential approach is given.

The paper is organized as follows. In Sect. II we remind briefly the basic statements of RHD, especially of the instant form of RHD. The wave functions of composite systems are defined. In Sect. III the formulae for the lepton decay constants are derived in the instant form of RHD. Corresponding expression is obtained from the method of parameterization of electroweak current matrix element. In Sect.IV the procedure of calculations is described. The wave functions of the pseudoscalar mesons are calculated using variational method. The model parameters are discussed. Sect. V is devoted to the discussion of the results. Comparison of the present results with the results of the other approaches and experimental data is given. Sect.VI contains the conclusions.

2. Basic statements of RHD

Let us discuss the general features of RHD in brief (this approach has been detailed in references [30, 31, 32]).

In the RHD employing constituent quarks [27], mesons are considered as bound states of a quark q and an antiquark $\bar{Q}$. In this formalism the interaction is introduced in the generators of the Poincaré group without violating the form of the commutation relations in the algebra of the generators. Technically, this is achieved by adding the interaction operator $\hat{V}$ to the operator $\hat{M}_0$ of mass of the system free from interaction: $\hat{M}_0 \;\to\; \hat{M}_I = \hat{M}_0 + \hat{V}$, $\hat{M}_0^2 = (p_1 + p_2)^2 = P^2$. Here $\hat{M}_I$ is the mass operator of the system of interacting particles. In the instant form of RHD, the Poincaré algebra is conserved under this modification of the mass operator, provided that $\hat{V}$ commutes with the operator $\hat{\vec{J}} = (\hat{J}_1\ ,\ \hat{J}_2\ ,\ \hat{J}_3)$ of the total angular momentum, with the operator $\hat{\vec{P}}$ of the total 3–momentum, and with operator $\vec{\bigtriangledown}_P$.

In the RHD formalism, the wave functions are calculated as eigenfunctions of the complete set of commuting operators. In the instant form of RHD, the complete set of commuting operators consists of the following operators:

$$\hat{M}_I = \hat{M}_0 + \hat{V}\ , \qquad \hat{J}^2\ , \qquad \hat{J}_3\ , \qquad \hat{\vec{P}}\ . \tag{2.1}$$

In this version of dynamics, the operators $\hat{J}^2$, $\hat{J}_3$, and operator $\hat{\vec{P}}$ do not involve interaction; that is, they coincide with the corresponding operator of the free system.

There exists a basis in which the above three interaction–free operators are diagonal. Thus, the calculation of the wave function of a composite system reduces to diagonalizing the operator $\hat{M}_I$ in (2.1).

In the RHD formalism, the Hilbert space of states of a composite system is the tensor product of two single–particle spaces: $\mathcal{H}_{q\bar{Q}} = \mathcal{H}_q \otimes \mathcal{H}_{\bar{Q}}$. For a basis in the space $\mathcal{H}_{q\bar{Q}}$, we can choose, for example, the set of vectors

$$|\,\vec{p}_1\,, m_1;\,\vec{p}_2\,, m_2\rangle = |\,\vec{p}_1\,m_1\rangle \otimes |\,\vec{p}_1\,m_2\,\rangle\ , \tag{2.2}$$

$$\langle\vec{p}, m\,|\,\vec{p}^{\,\prime}\,m'\rangle = 2p_0\,\delta(\vec{p}-\vec{p}^{\,\prime})\,\delta_{mm'}\ .$$

Here $\vec{p}_1$, $\vec{p}_2$ are 3–momenta of particles, m_1 , m_2 are spin projections on the z axis, $p_0 = \sqrt{\vec{p}^{\,2} + m_q^2}$, m_q is the constituent quark mass.

To diagonalize the operators $\hat{J}^2$, $\hat{J}_3$, and $\hat{\vec{P}}$ from the set (2.1), we make use of a basis in which center–of–mass motion is separated explicitly. We take it in the form:

$$|\,\vec{P},\ \sqrt{s},\ J,\ l,\ S,\ m_J\,\rangle\ . \tag{2.3}$$

Here $P_\mu = (p_1 + p_2)_\mu$, $P_\mu^2 = s$, $\sqrt{s}$ is the invariant mass of the two-particle system, l is the orbital angular momentum in the center–of–mass frame (C.M.S.), $\vec{S}^{\,2} = (\vec{S}_1 + \vec{S}_2)^2 = S(S+1)$, S is the total spin in C.M.S., J is the total angular momentum with the projection m_J. The quantities l and S are constructed as invariant degeneracy parameters. The bases given by (2.2) and (2.3) are related to each other through the Clebsch–Gordan expansion of Poincaré group (see e.g. [19]).

As in the basis (2.3) the operators $\hat{J}^2$, $\hat{J}_3$, $\hat{\vec{P}}$ in (2.1) are diagonal, one needs to diagonalize only the operator $\hat{M}_I$ in (2.1) in order to obtain the system wave function.

The corresponding composite–particle wave function has the form [20]:

$$\langle\vec{P}\,,\,\sqrt{s},\ J,\ l,\ S,\ m_J|\,\Psi\rangle = N_c\,\delta(\vec{P}-\vec{p}_c)\delta_{J_cJ}\delta_{m_{J_c}m_J}\,\varphi_{lS}^{J_c}(k)\ . \tag{2.4}$$

Here J_c , m_{J_c} , $\vec{p}_c$ are quantum numbers in state $|\,\Psi\rangle$. The explicit form of N_c will not be used.

In pseudoscalar mesons $J = l = S = 0$ and we use in equation (2.4) for simplicity the notation: $\varphi_{lS}^J(k)\ \rightarrow\ \varphi(k)$, $\varphi(k)$ is a phenomenological wave function normalized with the account of relativistic density of states [20]:

$$\varphi(k(s)) = \sqrt{\sqrt{s}(1-\eta^2/s^2)}u\,(k)\,k\ , \qquad n_c\int\,|u\,(k)|^2\,k^2\,dk = 1\,, \tag{2.5}$$

$$k = \frac{\left[s^2 - 2s\left(m_q^2 + m_Q^2\right) + \eta^2\right]^{1/2}}{2\sqrt{s}}\ .$$

m_q , m_Q are the masses of light and heavy quark respectively, $n_c = 3$ is the number of quark colors, $\eta = m_Q^2 - m_q^2$.

3. Calculation of the lepton decay constant of the pseudoscalar meson

Let us discuss now the calculation of the leptonic decay constants in our approach. The leptonic decay constant of the pseudoscalar meson f_P is determined by the matrix element of the electroweak current [16]:

$$\langle 0|J^\mu|\Psi\rangle\ = \langle 0|J^\mu|\vec{p}_c\rangle\ =\ i\,f_P\,p_c^\mu\,\frac{1}{(2\pi)^{3/2}}\ , \tag{3.6}$$

where p_c is 4-momentum of meson.

Let us expand the left-hand side of (3.6) in the basis given by (2.3). Since we have $J = S = l = 0$ for the pseudoscalar meson, the corresponding symbols are omitted in basis vectors. Taking into account the explicit form of the meson wave function (2.4), we can rewrite the equation (3.6) in the form

$$\int \frac{N_c}{N_{CG}}\, d\sqrt{s}\, \langle 0|J^\mu|\vec{p}_c, \sqrt{s}\rangle\, \varphi(k(s)) \;=\; i\, f_P\, p_c^\mu\, \frac{1}{(2\pi)^{3/2}}\;. \tag{3.7}$$

N_{CG} is the normalization factor of vectors in the basis (2.3). The explicit form of N_{CG} will not be used.

Following our method of construction of current transition matrix elements [20, 27, 34] it is necessary to extract from the current matrix element in (3.7) the reduced matrix elements (form factors) containing the dynamical information on the process and a part which defines the transformation properties under Lorentz transformation. In the case of pseudoscalar mesons our method has a simple form. The left–hand side of (3.7) can be represented as a functional that is defined on the space of test functions and that specifies a Lorentz covariant distribution (generalized function). This distribution can be represented as the product of a Lorentz covariant smooth function and a Lorentz invariant distribution [27]. Following [19, 20, 27], we can therefore break down the integrand in (3.7) into a covariant factor, which is a smooth function, and an invariant factor representing the distribution:

$$\frac{N_c}{N_{CG}}\, \langle 0|J^\mu|\vec{p}_c, \sqrt{s}\rangle \;=\; i\, G(s)\, B^\mu(s)\, \frac{1}{(2\pi)^{3/2}}\;. \tag{3.8}$$

The invariant form factor (reduced matrix element) $G(s)$ is a distribution, 4–vector $B^\mu(s)$ defines the transformation properties of matrix elements in (3.7). $G(s)$ contains all the dynamical information on the process. In what follows we shall formulate the physical approximations not in terms of current operator but in terms of form factor $G(s)$.

If the equality (3.7) is considered as the equality of functionals specifying distributions on the space of test functions, the explicit form of the vector B_μ is

$$B^\mu(s) \;=\; p_c^\mu\;. \tag{3.9}$$

To prove (3.9), it is sufficient to notice that the covariant factor on the right-hand side of (3.7) (4–vector p_c^μ) remains unchanged when we go over from one function belonging to the space of test functions to another. From (3.7)–(3.9), it follows that, for the leptonic decay constant, we have the integral representation

$$\int d\sqrt{s}\, G(s)\, \varphi(k(s)) \;=\; f_P\;. \tag{3.10}$$

The form factor $G(s)$ can generally be calculated within the Standard Model of electroweak interactions. In this study, however, we will restrict ourselves by calculating $G(s)$ in the approximation of four–fermion interaction. For $G(s)$, we take form factor that parameterizes the weak current of the free two-quark system as

$$\langle 0|J_0^\mu|\vec{P}, \sqrt{s}\rangle \;=\; i\, G_0(s)\, P^\mu\, \frac{1}{(2\pi)^{3/2}}\;. \tag{3.11}$$

The explicit form of the matrix element (3.11) is prescribed by the general method for parameterizing the matrix elements of current operators (see e.g. [20, 27]).

The representation (3.11) is analogous to the representation (3.6), but the constant f_P is replaced by the form factor, which depends on the invariant variable s.

To calculate $G_0(s)$, we expand (3.11) in single–particle basis (2.2). Equality (3.11) then reduces to the form

$$i\,G_0(s)\,P^\mu\,\frac{1}{(2\pi)^{3/2}} \;=\; \sum_{m_1,m_2,i_c}\int\frac{d\vec p_1}{2p_{10}}\frac{d\vec p_2}{2p_{20}}\langle 0|J_0^\mu|\vec p_1,m_1;\vec p_2,m_2\rangle$$

$$\times\langle\vec p_1,m_1;\vec p_2,m_2|\vec P,\sqrt{s}\rangle\;, \tag{3.12}$$

where $i_c\;=\;1,\,2,\,3$ is index of summation over quark color. The expressions for the Clebsch-Gordan coefficients of the Poincaré group $\langle\vec p_1,m_1;\vec p_2,m_2|\vec P,\sqrt{s}\rangle$ can be found in [19]. The matrix element of the current in the basis specified by (2.2) is determined by the standard expression for the matrix element of the leptonic–decay current [16]. As a result we have

$$\langle 0|J_0^\mu|\vec p_1,m_1;\vec p_2,m_2\rangle \;=\; \frac{1}{(2\pi)^3}\bar v(\vec p_2,m_2)\gamma^\mu(1+\gamma^5)u(\vec p_1,m_1)\;. \tag{3.13}$$

Integration in (3.12) is performed in the reference frame where $\vec P\;=0$. For $G_0(s)$, we then obtain

$$G_0(s) = \frac{n_c\,\sqrt{(\omega_{m_q}(k)+m_q)(\omega_{m_Q}(k)+m_Q)}}{2\sqrt{2}\pi\,(\omega_{m_q}(k)+\omega_{m_Q}(k))}$$

$$\times\left[1-\frac{k^2}{(\omega_{m_q}(k)+m_q)\,(\omega_{m_q}(k)+m_Q)}\right]\;, \tag{3.14}$$

here $\omega_{m_i}(k)=\sqrt{k^2+m_i^2}\;,\;i=q,Q$. The final expression for the lepton decay constants of pseudoscalar mesons is following:

$$\int d\sqrt{s}\,G_0(s)\,\varphi(s) = f_P\;. \tag{3.15}$$

Equation (3.15) can be reduced to the following form:

$$f_P = \frac{n_c}{\sqrt{2}\pi}\int g(k)\,k^2\,u(k)\,dk\;, \tag{3.16}$$

here

$$g\,(k) = \sqrt{\frac{(m_Q+m_q)^2-\big(\omega_{m_q}(k)-\omega_{m_Q}(k)\big)^2}{\omega_{m_Q}(k)\,\omega_{m_q}(k)\,\big(\omega_{m_q}(k)+\omega_{m_Q}(k)\big)}}\;. \tag{3.17}$$

Let us remark that expression for the lepton decay constant of pseudoscalar mesons (3.15) (or (3.16)) coincides with those ones in the light–front form [16] and the point form [25] of RHD and in the dispersion relations approach [23]. However, for example, the expressions for electromagnetic form factors are different in these approaches.

4. Calculations

In this work we use the variational method for the approximate calculation of function $u(k)$ in equations (2.4), (2.5) (see [36], too). It the case of RHD this method is reduced to the calculation of the minimum of the following functional:

$$M_P(\alpha\,,\,\beta\,,\,\gamma\,,\ldots) = \langle\Psi|\hat M_I|\Psi\rangle = \langle\Psi|\hat M_0|\Psi\rangle + \langle\Psi|\hat V|\Psi\rangle\;. \tag{4.18}$$

Here α, β, γ, ... are parameters including the parameters of trial function, M_P is meson mass.

The matrix elements of $\hat{M}_0$ and $\hat{V}$ are calculated in momentum and coordinate representations respectively:

$$\langle\Psi|\hat{M}_0|\Psi\rangle = n_c \int k^2 dk |u(k)|^2 \left(\sqrt{k^2+m_q^2}+\sqrt{k^2+m_{\bar{Q}}^2}\right) , \quad (4.19)$$

$$\langle\Psi|\hat{V}|\Psi\rangle = n_c \int r^2 dr |\psi_0(r)|^2 \tilde{V}(r) , \quad (4.20)$$

here $u(k)$ is phenomenological wave function (2.4), (2.5), $\tilde{V}(r)$ is the interaction operator in coordinate representation. Wave function $\psi_0(r)$ is determined as follows:

$$\psi_0(r) = \sqrt{\frac{2}{\pi}} \int k^2\, j_0(kr)\, u(k)\, dk , \quad n_c \int r^2 |\psi_0(r)|^2\, dr = 1 ,$$

where $j_0(x)$ is spherical Bessel function:

$$j_0(x) = \frac{\sin\, x}{x} .$$

For the interaction operator in coordinate representation $\tilde{V}(r)$ we use the smeared potential with Coulomb behavior at small distances, linear confinement and spin–spin interaction. The smearing is realized by the scheme [15, 37]:

$$\tilde{V}(r) = \int \rho(\vec{r}-\vec{r}\,')\, V(r)\, d\vec{r}\,' ,$$

$$\rho(\vec{r}-\vec{r}\,') = \frac{\sigma^3}{\pi^{3/2}} \exp(-\sigma^2(\vec{r}-\vec{r}\,')^2) . \quad (4.21)$$

Here σ is the smearing parameter.

This interaction incorporates wide varieties of models because the variation of the smearing parameter transforms the potential behavior significantly.

The operator of interaction has the following form:

$$\tilde{V}(r) = \tilde{V}_{Coulomb}(r) + \tilde{V}_{linear}(r) + \tilde{V}_{SS}(r) . \quad (4.22)$$

The smeared Coulomb part of interaction is represented in the following way:

$$\tilde{V}_{Coulomb}(r) = -\frac{4}{3}\sum_{k=1}^{3}\frac{\alpha_k}{r}\,\mathrm{erf}(\tau_k\, r) , \quad (4.23)$$

$$\mathrm{erf}(x) = \left(\frac{2}{\sqrt{\pi}}\right)\int_0^x \exp(-t^2)\, dt ,$$

here $\mathrm{erf}(x)$ is the error function.

The following parameterization of the running strong coupling constants was used to obtain the representation (4.23) [15, 37]:

$$\alpha_s(Q^2) = \sum_{k=1}^{3} \alpha_k\, \exp(-Q^2/4\gamma_k^2) , \quad (4.24)$$

here $\alpha_1 = 0.25$, $\alpha_2 = 0.15$, $\alpha_3 = 0.2$, $\gamma_1^2 = 1/4$, $\gamma_2^2 = 5/2$, $\gamma_3^2 = 250$,

$$\alpha_s^{critical} \;=\; \sum_k \alpha_k \;, \quad \alpha_s^{critical} \;=\; 0.60 \;, \quad \frac{1}{\tau_k} \;=\; \frac{1}{\gamma_k} \;+\; \frac{1}{\sigma} \;.$$

The confining part of interaction has the form:

$$\tilde{V}_{linear}(r) \;=\; \beta\, r \left[\frac{\exp(-\sigma^2\, r^2)}{\sqrt{\pi}\sigma r} \;+\; \left(1 \;+\; \frac{1}{2\sigma^2 r^2}\right) \mathrm{erf}(\sigma\, r) \right] + \omega_0 \;. \tag{4.25}$$

Spin-spin part of the interaction is following:

$$\tilde{V}_{SS}(r) \;=\; -\frac{32\sigma^3}{9\sqrt{\pi}\; m_q m_Q} \left(\vec{S}_q \vec{S}_Q\right) \exp(-\,\sigma^2\, r^2) \sum_k \alpha_k \,\mathrm{erf}(\gamma_k\, r) \;. \tag{4.26}$$

Here S_q, S_Q are spin operators of light and heavy quarks respectively.

As trial function in (4.19), (4.20) we use the wave function of the ground state of harmonic oscillator:

$$u(k) = N_{HO} \exp\left(-\frac{k^2}{2\, b^2}\right) \;. \tag{4.27}$$

Our relativistic approach to the calculation of the decay constant for B–, B_s–, D–, D_s–mesons contains the conventional set of the model parameters: parameter of the trial function (4.27) b, masses of quarks $m_u = m_d$, m_s , m_b , m_c as well as parameters of the interaction operator in (4.22), (4.23), (4.25), (4.26) β , σ and ω_0.

Let us discuss the choice of the values of these parameters in our calculation. The parameters of the trial functions b in (4.27) are determined by the minimum conditions for the functional (4.18) if all other parameters are fixed:

$$M_P\left(b, m_q, m_Q, \beta, \sigma, \omega_0\right) = \min\, M_P\left(b, m_q, m_Q, \beta, \sigma, \omega_0\right) \;. \tag{4.28}$$

The parameters of the interaction operator in the CQM are determined usually in a phenomenological way, from the description of meson spectra. The parameter of the linear part of interaction in (4.25) is

$$\beta = 0.18\ \mathrm{GeV}^2 \;. \tag{4.29}$$

This value β was used in the current calculations (see e.g. [15, 26, 38, 39]).

The values of smearing parameter σ in (4.21)-(4.26) are constrained by additional condition on the difference of meson masses in the states with spin $S = 1$ and spin $S = 0$:

$$M^{S=1}\left(b, m_q, m_Q, \beta, \sigma, \omega_0\right) - M^{S=0}\left(b, m_q, m_Q, \beta, \sigma, \omega_0\right) = \Delta M^{exp} \;. \tag{4.30}$$

We used the values for the ΔM^{exp} from reference [40].

The parameter ω_0 in (4.25) provides the fulfilling of the following equality:

$$\min\, M_P\left(b, m_q, m_Q, \beta, \sigma, \omega_0\right) = M_P^{exp} \;, \tag{4.31}$$

where M_P^{exp}- experimental value of the pseudoscalar meson mass [40].

We are coming now to the question of the choice of light quark masses in our approach. The mass of light quark $m_u = m_d$ and the mass of s-quark can be fixed from the description of the experimental lepton decays constants for the pion and kaon and experimental values of the pion and kaon masses.

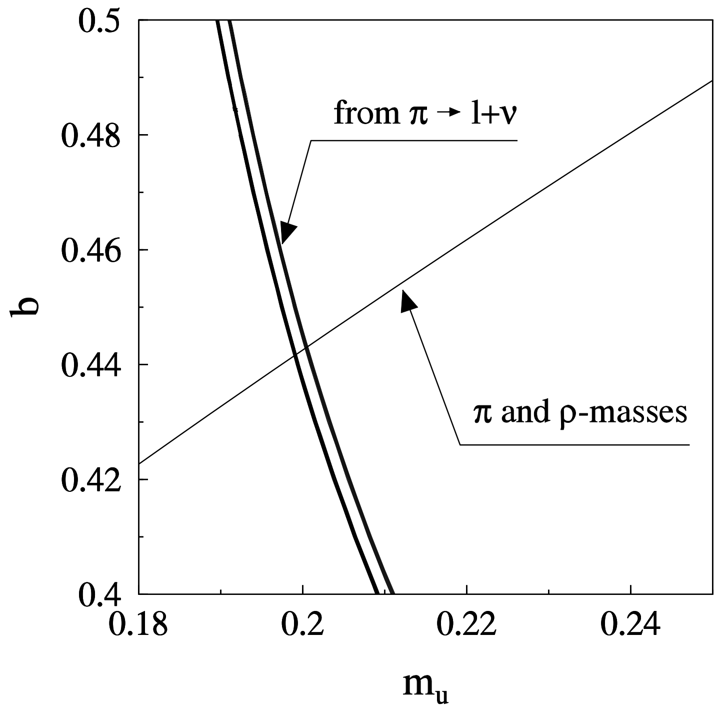

Рис. 1. Illustrative plots of the solution of the equalities (4.32) and (4.28) – (4.31) for $b(m_u)$ in the case of pion.

If one uses the expressions (3.14), (3.15) for the calculation of the pion lepton decay constant f_π then the condition

$$f_\pi = f_\pi^{exp} = 130.7 \ \pm 0.1 \pm 0.36 \text{ MeV} , \tag{4.32}$$

determines the implicit function $b(m_u)$. The experimental errors in (4.32) give an acceptable values for b and m_u to satisfy the equality (4.32) (the domain between steeply two sloped curves in figure 1). Pion wave function (4.27) is calculated by variational method with interaction (4.21) – (4.26). Let us solve the system of equations (4.28) – (4.31) for the pion parameters b , σ and ω_0 with different values of mass m_u and following experimental values [40]:

$$M^\pi_{exp} = 139.56995 \pm 0.00035 \text{ MeV} ,$$
$$\Delta M^{exp} = M^\rho - M^\pi = 627.33 \ \pm \ 0.8 \text{ MeV} , \tag{4.33}$$

here M^π , M^ρ are pion and ρ-meson masses respectively. The solution of the equations (4.28) – (4.31) depends on the experimental errors in (4.33) weakly. Values b as the solution of the equations (4.28) – (4.31) with different m_u determine the function $b(m_u)$ (increasing curve in figure 1).

So, a simultaneous solution of equations (4.32) and (4.28) – (4.31) exists for following light–quark masses only (see figure 1):

$$m_u = m_d = 0.200 \ \pm \ 0.01 \text{ GeV} . \tag{4.34}$$

Analogous procedure can be performed for kaon using (4.32):

$$f_K = f_K^{exp} = 160.6 \ \pm 1.4 \text{ MeV} , \tag{4.35}$$

and experimental values of the corresponding meson masses from reference [40]:

$$M_{exp}^K = 493.677 \ \pm \ 0.016 \text{ MeV} ,$$

$$\Delta M^{exp} = M^{K^*} - M^K = 397.944 \ \pm 0.24 \text{ MeV} , \tag{4.36}$$

here M^K , M^{K^*} are masses of the scalar K-meson and K^*-meson. The case of the s-quark mass is illustrated in figure 2. Taking into account (4.34) we obtain:

$$m_s = 0.380 \ \pm \ 0.100 \text{ GeV} . \tag{4.37}$$

It is interesting to remark that our value of mass for u- and d-quarks is close to that obtained by appreciably model independent way in reference [41]. Let us note that the masses of $u-$ and s-quarks (4.34), (4.37) give for $SU(3)$–breaking parameter:

$$(m_s - m_u)/m_u = 0.9 .$$

This value is considerably greater than in reference [42]: $\simeq 0.37$.

The following values for the heavy quark masses are the most used in current calculations:

$$1.30 \le m_c \le 1.88 \text{ GeV} , \qquad 4.60 \le m_b \le 5.28 \text{ GeV} . \tag{4.38}$$

The m_c and m_b were varied in our calculations in these intervals. Uncertainty in the values of quark masses (4.34), (4.37), (4.38) is the main source of the theoretical uncertainty in the calculation of the lepton decay constants in our work. Uncertainties of our results associated with uncertainties in the values of meson masses are insignificant.

The results of our calculation of the constants f_B, f_{Bs}, f_D, f_{Ds} are given in 1. Presented values are obtained using the mean values of parameters in the intervals (4.34), (4.37) and (4.38).

5. Results

Let us discuss the results of calculation. The firm experimental data exist for the value of f_{Ds} only. Therefore the calculation of this value can serve to test our theoretical predictions for other lepton constants. This statement is particularly important for the constant f_B because the measurement of f_B from $B \to l \, \bar{\nu}_l$ decay is currently not feasible [44]. Our result for f_{Ds} coincides with experiments in the limits of experimental errors. It should be remarked that our f_{Ds} is in good agreement with lattice and sum rules calculations.

The last remark is true for the constant f_B obtained in this paper, too.

Our constant f_{Bs} is in reasonably good agreement with lattice calculations, but our result correlates poorly with the recent sum rules calculation [13].

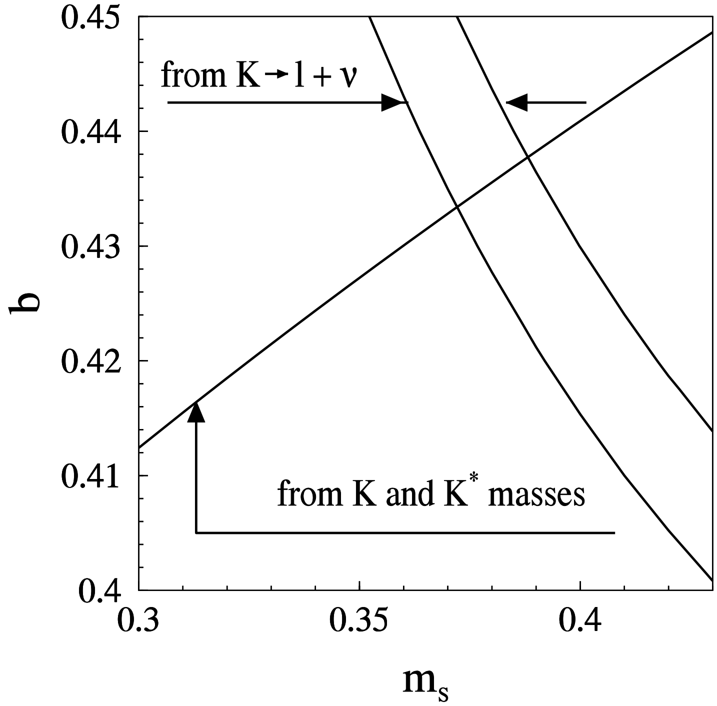

Рис. 2. Illustrative plots of the solution of the equalities (4.32) and (4.28) – (4.31) for $b(m_s)$ in the case of kaon.

Finally, our value of f_D agrees closely with an average over different calculations on the lattice [9, 10] and with that in sum rules [12]. The only experimental result for f_D [43] has large errors and corresponding minor criticality for existing calculations.

Let us remark that obtained in present paper ratios of the lepton decay constants are in good agreement with the lattice calculations [8] (taking into account our theoretical uncertainty):

$$\frac{f_{B_s}}{f_B} = 1.203(29)(28)\left(^{+38}_{-0}\right) \qquad \frac{f_{D_s}}{f_D} = 1.182(39)(25)\left(^{+41}_{-0}\right)\ . \tag{5.39}$$

Our approach gives the model independent estimation for the lepton decay constant in the CQM with the help of equations (3.16),(3.17). Using the fact that function (3.17) in equation (3.16) is monotonically decreasing we can obtain the upper and lower limits for the lepton decay constant:

$$\frac{n_c}{\sqrt{2}\pi}\, g(m_Q) \int k^2\, u(k)\, dk \;\leq\; f_P \;\leq\; \frac{n_c}{\sqrt{2}\pi}\, g(m_q) \int k^2\, u(k)\, dk\ . \tag{5.40}$$

Таблица 1. The lepton decay constants of heavy–light pseudoscalar mesons. Presented values were obtained with the mean values of parameters in the intervals (4.34), (4.37) and (4.38). The values are given in MeV. The stared theoretical values are averages over different calculations. The stared experimental value is a current world average.

	f_B	f_{Bs}	f_D	f_{Ds}
Present paper	145 ± 2	189 ± 4	194 ± 6	246 ± 12
Lattice	160^{+59}_{-25} [2]	185^{+46}_{-10} [2]	194^{+68}_{-11} [2]	212^{+50}_{-11} [2]
calculations	147^{+34}_{-38} [3]	175^{+40}_{-36} [3]	192^{+42}_{-19} [4]	210^{+51}_{-19} [4]
	157^{+69}_{-20} [4]	171^{+71}_{-21} [4]	211^{+16}_{-26} [6]	240^{+30}_{-25} [5]
	179^{+52}_{-27} [6]	204^{+52}_{-16} [6]	$225\pm30^*$ [9, 10]	231^{+20}_{-13} [6]
	204^{+81}_{-37} [8]	187 ± 34 [7]		223^{+110}_{-108} [7]
	$200\pm30^*$ [9, 10]	242^{+81}_{-43} [8]		
Sum rules	$178\pm42^*$ [12]	232 ± 25 [13]	$188\pm48^*$ [12]	231 ± 24 [11]
			195 ± 20 [12]	
Experiments			$300^{+180+80}_{-150-40}$ [43]	$280\pm48^*$ [40]
				$286\pm44\pm41$ [44]

The nonrelativistic expression for the lepton decay constant can be obtained from the equation (3.15) by taking the nonrelativistic limit which gives the standard form for the CQM in terms of coordinate space wave function in the origin:

$$f_P^{NR} = \frac{\sqrt{2}\, n_c}{\sqrt{m_q + m_Q \pi}} \int k^2\, u(k)\, dk = \frac{n_c}{\sqrt{\pi}\,\sqrt{m_q + m_Q}} \psi_0(0)\ . \tag{5.41}$$

So, we have the following model independent estimation from equations (5.40) and (5.41):

$$\frac{1}{2}\sqrt{m_q + m_Q}\, g(m_Q)\ \leq\ \frac{f_P}{f_P^{NR}}\ \leq\ \frac{1}{2}\sqrt{m_q + m_Q}\, g(m_q)\ . \tag{5.42}$$

The upper and lower limits are independent on the model of quark–antiquark interaction in the heavy–light mesons.

It is possible to obtain the approximate model independent estimation for the relativistic corrections for the lepton decay constant in the framework of the CQM:

$$f_P = f_P^{NR} + \Delta\, f_P^R\ . \tag{5.43}$$

Corresponding estimation is given by following inequalities:

$$\frac{1}{2}\sqrt{m_q + m_Q}\, g(m_Q) - 1\ \leq\ \frac{\Delta\, f_P^R}{f_P^{NR}}\ \leq\ \frac{1}{2}\sqrt{m_q + m_Q}\, g(m_q) - 1\ . \tag{5.44}$$

The relativistic correction $\Delta\, f_P^R$ has the negative sign.

Let us discuss briefly the relativistic effects obtained in our calculations. Corresponding estimation can be obtained by comparison of equations (3.15) and (5.41). The relativistic effects are very significant and have following values for the calculated constants: for f_B and f_{Bs} they are 28% and 24%, for the more light mesons f_D and f_{Ds} they are 43% and 40%, respectively.

It is interesting to calculate the lepton decay constants in the limit $m_Q \to \infty$ (see [45] also). The corresponding asymptotic estimation of the integral representation (3.15) with wave function (4.27) gives the following expression for the leading term:

$$f_P \sim \frac{\sqrt{2\,n_c}}{\pi^{3/4}} \frac{b^{3/2}}{m_Q^{1/2}}\,. \tag{5.45}$$

Using the values parameters listed above we obtain: $f_{B\infty} = 146$ MeV, $f_{Bs\infty} = 182$ MeV, $f_{D\infty} = 256$ MeV, $f_{Ds\infty} = 321$ MeV. One can see that the leading term gives good description for B, B_s– mesons, while it is not true for D, D_s– mesons (compare with 1).

It is important to emphasize that the sensibility of our results to the variation of quark masses in the intervals (4.34), (4.37) and (4.38) is weak. The change of the quark masses in this interval gives rise to the change of the lepton decay constants for B-meson not more than 1.4%, for B_s-meson – 2.1%, D-meson – 3.1%, D_s-meson – 4.9%.

6. Conclusion

In the present paper the lepton decay constants of the heavy–light mesons B, B_s, D, D_s are calculated in the framework of the instant form of relativistic Hamiltonian dynamics. Interaction between constituent quarks is taken in the form of the smeared potential with linear rise at large distances, Coulomb behavior at small distances and spin-spin interaction. Wave functions of mesons in sense of RHD are calculated by variational method. Results are in good agreement with lattice calculations and existing experiments.

The importance of relativistic corrections is established for all the mesons. For the B–, B_s–mesons it is obtained that the corrections in the inverse mass of the heavy quark $1/m_Q$ are small in the limit $m_Q \to \infty$.

Model independent constraints are obtained for the relativistic corrections to the lepton decay constants in the constituent quark model. It is obtained that the values of these corrections in the different version CQM can be varied in the wide limits.

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КОНСТАНТЫ ЛЕПТОННЫХ РАСПАДОВ ПСЕВДОСКАЛЯРНЫХ МЕЗОНОВ В РЕЛЯТИВИСТСКОЙ ГАМИЛЬТОНОВОЙ ДИНАМИКЕ

© 2003 В.В. Андреев[1], А.Ф. Крутов[2], О.И. Шро[3], В.Е. Троицкий[4]

Аннотация

В рамках мгновенной формы релятивистской гамильтоновой динамики (РГД) выполгнено вычисление констант лептонных распадов псевдоскалярных B -, B_s -, D -, D_s-мезонов, содержащих один тяжелый кварк. Взаимодействие между кварками описывается при помощи "размазанного"потенциала с кулоновским поведением на малых расстояниях, линейным конфайнментом и спин-спиновым взаимодействием. Волновые функции в смысле РГД вычисляются при помощи вариационного метода. Массы легких кварков фиксируются из описания констант лептоного распада пиона и каона. Производится оценка теоретической неопределенности расчетов за счет разброса значений масс кварков и тяжелых мезонов. Результаты расчетов находятся в хорошем согласии с эксперементами и результатами расчетов на решетках.

[1]Андреев Виктор Васильевич, эл.почта: andreev@gsu.unibel.by, кафедра теоретической физики Гомельского государственного университета, Беларусь, 246019, Гомель, ул. Советская, 104.

[2]Крутов Александр Федорович, эл.почта: krutov@ssu.samara.ru, кафедра общей и теоретической физики Самарского государственного университета, Россия, 4430111, Самара, ул. Академика Павлова, 1.

[3]Шро Олег Иванович, эл.почта: oshro@ssu.samara.ru, кафедра вычислительной математики Самарского государственного университета, Россия, 4430111, Самара, ул. Академика Павлова, 1.

[4]Троицкий Вадим Евгеньевич, эл.почта: troitsky@theory.sinp.msu.ru, Научно-исследовательский институт ядерной физики им. Д.В.Скобельцына, Московский государственный университет, Россия, 119899, Москва, Воробьевы горы.