COMPUTER MODELING OF PHYSICAL PROCESSES, DEVICES, SYSTEMS IN INDUSTRY AND EDUCATION

RADIATIVE DECAYS OF VECTOR MESONS IN POICARE-COVARIANT QUARK MODEL

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Radiative decays of vector mesons is a handy tool for studying the structure of hadrons as these processes are "purely" hadrons and don't require additional relations of electroweak theory. In our work, the calculation of the form-factor of the radiative decay conducted within the constituent relativistic quark model based on the point form of Poincare-invariant quantum mechanics or Relativistic Hamiltonian dynamics (**RHD**): work in this form **RHD** has several advantages in the calculations, one of which is a match 4-rates for systems with and without interaction.

$V \to P\gamma$ **decay in Relativistic Hamiltonian dynamics.** The expression for the decay constants can be parameterized using the 4-velocities of the initial and final meson by the following expression:

$$F_{VP\gamma}(q^2)K^{\alpha}(\mu)=(2\pi)^3\ \frac{\sqrt{4V_0V_0^{/}}}{\sqrt{MM^{/}}}\ {}_P\langle\vec{Q}'|J^{\alpha}|\vec{Q}\rangle_V, \qquad (1)$$

where $K^{\alpha}(\mu)=i\varepsilon^{\alpha\nu\rho\sigma}\varepsilon_{\nu}(\mu)V_{\rho}V_{\sigma}^{/}$. This parameterization is convenient for a point form of **RHD**.

In this paper, we consider mesons $\mathrm{V}(Q,M)$ and $\mathrm{P}(Q^{/},M^{/})$ as relativistic constituent quark q and anti-quark $\bar{Q}$ system in the framework of Poincare-invariant quantum mechanics. In this approach, this decay is caused by the emission of a γ-quantum by the quark, entering the meson V. Since the Poincare-invariant quantum mechanics allows to relate the state vector mesons with the state vector of its constituent quarks $p_1=(\omega_{m_q}(p_1),\vec{p}_1)$ and $p_2=(\omega_{m_{\bar{Q}}}(p_2),\vec{p}_2)$, we shall construct a basis of the direct product of two quark masses m_q and $m_{\bar{Q}}$ with helicity λ_1 and λ_2:

$$|\,\vec{p}_1,\lambda_1\rangle|\,\vec{p}_2,\lambda_2\rangle\equiv|\,\vec{p}_1,\lambda_1;\vec{p}_2,\lambda_2\rangle. \qquad (2)$$

Using the Clebsch-Gordan decomposition of the Poincare group for the scheme with «L-S» scheme [1], write down the initial and final condition using the full and relative momentum of the two quarks [2] :

$$|\vec{Q}\rangle_V=\int d\vec{k}\sqrt{\frac{\omega_{m_q}(p_1)\omega_{m_{\bar{Q}}}(p_2)}{\omega_{m_q}(k)\omega_{m_{\bar{Q}}}(k)V_0}}\,\frac{\Psi^{\mu}(k)}{2\sqrt{\pi}}\sum_{\lambda_1,\lambda_2}\sum_{\nu_1,\nu_2}C^{\frac{1}{2},\frac{1}{2},1}_{\nu_1,\nu_2,\mu}D^{1/2}_{\lambda_1,\nu_1}(\vec{n}_{W_1})D^{1/2}_{\lambda_2,\mu-\nu_1}(\vec{n}_{W_2})|\,\vec{p}_1,\lambda_1;\vec{p}_2,\lambda_2\rangle, \qquad (3)$$

$$|\vec{Q}'\rangle_P=\int d\vec{k}'\sqrt{\frac{\omega_{m_q}(p_1')\omega_{m_{\bar{Q}}}(p_2')}{\omega_{m_q}(k')\omega_{m_{\bar{Q}}}(k')V_0'}}\,\frac{\Phi(k')}{2\sqrt{\pi}}\sum_{\lambda_1',\lambda_2'}\sum_{\nu_1',\nu_2'}C^{\frac{1}{2},\frac{1}{2},0}_{\nu_1',\nu_2',0}D^{1/2}_{\lambda_1',\nu_1'}(\vec{n}_{W_1}')D^{1/2}_{\lambda_2',-\nu_1'}(\vec{n}_{W_2}')|\,\vec{p}_1',\lambda_1';\vec{p}_2',\lambda_2'\rangle, \qquad (4)$$

with the Clebsch-Gordon coefficients:

$$C^{\frac{1}{2},\frac{1}{2},1}_{\nu_1,\nu_2,\mu}=\frac{\sqrt{3+4\nu_1\nu_2}}{2}\delta_{\mu,\nu_1+\nu_2},\ C^{\frac{1}{2},\frac{1}{2},0}_{\nu_1',\nu_2',0}=\sqrt{2}\nu_1^{/}\delta_{\nu_1^{/},-\nu_2^{/}}. \qquad (5)$$

Compliance with the requirements of the Poincare-invariance in the framework of the point form RGD in expressions (3) and (4) has led to the appearance of the wave functions of vector $\Psi^{\mu}(k)$ and scalar $\Phi(k')$ meson as bound systems, which, given the number of quark colors normalized expression

$$N_C\int_0^{\infty}d\vec{k}\vec{k}^2|\Psi^{\mu}(k)|=N_C\int_0^{\infty}d\vec{k}'\vec{k}'^2|\Phi(k')|=1\ . \qquad (6)$$

Substitution of the electromagnetic current operator

$$\hat{J}^{\mu} = e\ \hat{\bar{\psi}}(x)\Gamma^{\alpha}\hat{\psi}(x) \qquad (7)$$

with [3]

$$\Gamma^{\alpha} = F_1(q^2)\gamma^{\alpha} + \frac{i\sigma^{\alpha\beta}}{2m} q_{\beta} F_2(q^2) \qquad (8)$$

in the expression (1) using the expression (3), (4) and (5) leads us to

$$F_{VP\gamma}(q^2) = \frac{1}{2\pi}\sum_{\lambda_1,\lambda_2}\sum_{\nu_1,\nu_2}\sum_{\lambda_1',\lambda_2'}\sum_{\nu_1',\nu_2'}\int\int d\vec{k}d\vec{k}' \sqrt{\frac{\omega_{m_q}(p_1)\omega_{m_{\bar{Q}}}(p_2)M}{\omega_{m_q}(k)\omega_{m_{\bar{Q}}}(k)}}\sqrt{\frac{\omega_{m_q}(p_1')\omega_{m_{\bar{Q}}}(p_2')M'}{\omega_{m_q}(k')\omega_{m_{\bar{Q}}}(k')}}\sqrt{\frac{3+4\nu_1\nu_2}{2}}\nu_1'\delta_{\mu-\nu_1,\nu_2}\delta_{\nu_1',-\nu_2'}$$
$$\frac{(MM')^{-1}}{(KK^*)}\Psi(k)\Phi(k')(e_q\frac{D^{*1/2}_{\lambda_1',\nu_1'}(\vec{n}'_{W_1})\bar{u}_{\lambda_1'}(\vec{p}_1',m_q)(\Gamma K^*)D^{1/2}_{\lambda_1,\nu_1}(\vec{n}_{W_1})u_{\lambda_1}(\vec{p}_1,m_q)}{\sqrt{4\omega_{m_q}(p_1')\omega_{m_q}(p_1)}}D^{*1/2}_{\lambda_2',\nu_2'}(\vec{n}'_{W_2})\langle\vec{p}_2',\lambda_2'|\vec{p}_2,\lambda_2\rangle D^{1/2}_{\lambda_2,\nu_2}(\vec{n}_{W_2})+ \qquad (9)$$
$$+e_{\bar{Q}}\frac{D^{1/2}_{\lambda_2,\nu_2}(\vec{n}_{W_2})\bar{\upsilon}_{\lambda_2}(\vec{p}_2,m_{\bar{Q}})(\tilde{\Gamma}K^*)D^{*1/2}_{\lambda_2',\nu_2'}(\vec{n}'_{W_2})\upsilon_{\lambda_2'}(\vec{p}_2',m_{\bar{Q}})}{\sqrt{4\omega_{m_{\bar{Q}}}(p_2)\omega_{m_{\bar{Q}}}(p_2')}}D^{*1/2}_{\lambda_1',\nu_1'}(\vec{n}'_{W_1})\langle\vec{p}_1',\lambda_1'|\vec{p}_1,\lambda_1\rangle D^{1/2}_{\lambda_1,\nu_1}(\vec{n}_{W_1})).$$

To further simplification of expression (9) we use the transformation formula Dirac bispinors [1]

$$\sum_{\sigma} D^{1/2}_{\sigma,\lambda}(\vec{n}_W(\vec{k},\vec{Q}))u_{\sigma}(\vec{p},m) = B(\vec{u}_Q)u_{\lambda}(\vec{k},m),$$
$$\sum_{\sigma}\bar{u}_{\sigma}(\vec{p},m)D^{*1/2}_{\sigma,\lambda}(\vec{n}_W(\vec{k},\vec{Q})) = \bar{u}_{\lambda}(\vec{k},m)B^{-1}(\vec{u}_Q), \qquad (10)$$

and the transformation law of the state vectors

$$\sum_{\sigma} D^{1/2}_{\sigma,\lambda}(\vec{n}_W)|\vec{p},\sigma\rangle = \sqrt{\frac{\omega_m(k)}{\omega_m(p)}}U(\vec{u}_Q)|-\vec{k},\lambda\rangle,$$
$$\sum_{\sigma}\langle\vec{p},\sigma|D^{*1/2}_{\sigma,\lambda}(\vec{n}_W) = \sqrt{\frac{\omega_m(k)}{\omega_m(p)}}\langle-\vec{k},\lambda|U^{+}(\vec{u}_Q). \qquad (11)$$

After a series of simplifications of the expression (9), we finally obtain:

$$F_{VP\gamma}(q^2) = \frac{1}{4\pi}\sum_{\nu_1,\nu_1'}\int\int d\vec{k}d\vec{k}'\sqrt{\frac{3+4\nu_1(\mu-\nu_1)}{2}}\nu_1'\Psi(k)\Phi(k')(\sqrt{MM'})^{-1}$$
$$(\sqrt{\frac{1}{\omega_{m_q}(k)\omega_{m_q}(k')}}e_q\bar{u}_{\nu_1'}(\vec{k}',m_q)B^{-1}(\vec{u}_{Q'})\frac{(\Gamma K^*)}{(KK^*)}B(\vec{u}_Q)u_{\nu_1}(\vec{k},m_q)\langle-\vec{k}',-\nu_1'|U^{+}(\vec{u}_{Q'})U(\vec{u}_Q)|-\vec{k},\mu-\nu_1\rangle+$$
$$+\sqrt{\frac{1}{\omega_{m_{\bar{Q}}}(k)\omega_{m_{\bar{Q}}}(k')}}e_{\bar{Q}}\bar{\upsilon}_{\mu-\nu_1}(-\vec{k},m_{\bar{Q}})B^{-1}(\vec{u}_{Q'})\frac{(\Gamma K^*)}{(KK^*)}B(\vec{u}_Q)\upsilon_{-\nu_1'}(-\vec{k}',m_{\bar{Q}})\langle\vec{k}',\nu_1'|U^{+}(\vec{u}_{Q'})U(\vec{u}_Q)|\vec{k},\nu_1\rangle) . \qquad (12)$$

Calculation of the form factor $F_{VP\gamma}(q^2)$ in the generalized Breit system. For generalized Breit system we have

$$V_{\bar{Q}} + V_{\bar{Q}'} = 0 \qquad (13)$$

and, as a consequence,

$$B(u_{\bar{Q}}) = B(u_{\bar{Q}'}), \qquad (14)$$

where the boost operator $B(u_{\bar{Q}})$ have properties $B^{-1}(u_{\bar{Q}}) = B(-u_{\bar{Q}})$ and $B(u_{\bar{Q}})B(u_{\bar{Q}}) = B(\upsilon_{\bar{Q}}) = V_{\bar{Q}}\gamma^0$.

Using the vectors

$$\vec{k}_1 = \vec{k} + \vec{\upsilon}_{\bar{Q}}((\varpi+1)\omega_{m_q}(k) + \sqrt{\varpi^2-1}\cdot|\vec{k}|\cos\theta),$$
$$\vec{k}_2 = \vec{k} - \vec{\upsilon}_{\bar{Q}}((\varpi+1)\omega_{m_{\bar{Q}}}(k) - \sqrt{\varpi^2-1}\cdot|\vec{k}|\cos\theta) \qquad (15)$$

and the transformation law of the state vectors, the expression (12) simplifies to

$$F_{VP\gamma}(q^2)=\frac{1}{4\pi}\sum_{\nu_1,\nu_1'}\int d\vec{k}\sqrt{\frac{3+4\nu_1(\mu-\nu_1)}{2}}\nu_1'\Psi(k)(\sqrt{MM'})^{-1}$$

$$(\sqrt{\frac{\omega_{m_{\bar{Q}}}(k_2)}{\omega_{m_q}(k)\omega_{m_q}(k_2)\omega_{m_{\bar{Q}}}(k)}}e_q\bar{u}_{\nu_1'}(\vec{k}_2,m_q)\frac{(\Gamma K^*)}{(KK^*)}V_{\bar{Q}}\gamma^0 u_{\nu_1}(\vec{k},m_q)D^{1/2}_{-\nu',\mu-\nu_1}(-\vec{n}_{W_2}(\vec{k},\vec{\upsilon}_{\bar{Q}}))\,\Phi(k_2)+ \qquad (16)$$

$$+\sqrt{\frac{\omega_{m_q}(k_1)}{\omega_{m_{\bar{Q}}}(k)\omega_{m_{\bar{Q}}}(k_1)\omega_{m_q}(k)}}e_{\bar{Q}}\bar{\upsilon}_{\mu-\nu_1}(-\vec{k},m_{\bar{Q}})\frac{(\Gamma K^*)}{(KK^*)}V_{\bar{Q}}\gamma^0\upsilon_{-\nu_1'}(-\vec{k}_1,m_{\bar{Q}})D^{1/2}_{\nu',\nu_1}(\vec{n}_{W_1}(\vec{k},\vec{\upsilon}_{\bar{Q}}))\,\Phi(k_1))$$

or, using the explicit form of the operator Γ^{α},

$$F_{VP\gamma}(q^2)=e_q((1+\kappa_q)I_1+\kappa_q I_2)+e_{\bar{Q}}((1+\kappa_{\bar{Q}})I_1+\kappa_{\bar{Q}}I_2) \qquad (17)$$

where κ_q and $\kappa_{\bar{Q}}$, accordingly, the anomalous magnetic moments of the quark and antiquark, a $I_1=I(m_q,m_{\bar{Q}}), I_2=I(m_{\bar{Q}},m_q)$ - integrals of expression (16).

Choosing the wave functions of pseudoscalar and vector mesons in the form

$$\Phi(k)=\frac{2}{\pi^{1/4}\beta^{3/2}}\cdot\exp(-\frac{k^2}{2\beta^2}) \qquad (18)$$

for the parameters β, obtained on the basis of experimental data on leptonic decays of hadrons [4], carrying out numerical integration, we find that for certain values of the anomalous magnetic moments: κ_u = 0,1404 , κ_d = 0.1114, κ_s = 0,1957 (in natural units), we describe the experimentally known decay widths $\rho^+\to\pi^+\gamma$, $K^{*0}\to K^0\gamma$ and $K^{*+}\to K^+\gamma$.

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SOME METHODS OF NUMERICAL SOLUTION QUASI-LINEAR HEAT CONDUCTION PROBLEMS

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Problem of quasi-linear heat conduction is not new. Some information about this subject you can find in [1].

The questions of finding the approximate (numerical) solution of linear problems are studied in details. However, the question of finding the numerical solution of second order quasi-linear problems of parabolic type is studied much lesser. Level of accuracy of the approximate solutions of such problems is not more than 1E-3 – 1E-4. First of all, level of accuracy depends on the period of time where an approximate solution is sought.

In this paper, you can find the solution of number quasi-linear problems of parabolic type. We tried to largely remove the existing restrictions on the maximum achievable accuracy by these problems.

In these problems, we tried to largely remove the existing restrictions on the maximum achievable accuracy.

Material and methods. We know that the quasi-linear equation of the heat conduction is:

$$u_t^{'}=\frac{\partial}{\partial x}(K(x,t,u)\frac{\partial u}{\partial x})+f(x,t,u). \qquad (1)$$

The problem is to find an approximate solution of equation (1) that satisfies the initial and edge conditions: