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TRANSLATION

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TRANSLATION

To the theory of total reflection*

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The effect of total reflection is of great fundamental interest and it has been the objective of many theoretical and experimental papers (e.g. [1]). However, all research is limited to the case that the incident light is linearly polarized perpendicular or in parallel to the plane of incidence. Only Wiegrefe alone [5] has considered the case that the oscillation azimuth χ of linear polarized incident wave is different from 0 or $\pi/2$. Already in this case some fundamental features of this phenomenon, which are not observed in special cases $\chi = 0$ or $\pi/2$, are revealed. There are, however, mistakes in this article [5], and its results remained almost unnoticed[1]. A general case of total reflection for arbitrary elliptical incident polarization has not been approached yet. Meanwhile, as discussed below, its consideration allows one to discover some fundamental, previously unknown properties of total reflection.

As a consequence of the linearity of Maxwell equations and boundary conditions, one can always expand incident, reflected and refracted fields into a sum of corresponding components that are parallel and perpendicular to the plane of incidence. In case of partial reflection on the boundary between transparent isotropic media, an analogous representation also holds for the energy density w and the Poynting vector $\mathbf{P}$. However, in the general case, such a decomposition for w and $\mathbf{P}$ is impossible, as soon as they are quadratically dependent on $\mathbf{E}$ and $\mathbf{H}$. It is this reason that is responsible for the vital distinction of total reflection in the general case of incident light polarization from the linear polarization case at $\chi = 0$ or $\pi/2$.

The relations of interest are expressed in the simplest and most compact form when all calculations are made in vector form and not in component from. The Maxwell equations for plane waves[2]

$$\mathbf{E} = \mathbf{E}_0 e^{i\phi}, \qquad \mathbf{H} = \mathbf{H}_0 e^{i\phi}, \qquad \phi = \omega\left(t - \frac{1}{c}\mathbf{mr}\right) \tag{1}$$

in non-magnetic media take the form[3]

$$\mathbf{D} = \varepsilon\mathbf{E} = -[\mathbf{mH}], \qquad \mathbf{H} = [\mathbf{mE}] \qquad (\mathbf{m}^2 = \varepsilon). \tag{2}$$

Here, $\mathbf{m} = n\mathbf{n}$ is the refraction vector [3, 4]; n is the index of refraction; $\mathbf{n}$ is the unit wave normal vector. The electric and magnetic energy density and Poynting vector are expressed as

$$w_e = \frac{\varepsilon}{32\pi}(\mathbf{E} + \mathbf{E}^*)^2, \qquad w_m = \frac{1}{32\pi}(\mathbf{H} + \mathbf{H}^*)^2, \tag{3}$$

$$\mathbf{P} = \frac{c}{16\pi}[\mathbf{E} + \mathbf{E}^*, \mathbf{H} + \mathbf{H}^*]. \tag{4}$$

Denoting the incident, reflected and refracted waves respectively by indices 0, 1, 2, one can write the geometrical laws of reflection and refraction as

$$[\mathbf{m}_0\mathbf{h}] = [\mathbf{m}_1\mathbf{h}] = [\mathbf{m}_2\mathbf{h}] = \mathbf{a}, \tag{5}$$

where $\mathbf{h}$ is the unit normal to the interface. Hence it follows that

$$\mathbf{m}_i = [\mathbf{ha}] + \eta_i\mathbf{h}, \qquad \eta_i = \mathbf{m}_i\mathbf{h}_i, \qquad \eta_1 = -\eta_0, \qquad \eta_2 = \sqrt{n_2^2 - \mathbf{a}^2} \tag{6}$$

($n_0 = n_1$, n_2 are the indices of refraction of either media). One can write an electric field for each of three waves as

$$E_i = A_i\mathbf{a} + B_i[\mathbf{n}, \mathbf{a}]. \tag{7}$$

* Fedorov F I 1955 To the theory of total reflection *Doklady Akademii Nauk SSSR*, **105**, # 3, 465–8; received 9 December 1949. Presented by academician Lebedev A A on 27 May 1955. Translated by D I Pustakhod 2012.

[1] Paper [5], for example, is not cited in the review [1], its results are also overlooked in the well-known M Born's monograph [2], which contains a misstatement in connection with this (see footnote 4).

[2] Translators note: in the original article the expression for the phase ϕ appears to be misprinted and does not contain the scalar product **mr** between the unit wave vector **m** and the position vector **r**. This has been amended in the translation for the sake of clarity.

[3] Translators note: Fedorov uses both [**ab**] and [**a**, **b**] to denote the vector or cross product $\mathbf{a} \times \mathbf{b}$.

Fresnel equations for amplitudes A_i, B_i have the form

$$\frac{A_0}{\mathbf{a}[\mathbf{m}_1\mathbf{m}_2]} = \frac{A_1}{\mathbf{a}[\mathbf{m}_2\mathbf{m}_0]} = \frac{-A_2}{\mathbf{a}[\mathbf{m}_0\mathbf{m}_1]}, \tag{8}$$

$$\frac{B_0/A_0}{\mathbf{n}_0\mathbf{n}_2} = \frac{B_1/A_1}{\mathbf{n}_1\mathbf{n}_2} = \frac{B_2/A_2}{\mathbf{n}_2\mathbf{n}_2}. \tag{9}$$

Total reflection occurs when $n_2^2 - \mathbf{a}^2 \leq 0$, from which follows that

$$\mathbf{m}_2 = \mathbf{m}' + \mathrm{i}\mathbf{m}'' = [\mathbf{ha}] - \mathrm{i}\eta\mathbf{h}, \qquad \eta = +\sqrt{\mathbf{a}^2 - n_2^2}. \tag{10}$$

In this case the complex vector $\mathbf{m}_2$ will be nonlinear ($[\mathbf{m}_2\mathbf{m}_2] \neq 0$), whereas the refracted wave will be non-uniform [4].

From (2) and (4) we have general relations for the energy density and Poynting vector of a non-uniform wave in an isotropic non-magnetic dielectric:

$$w = w_e + w_m = w' + w'', \quad w'' = \frac{\varepsilon}{16\pi}(\mathbf{E}^2 + \mathbf{E}^{*2}), \tag{11}$$

$$w' = \frac{1}{16\pi}((\varepsilon + |\mathbf{m}|^2)|\mathbf{E}|^2 - |\mathbf{m}\mathbf{E}^*|^2), \tag{12}$$

$$\mathbf{P} = \mathbf{P}' + \mathbf{P}'', \quad \mathbf{P}'' = \frac{c}{16\pi}(\mathbf{E}^2 \cdot \mathbf{m} + \mathbf{E}^{*2} \cdot \mathbf{m}^*), \tag{13}$$

$$\mathbf{P}' = \frac{c}{16\pi}(|\mathbf{E}|^2(\mathbf{m} + \mathbf{m}^*) - [\mathbf{m} - \mathbf{m}^*, [\mathbf{EE}^*]]). \tag{14}$$

Obviously, quantities w' and $\mathbf{P}'$ do not contain the phase factor $\mathrm{e}^{\mathrm{i}\phi}$, while w'' and $\mathbf{P}''$ contain $\mathrm{e}^{\pm\mathrm{i}\phi}$. Therefore, for mean values $\overline{w}$ and $\overline{\mathbf{P}}$ we have: $\overline{w} = w'$, $\overline{\mathbf{P}} = \mathbf{P}'$. It can be shown that linear polarization is determined by the constraint $[\mathbf{EE}^*] = 0$, whereas circular polarization is determined by $\mathbf{E}^2 = 0$ [4]. But in general the oscillation ellipse semiaxes are equal in magnitude and direction with the real and imaginary components of vector

$$\mathbf{E}_\mathrm{r} = \sqrt{\frac{|\mathbf{E}^2|}{\mathbf{E}^2}}\mathbf{E}. \tag{15}$$

According to (13), vector $\mathbf{P}''$ traces an ellipse in the plane, parallel to the plane of complex vector $\mathbf{m}_2 = \mathbf{m}' + \mathrm{i}\mathbf{m}''$, i.e. the plane of incidence. Using (15), one can make sure that the ellipse semiaxes are proportional and parallel to $\mathbf{m}'$ and $\mathbf{m}''$. Hence, the total energy flux vector in the second medium traces a cone twice during one period, which points into the same direction from the plane of incidence as the vector $\mathbf{P}'$. From (14) it follows that in the general case of total reflection the average energy flux in the refracted wave is not parallel to the plane of incidence: it has a perpendicular component associated with the term $[\mathbf{m}_2 - \mathbf{m}_2^*, [\mathbf{EE}^*]]$.[4] This component is equal to zero for the common reflection ($\mathbf{m}_2 = \mathbf{m}_2^*$) and in the case that $A_0 = 0$ or $B_0 = 0$. Moreover, it disappears at $[\mathbf{EE}^*] = 0$, i. e. when vector $\mathbf{E}_2$ is linear [4]. According to (7) and (9) this lateral flux also equals zero with the constraint $\frac{B_0^*/A_0^*}{B_0/A_0} = \frac{\mathbf{m}_2^*\mathbf{m}_0}{\mathbf{m}_2\mathbf{m}_0}$, which defines only the phase difference of the components A_0 and B_0, while the modules ratio of these components is unrestricted. Wiegrefe [5] was the first to pay attention to the presence of a lateral flux in total reflection, but only for the case of linearly polarized incident light[5]. More generally, at a given incident wave energy and angle of incidence the lateral flux peaks at $\frac{B_0^*/A_0^*}{B_0/A_0} = -\frac{\mathbf{m}_2^*\mathbf{m}_0}{\mathbf{m}_2\mathbf{m}_0}$, i.e. at some elliptical polarization of the incident light. In the case of linear polarization of the incident wave at $\chi = 45°$, the lateral energy flux through a stripe of 1 cm width, that stretches in the second medium from the interface to infinity and is parallel to the plane of incidence, equals

$$S_{2\,\mathrm{side}} = S_0 \frac{\lambda_0}{2\pi} \frac{\sin 2\psi\sqrt{\sin^2\psi - n^2}}{(1 - n^2)(\tan^2\psi - n^2)}. \tag{16}$$

Here, S_0 is the incident wave energy flux through a perpendicular area element of 1 cm^2; λ_0 is the optical wavelength in the first medium expressed in centimeters; ψ is the angle of incidence; $n = n_2/n_1$ is the relative index of refraction. The lateral energy flux specified should lead to a specific lateral light pressure, as far as the lateral flux in the incident wave is nil, and therefore a corresponding component of electromagnetic field momentum is not conserved. However, in view of the fact that according to (16), $S_{2\,\mathrm{side}}/S_0 \sim 10^{-5}$ for visible light, it is difficult to detect this effect in practice.

It should be noted that as a consequence of the presence of this lateral component, the reflected beam in the general case of total reflection must be displaced not only along the plane of incidence, which was confirmed by the experiments of Goos and Hänchen [9], but also in a direction orthogonal to the mentioned plane as well.

It follows from equations (14) and (10), that $\mathbf{P}_2'\mathbf{h} = \mathbf{P}_2\mathbf{h} = 0$. Therefore, the mean energy flux into the second medium is equal to zero, which allows one to speak about the total reflection. Under these conditions the field presence in the second medium in case of an infinite wave in time and space is attributable to the term $\mathbf{P}_2''$ (13), giving an alternating energy flux through the interface, which averages out to zero. However, in the case of a circularly polarized refracted wave ($\mathbf{E}_2^2 = \mathbf{E}_2^{*2} = 0$) $\mathbf{P}_2'' = 0$ and, therefore, $\mathbf{P}_2'\mathbf{h} = 0$, i.e. not only the average flux, but an instant energy flux through the interface is equal to zero as well. Here, the common explanation of the field presence in the second medium becomes entirely inconsistent, showing the fundamental inadequacy of the total reflection theory, which ignores the boundedness of an incident wave in space or time. From (7) to (9) and (15) it can easily be shown, that this particular case occurs at an elliptical polarization of an incident light such that the ratio of the oscillation ellipse

[4] In Born's monograph [2] it is mistakenly stated, that in total reflection the energy flux in the second medium is directed in parallel to the plane of incidence (p 62).

[5] In article [5] it is mistakenly stated, that lateral energy flux is always directed to the left from the plane of incidence, regardless of the direction of polarization of linearly polarized incident light (p 470). In fact it follows from (14), (7), and (9), that the direction of the lateral flux component reverses as the incident light oscillation azimuth changes its sign.

semiaxes equals the relative index of refraction and, therefore, is independent of the angle of incidence. An angle χ between the major axis of oscillation ellipse of the incident wave and the incidence plane normal (**a**) is determined by

$$\tan 2\chi = \pm \frac{2\eta_0 \eta n_1 n_2}{\eta_0^2 n_2^2 - \eta^2 n_1^2}. \tag{17}$$

$\chi = 0$ in case of incidence at a critical angle of total reflection, and $\chi = \pi/2$ at glancing incidence. The phase difference δ of the components A_0 and B_0 ($B_0/A_0 = |B_0/A_0|\mathrm{e}^{\mathrm{i}\delta}$) is given by $\tan\delta = \pm a^2/(\eta_0 \eta)$. In this case the oscillation ellipse of the reflected wave has the same size, shape and direction of circulation as that of the incident wave, differing only in sign of χ.

The dependence of the depth of light penetration into the second medium from the incident wave polarization in total reflection was discovered in the experiments of Quincke [6] and Gall [7]. This dependency is entirely explained on the basis of Eichenwald theory (e.g. see [8]), as far as Quincke and Gall considered the standard case of linearly polarized incident light at $\chi = 0$ and $\pi/2$.[6] It is evident, that an analogous experimental study for the special case specified above of elliptical polarization of the incident wave is of much interest.

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[6] In the review [1] it is mistakenly stated, that this issue still remains unsolved (p 459–60). In fact, it has long been resolved (e.g. see [8]).