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ELECTRODYNAMICS AND WAVE PROPAGATION

Transformation of the Polarization of Electromagnetic Waves by Helical Radiators

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Abstract—Transformation of the polarization of an electromagnetic wave by a 2D array of double-turn metal helices is investigated both theoretically and experimentally. Electric and magnetic moments induced in an isolated helix by an incident linearly polarized wave are determined. A universal relationship between the projections of the induced moments onto the axis of a helix is derived. The helix parameters that are optimal for radiation of a circularly polarized wave are calculated. A 2D array composed of double-turn helical radiators with optimum parameters is manufactured. Experimental verification of the obtained theoretical results is performed.

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INTRODUCTION

Artificial composite media possessing chiral properties in the microwave band have been intensively studied in the last 15 years [1–11]. It was assumed that artificial chiral materials can be used for the development of nonreflecting coatings of metal surfaces. Investigations of the possible application of artificial chiral materials for lowering the reflection of electromagnetic waves were described in [4–7]. However, in [9], it was deduced that chirality is not important for the formation of nonreflecting coatings. Moreover, it was revealed that the use of nonchiral absorbing media is another way of substantially lowering the intensity of reflected electromagnetic waves at some frequency. The authors of paper [9] came to this conclusion after calculation of the electromagnetic waves scattered by metal helices situated in a dielectric medium. This medium can be created artificially via placement of metal helical wire inclusions into a dielectric material [11].

The purpose of this study is to investigate the interaction of microwave electromagnetic radiation and a 2D array composed of metal helices having precalculated optimum parameters and to demonstrate that such structures can be used to transform the polarization of microwave electromagnetic waves, for example, to obtain a circularly polarized wave.

Most studies analyzing this problem consider cylindrical helical antennas connected to a feeder (active helices). These devices radiate in the axial mode with elliptic polarization of the radiated field [12]. In this paper, we analyze and calculate characteristics of a 2D array composed of passive double-turn helices. Unlike known helical elements, this element ensures formation, due to radiation of the coupled components of the electric dipole moment and the magnetic moment, of a circularly polarized wave in the direction perpendicular to the axial direction.

The device whose structure is closest to the analyzed array is a polarization transformer containing a dielectric layer and a lattice structure (grating array) made of identical conducting elements [13]. In this device, the dielectric layer serves as a board whose one side contains a grating of identical conducting elements shaped as meander lines. The lines are placed at an angle of 45° with the plane of a linearly polarized wave and are parallel to each other. In addition, the grating has parameters that change the phases of two components of the electric intensity vector of the field of an electromagnetic wave passing through an antenna polarizer. The device ensures generation of the circularly polarized electromagnetic wave only when the incident wave passes through a polarizer. As a result, it cannot be used in reflecting systems.

In the method analyzed in this paper, a linearly polarized electromagnetic wave is transformed into a circularly polarized wave irrespective of the orientation of the polarization plane of the incident linearly polarized electromagnetic wave for a specified direction of propagation of the incident wave.

A circularly polarized wave is formed owing to radiation of the coupled electric dipole moments and the magnetic moments of each helix. The magnitudes of the contributions of these moments to the reflected wave are equal.

1. CALCULATION OF THE ELECTRIC DIPOLE MOMENT AND THE MAGNETIC MOMENT THAT ARE INDUCED IN A HELIX BY AN INCIDENT WAVE

Characteristics of the electromagnetic radiation scattered by a helical element depend on the ratio of geometric dimensions of this helix and the wavelength. Let us consider the case when linear geometric dimensions of a helix are substantially less than the length of an incident wave. This circumstance allows us to apply the dipole approximation of the radiation theory [14].

Let us find the electric dipole moment and magnetic moment of a helix in this case. In each helix, the main condition for the manifestation of gyrotropic properties is the simultaneous formation of a electric dipole moment and a magnetic moment that are coupled with each other and induced by an external field.

It is necessary to calculate all components of the electric dipole moment and the magnetic moment in a helix. The polarization of the radiated wave depends on the relation between these moments.

Let l be the coordinate measured along a helix, L be the length of a helix, and $\vec{S}(l)$ be the shift of the conductance electrons along a helix. Then, the expression for vector $\vec{p}$ of the dipole moment can be written as

$$\vec{p} = \int_{(V)} QN_e\vec{S}(l)dV = -eN_eS_w\int_{-L/2}^{L/2}\vec{S}(l)dl, \qquad (1)$$

where $Q = -e$ is the electron charge, $dV = S_w dl$ is the volume of the element of a helix, S_w is the sectional area of the wire, and N_e is the volume density of conductance electrons. The magnetic moment of a helix can be calculated in a similar way. With allowance for the geometric dimensions of a helix, it is possible to calculate projections of these moments onto the coordinate axes.

Let us consider a helix that has turns of radius r and length L. The helix height is $H = hN_h$, the helix lead angle relative to the plane perpendicular to the helix axis is α, the helix axis coincides with the $0x$ axis, and the number of helix turns is N_h (see Fig. 1).

Let us introduce relative torsion of a helix, q, which is related to helix pitch h as

$$h = \frac{2\pi}{|q|}. \qquad (2)$$

The sign of parameter q determines the direction of rotation of a helix in space. If $q > 0$, a helix forms a right-handed screw (see Fig. 1).

Let us consider the main resonance, which arises when the helix length is approximately equal to the wavelength of the incident radiation. In this case, the current strength monotonically decreases with distance from the helix center and vanishes at the helix ends. A model with the current linearly decreasing with distance from the helix center to the ends was considered in [15, 16].

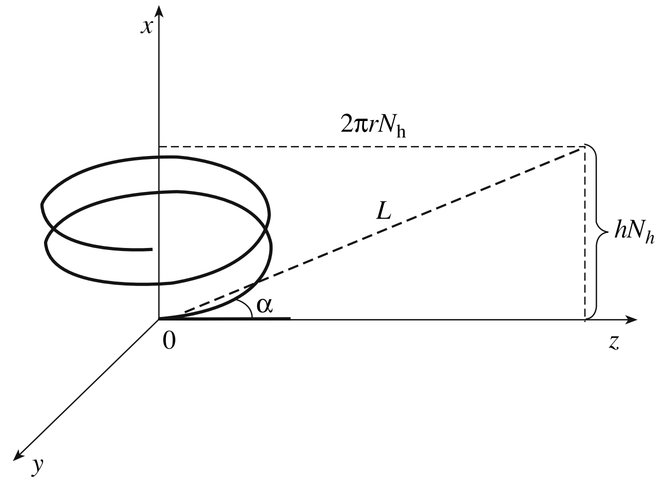

Fig. 1. Helix and the schematic view of its unraveled form.

However, the model with a harmonic decrease of the current with distance from the helix center to the ends is more accurate. This behavior corresponds to a steady-state standing-wave oscillation with zero current at the helix ends. The maximum intensity corresponds to oscillations with the half-wave present along the helix length. A harmonic dependence of the current on the coordinate is considered below. This example is important because an arbitrary distribution of the helix current can be reduced (with the use of the Fourier analysis) to a harmonic dependence of the current on the coordinate.

It is shown that the y components of the electric dipole moment and the magnetic moment of the helix tend to zero irrespective of the number of helix turns. This property is due to the symmetry of the current distribution relative to the helix center.

As the number of turns increases, the magnitudes of the z components of the electric dipole moment and the magnetic moment decrease relative to the magnitudes of the x components of these moments. Thus, the moment components directed along the helix axis play the main role.

In order to reveal general regularities, we consider the monochromatic time dependence of the shift of conductance electrons,

$$S(x, t) = S(x)\exp(-i\omega t), \qquad (3)$$

where ω is the cyclic frequency of the current in a helix. In this case, we have the following relationship between shift S of conductance electrons and current strength $\mathcal{I}$:

$$S = -\frac{i}{eN_e\omega S_w}\mathcal{I}. \qquad (4)$$

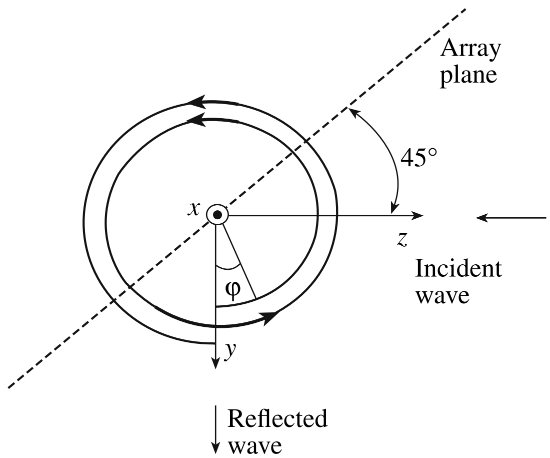

Fig. 2. Current distribution over a double-turn helix. The arrows show the current direction, with the current strength being proportional to the arrow length, and φ is the polar angle.

Relationships (1) and (4) can be used to obtain the following expression for the x component of the electric dipole moment of a helix:

$$p_x = \frac{i}{\omega}\int_{x_1}^{x_2} \mathcal{I}(x)dx. \quad (5)$$

This expression is well known [17–19].

In addition, using relationships (1) and (5) and taking into account the geometric parameters of a helix, we calculate the x component of the magnetic moment:

$$m_x = \frac{1}{2}r^2 q\int_{x_1}^{x_2} \mathcal{I}(x)dx. \quad (6)$$

If follows from expressions (5) and (6) that the relationship between the projections of moments onto the helix axis is given by the formula

$$p_x = \frac{2i}{\omega r^2 q}m_x. \quad (7)$$

This relationship is universal because it is independent of the current distribution along a helix. The x components of the helix moments play the main role in radiation of a circularly polarized wave in the direction perpendicular to the helix axis.

We tested relationship (7) for three particular cases of the current distribution along a helix: (i) the uniform distribution, (ii) the distribution with a linear decrease of the current amplitude from the helix center to its ends, and (iii) a harmonic dependence of current on the coordinate.

The universal character of relationship (7) likewise should be treated in a more general sense. In an artificial gyrotropic structure, the current flowing in a helix can vary not only under the direct influence of an incident electromagnetic wave but also under the influence of other helices forming the structure. However, in all variations, components p_x of the electric dipole moment and m_x of the magnetic moment vary consistently and relationship (7) remains valid. Therefore, the geometric parameters of a helix that are presented below ensure radiation of a circularly polarized wave even in the case of a substantial increase in the concentration of helical elements in the artificial structure.

2. CALCULATING THE HELIX PARAMETERS REQUIRED FOR RADIATION OF A CIRCULARLY POLARIZED WAVE UNDER RESONANCE CONDITIONS AND CONSIDERING THE NUMBER OF HELIX TURNS

In the dipole approximation, the intensity of the electric filed of the radiated wave has the form [14, 20]

$$\vec{E}(\vec{R}, t) = \frac{\mu_0}{4\pi R}\left([[\ddot{\vec{p}}, \vec{n}]\vec{n}] + \frac{1}{c}[\vec{n}, \ddot{\vec{m}}]\right), \quad (8)$$

where $\vec{R}$ is the radius vector passing from a helix to the observation point, μ_0 is the magnetic constant, R is the distance from a helix to the observation point, $\vec{n}$ is the unit vector of the wave normal, c is the velocity of light in vacuum, and points above the vectors denote time differentiation.

Let us consider the wave radiated by a helix in the direction of the y axis. In this case, the wave exciting the helix propagates along the z axis.

In the experiment described below, the helices are placed on a plane made of a radio transparent material (Styrofoam). Therefore, if the angle of incidence is 45°, the wave reflected from the array is formed only by the waves radiated by the helices in the direction of the y axis. All radiated waves have the same phase and polarization. This circumstance causes mutual amplification of these waves. Hence, the wave reflected from the array has the same polarization as the waves radiated by each helix in the direction of the y axis.

This geometry of the experiment simplifies investigation of a radiated wave because the intensity of the wave radiated by one helix is substantially lower than the intensity of the incident wave (see Fig. 2).

Let the orientation of the receiving antenna be specified by unit vector $\vec{c}_0$ lying in plane XOZ and forming angle θ with the x axis (i.e., with the electric intensity vector of the field of the incident wave). Then,

$$\vec{c}_0 = \cos\theta\vec{x}_0 + \sin\theta\vec{z}_0, \quad (9)$$

$$\vec{n} = \vec{y}_0, \quad (10)$$

where $\vec{x}_0$, $\vec{y}_0$, and $\vec{z}_0$ are the unit vectors directed along the x, y, and z axis, respectively (basis vectors of the Cartesian coordinate system).

In this case, the intensity of the signal recorded by the receiving antenna is proportional to the following quantity:

$$I = \langle(\vec{E}\vec{c}_0)^2\rangle_t, \tag{11}$$

where angle brackets denote time averaging. For convenience of the subsequent calculations, we represent nonzero components of the electric and magnetic moments of a helix in the form

$$p_x = p_{x_0}S_0, \quad p_z = p_{z_0}S_0, \tag{12}$$

$$m_x = im_{x_0}S_0, \quad m_z = im_{z_0}S_0. \tag{13}$$

Using formulas (8)–(10), (12), and (13), we calculate intensity (11) of the signal recorded by the receiving antenna:

$$I = \frac{\mu_0^2\omega^4}{32\pi^2R^2}|S_0|^2\left(\left(p_{x_0}^2 + \frac{1}{c^2}m_{z_0}^2\right)\cos^2\theta + \left(p_{z_0}^2 + \frac{1}{c^2}m_{x_0}^2\right)\sin^2\theta + \left(p_{x_0}p_{z_0} - \frac{1}{c^2}m_{x_0}m_{z_0}\right)\sin 2\theta\right). \tag{14}$$

Let us study the possibility of radiation of a circularly polarized wave by such a helix. This phenomenon is possible if

$$|p_z| \ll |p_x|, \quad \frac{1}{c}|m_z| \ll |p_x|.$$

Then, we obtain from (14) the condition for radiation of a circularly polarized wave:

$$|p_x| = \frac{1}{c}|m_x|. \tag{15}$$

If condition (15) is satisfied, gyrotropic properties of a helix are the most pronounced because the electric field of the incident wave excites in the helix not only an electric dipole moment but also a magnetic moment, which is at least of the same importance.

In order to determine the values of the helix parameters that ensure radiation of a circularly polarized wave, we use universal relationship (7), which was derived above for the components of an electric dipole moment and a magnetic moment of a helix in the case of an arbitrary current distribution, and the condition of main frequency resonance:

$$\frac{\lambda}{2} = L, \tag{16}$$

where λ is the wavelength of the incident electromagnetic radiation.

Taking into account the relationship between geometric parameters of a helix,

Optimum values of the helix lead angle (α) that correspond to radiation of a circularly polarized wave for different values of the number of turns (N_h)

N_h	1	2	3	4	5	6	7	8
α, deg	13.65	7.1	4.75	3.6	2.9	2.4	2.0	1.8

$$L\cos\alpha = 2\pi rN_h, \tag{17}$$

we obtain the following trigonometric equation for helix lead angle α:

$$4N_h\tan\alpha = \cos\alpha \tag{18}$$

or

$$\sin^2\alpha + 4N_h\sin\alpha - 1 = 0. \tag{19}$$

Since angle α takes positive values, we can write the expression for the roots of Eq. (19) in the form

$$\alpha = \arcsin(-2N_h + \sqrt{4N_h^2 + 1}). \tag{20}$$

Values of the helix lead angle at which helices with different numbers of turns can radiate a circularly polarized wave are listed in the table.

As follows from the table, radiation of a circularly polarized wave is possible for both odd and even numbers of helix turns. Experimental results show that the intensity of the wave radiated by a helix rapidly decreases as the number of turns increases, i.e., as the helix lead angle decreases. Hence, the optimum number of turns is $N_h = 1$ or 2.

For a helix containing one turn, it is necessary to exclude the influence of components p_z and m_z of electric and magnetic moments that are orthogonal to the helix axis. Therefore, in order to obtain a circularly polarized wave, the ends of a single-turn helix should be directed toward the incident wave.

The current distribution along a helix with two turns is more symmetric, and a circularly polarized wave is radiated for any orientation of the helix ends relative to the vector of the incident wave.

Using the obtained results, we find values of the helix parameters ensuring radiation of a circularly polarized wave in the case when a helix is excited by a linearly polarized wave with the frequency $\nu = 3$ GHz. The length of the helix wire should correspond to the condition of the main frequency resonance: $L = 5$ cm. Using the table, we choose the helix lead angle that is realized at $N_h = 2$: $\alpha = 7.1°$. The helix radius can be calculated from formula (17): $r = 3.95 \times 10^{-3}$ m. The helix pitch is found from the relationship $h = \frac{L\sin\alpha}{N_h} = 3.1 \times 10^{-3}$ m.

In order to test the theoretical calculations performed above and create a polarization transformer radiating a circularly polarized wave, we manufactured

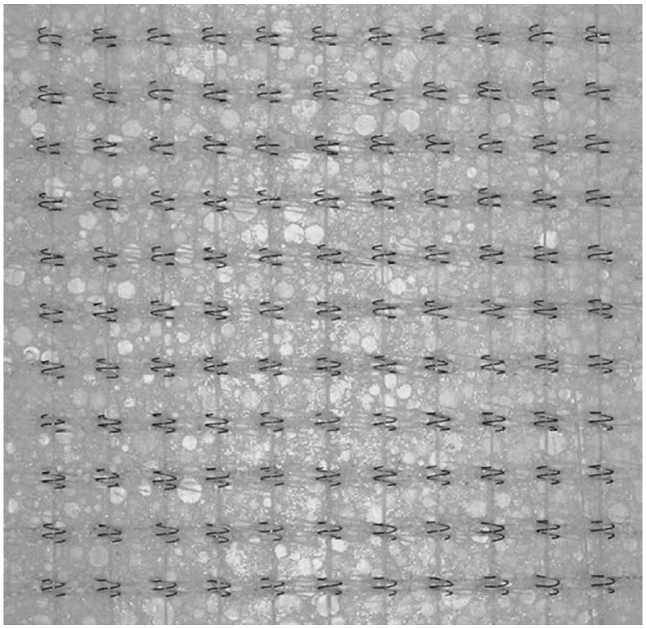

Fig. 3. Two-dimensional gyrotropic array composed of double-turn helical radiators fastened on a Styrofoam plate.

a prototype containing double-turn helical elements having the above-mentioned parameters (Fig. 3).

3. EXPERIMENTAL INVESTIGATION OF THE ELECTROMAGNETIC RADIATION REFLECTED BY A TWO-DIMENSIONAL GYROTROPIC STRUCTURE

In order to measure the polarization characteristic, a method based on the use of the receiving antenna with a linear polarization of the field (a horn antenna) was applied. The axial ratio of the electromagnetic wave reflected from the prototype of a 2D gyrotropic array was studied as a function of the frequency of the incident radiation. The measurements were performed in the frequency interval 2.6–3.9 GHz.

Axial ratio K of the reflected wave was calculated directly from the polarization pattern as the ratio of the minimum and maximum values of the signal level determined from readings of the receiver's display.

The results of this study are shown in the form of a plot in Fig. 4. As seen from this figure, the maximum value of the axial ratio lies in the frequency range 2.8–2.9 GHz. The theoretical calculation performed for this array assumes a circular polarization of the reflected wave at a frequency of the incident radiation of 3 GHz. This shift of the observed resonance frequency relative to the calculated value can be explained in terms of the slowing of electromagnetic waves in helical elements of the 2D array. This lowering of the wave velocity may be the result of induction of substantial electric dipole moments and magnetic moments in helices.

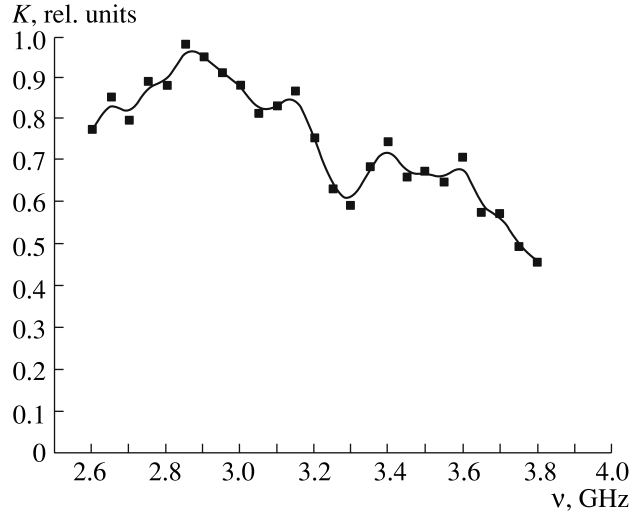

Fig. 4. Axial ratio K of the reflected wave vs. frequency ν of the incident linearly polarized wave.

The incident linearly polarized wave can be treated as a superposition of two circularly polarized waves with right-hand and left-hand circular polarizations. At the resonance frequency, a right-handed helix with the found optimum parameters radiates only the wave having the left-hand circular polarization and does not interact with the wave having the opposite polarization. Hence, a right-handed helix with the same parameters that has the appropriate orientation can be considered an oscillator orthogonal with respect to the wave having the right-hand circular polarization at the resonance frequency.

CONCLUSIONS

We have developed theoretical grounds for the design of transformers of the electromagnetic-wave polarization that are based on composite media with helical structures, including transformers of a linearly polarized wave into a circularly polarized wave.

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