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JOULE-THOMSON EXPANSION: CHARGED ANTI-DE SITTER BLACK HOLE AND DIETERICI II FLUID

The papers of Bekenstein, Hawking and Page [1–3] are the basis of black hole thermodynamics. The presence of electric charge in the latter [4] revealed an analogy between the phase diagrams of black holes and van der Waals liquids. In the last ten years, the emphasis has been on the analysis of the behavior of black holes and real liquids [5], within the framework of the Joule-Thomson process [6]. In this paper, we will consider and qualitatively compare the Joule-Thomson processes in a charged AdS black hole and a Dieterici II liquid.

In the Joule-Thomson process, a substance with enthalpy H = const moves from a region of high pressure to a region of lower pressure, which is accompanied by a change in its temperature. The effect at $dT < 0$ is positive, at $dT > 0$ is negative. The derivative describing the process has the form, where the sign of λ coincides with the sign of the effect

$$\left(\frac{\partial T}{\partial P}\right)_H = -\frac{\lambda}{c_P}\left(\frac{\partial V}{\partial P}\right)_T, \qquad \lambda = V\left(\frac{\partial P}{\partial V}\right)_T + T\left(\frac{\partial P}{\partial T}\right)_V, \tag{1}$$

The spacetime geometry of a charged AdS (anti-de Sitter) black hole is determined by the metric

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2,$$

where $G = \hbar = k_B = c = 1,\ d\Omega^2 = d\theta^2 + \sin^2(\theta)d\varphi^2$ and the function $f(r)$ is written as

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}. \tag{2}$$

In (2) l, M and Q are the anti-de Sitter radius, mass and charge of the black hole. In this case, the radius of the event horizon is found as the largest root of equation (2) at $f(r_+) = 0.$ The mass of the black hole in our case is considered as enthalpy H [7] and has the form

$$M = \frac{r_+}{2}\left(1 + \frac{Q^2}{r_+^2} + \frac{r_+^2}{l^2}\right). \tag{3}$$

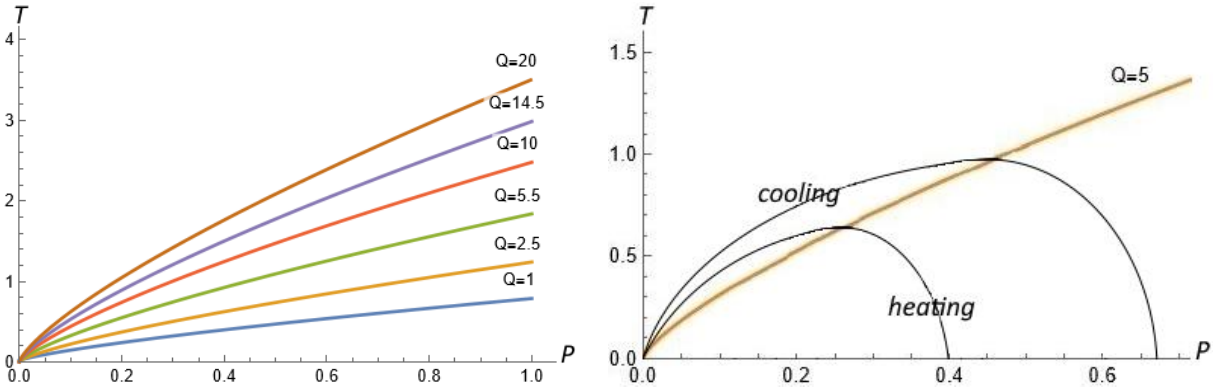
The above leads to the equation of state (ES) in the form $P = P(V, T; Q)$

$$P = \frac{T}{2r_+} - \frac{1}{8\pi r_+^2} + \frac{Q^2}{8\pi r_+^4}, \qquad r_+ = \left(\frac{3V}{4\pi}\right)^{1/3},$$

which, together with (1), defines a family of inversion curves parameterized by the charge Q, and the regions of positive and negative effects correspond to the sign of λ, but in this case it is more convenient to determine them based on the behavior of isenthalpic curves (3)

$$T_i = \frac{\sqrt{P_i}}{\sqrt{2\pi}} \frac{\left(1+16\pi P_i Q^2 - \sqrt{1+24\pi P_i Q^2}\right)}{\left(-1+\sqrt{1+24\pi P_i Q^2}\right)^{3/2}}. \quad (4)$$

Figure 1 shows: a family of inversion curves for different values of charge Q (a); cooling and heating zones with isenthalpic curves (b).



a b

a – with $Q=1;\ 2,5;\ 5,5;\ 10;\ 14,5;\ 20;$
b – with $Q=5$, cooling and heating regions and isenthalpic curves

Figure 1 – Inversion curves of a charged AdS black hole ($a=b=k_B=1$, following [5])

For the Dieterici II liquid, the ES with $\upsilon = V/N$, where N – the number of particles, is written as

$$P = \frac{k_B T}{\upsilon - b} - \frac{a}{\upsilon^{5/3}}. \quad (5)$$

The ES (5) and definition (1) give an explicit form of the process parameter

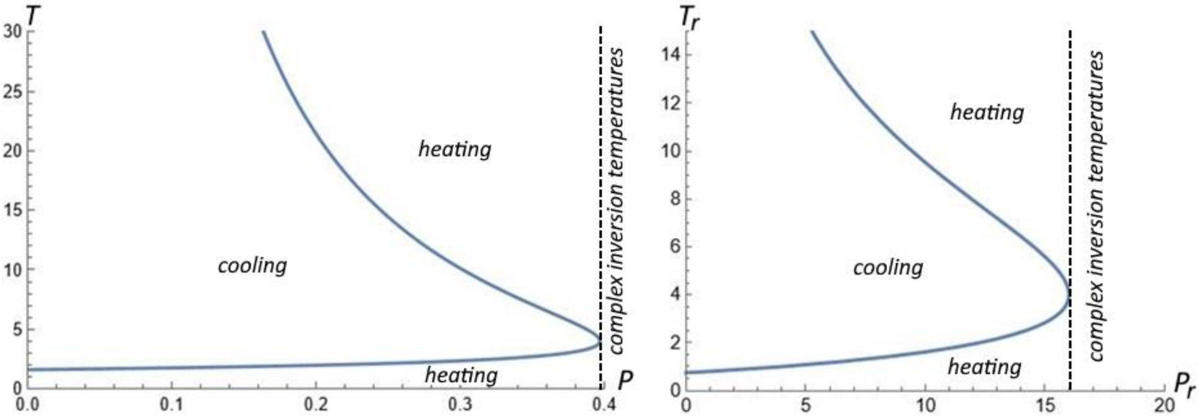
$$\lambda = \frac{5a}{3\upsilon^{5/3}} - \frac{k_B T b}{(\upsilon - b)^2},$$

which, under the condition of inversion $\lambda = 0$ and $T = T_i$, leads to a system of equations for the inversion temperature

$$T_i = \frac{5a}{3k_B b}\frac{(\upsilon - b)^2}{\upsilon^{5/3}}; \qquad T_i = \left(P + \frac{a}{\upsilon^{5/3}}\right)(\upsilon - b)\frac{1}{k_B}. \quad (6)$$

System (6) does not allow one to obtain an explicit analytical expression $T_i = T_i(P)$ for the inversion curve, but its graphical form, shown in Figure 2 (a), is easy to determine. The cooling and heating regions are also indicated, which are obvious from the condition $\lambda_{T\to\infty} < 0$. Also assumed, following [4], that $k_B = a = b = 1$. In the paper [8], an inversion curve for the Dieterici equation II was obtained in terms of the reduced variables. It is shown in Figure 2 (b). In this case, the system of equations for the inversion curve is the next

$$T_{i,r} = \frac{4(4V_r - 1)^2}{9V_r^{5/3}}; \qquad T_{i,r} = \frac{1}{15}\left(P_r + \frac{4}{V_r^{5/3}}\right)(4V_r - 1).$$



a b

a – from equations (6) under condition $a = b = k_B = 1$;
b – reduced form, following [8]

Figure 2 – Inversion curve for Dieterici II fluid

The inversion curves obtained in the work allow only qualitative assessments of the similarity of the behavior of the objects under study due to the special scaling of the scales, for example $a = b = k_B = 1$, according to [5]. However, it is clear that the regions of the positive effect have significant overlapping areas at certain values of Q, which indicates an analogy not only between the phase diagrams of black holes and van der Waals liquids [4], but also between the behavior of charged AdS black holes and Dieterici II liquids during Joule-Thomson expansion.

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