Polarizability of Nucleon in Quantum-Field Approach

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Abstract

On the basis of the relativistic gauge-invariant approach, the solutions of the electromagnetic equations by the covariant method of Green functions and the effective Lagrangians the low-energy Compton scattering amplitudes are determined. Calculations of magnetic and electric quasi-static polarizabilities of spinor particle were evaluated on the based on matrix elements calculation for Compton scattering amplitudes.

Introduction

At present there are many electrodynamic processes on the basis of which experimental data on hadrons polarizabilities can be obtained. In this context, there is a task of covariant determination of the polarizabilities contribution to the amplitudes and cross-sections of electrodynamic hadron processes [1],[2]. This problem can be solved in the framework of theoretical-field covariant formalism of the interaction of electromagnetic fields with hadrons with account for their polarizabilities. In the papers [3]-[6] one can find covariant methods of obtaining the Lagrangians and equations describing interaction of the electromagnetic field with hadrons, in which electromagnetic characteristics of these particles are fundamental. Effective field Lagrangians describing the interaction of low-energy electromagnetic field with nucleons based on expansion in powers of inverse mass

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of the nucleon have been widely used recently [7]. In Ref. [8] on the basis of correspondence principle between classical and quantum theories an effective covariant Lagrangian describing the interaction of electromagnetic field with particles of spin is presented in the framework of field approach with account for particles polarizabilities.

In this paper, in the framework of the covariant theoretical-field approach based on the effective Lagrangian presented in [8] a set of equations describing the interaction of electromagnetic field with hadrons of spin is obtained taking into account their polarizabilities and anomalous magnetic moments. Using the Greens function method for solving electrodynamic equations [9]-[12], amplitude of Compton scattering on the particles of spin is obtained with account for their polarizabilities. Structures of the amplitude that are similar to polarizabilities, but are caused by electromagnetic interactions, are obtained. The analysis of these structures contributions to hadrons polarizability is performed.

1 The covariant equations of interaction of an electromagnetic field with a nucleon taking into account polarizabilities

To determine the covariant equations describing the electromagnetic field interaction with nucleon taking into account anomalous magnetic moments and polarizabilities we use the following effective Lagrangian:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\overline{\Psi}\left(i\overrightarrow{\widehat{D}} - m\right)\Psi - \frac{1}{2}\overline{\Psi}\left(i\overleftarrow{\widehat{D}} + m\right)\Psi. \quad (1)$$

The following notations were introduced:

$$\overrightarrow{\widehat{D}} = \eta_{\sigma\nu}\gamma^{\sigma}\overrightarrow{\partial}^{\nu} + \frac{ie\kappa}{4m}\sigma^{\mu\nu}F_{\mu\nu} + ie\widehat{A}, \quad (2)$$

$$\overleftarrow{\widehat{D}} = \overleftarrow{\partial}^{\nu}\gamma^{\sigma}\eta_{\sigma\nu} - \frac{ie\kappa}{4m}\sigma^{\mu\nu}F_{\mu\nu} - ie\widehat{A}, \quad (3)$$

$$\eta_{\sigma\nu} = g_{\sigma\nu} + \frac{2\pi}{m}\left[\alpha F_{\sigma\mu}F^{\mu}_{\nu} + \beta\widetilde{F}_{\sigma\mu}\widetilde{F}^{\mu}_{\nu}\right]. \quad (4)$$

If we substitute expressions (2)-(4) into (1), the effective Lagrangian will have the form:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\overline{\Psi}\overleftrightarrow{\widehat{\partial}}\Psi - m\overline{\Psi}\Psi - e\overline{\Psi}\widehat{A}\Psi - \frac{e\kappa}{4m}\overline{\Psi}\sigma^{\mu\nu}\Psi F_{\mu\nu} + K_{\sigma\nu}\Theta^{\sigma\nu}, \quad (5)$$

where

$$K_{\sigma\nu} = \frac{2\pi}{m}\left[\alpha F_{\sigma\mu}F^{\mu}_{\nu} + \beta \widetilde{F}_{\sigma\mu}\widetilde{F}^{\mu}_{\nu}\right], \quad \Theta^{\sigma\nu} = \frac{i}{2}\overline{\Psi}\gamma^{\sigma}\overleftrightarrow{\partial}^{\nu}\Psi, \quad \overleftrightarrow{\partial}^{\nu} = \overrightarrow{\partial}^{\nu} - \overleftarrow{\partial}^{\nu}.$$

We separate the part related to nucleon polarizabilities in the Lagrangian (5)

$$L^{(\alpha,\beta)} = -\frac{2\pi}{m}\left[\alpha F_{\mu\sigma}F^{\mu}_{\nu} + \beta \widetilde{F}_{\mu\sigma}\widetilde{F}^{\mu}_{\nu}\right]\Theta^{\sigma\nu},$$

$$\widetilde{F}_{\mu\sigma}\widetilde{F}^{\mu\nu} = F_{\mu\sigma}F^{\mu\nu} - \frac{1}{2}\delta^{\nu}_{\sigma}F_{\mu\rho}F^{\mu\rho}, \tag{6}$$

$$L^{(\alpha,\beta)} = -\frac{2\pi}{m}\left[(\alpha+\beta)\,F_{\mu\sigma}F^{\mu\nu}\Theta^{\sigma}_{\nu} - \frac{\beta}{2}\Theta^{\sigma}_{\sigma}F_{\mu\nu}F^{\mu\nu}\right]. \tag{7}$$

Expression for the Lagrangian (7) is consistent with the effective Lagrangian presented in [13]. Formula (7) is a relativistic field-theoretic generalization of the non-relativistic relation

$$\mathrm{H} = -L^{(\alpha,\beta)} = -2\pi\left(\alpha\vec{E}^{2} + \beta\vec{H}^{2}\right),$$

which corresponds to the polarizabilities of induced dipole moments in a constant electromagnetic field [14]. In the case of a variable electromagnetic field the signs of polarizabilities in the Lagrangian (in the non-relativistic approximation) will change [15]. However, the structure of tensor contraction in (7) does not change.

In order to obtain the equations for interaction of the electromagnetic field with nucleons, we use the effective Lagrangian (1) and Euler-Lagrange equations:

$$\partial_{\mu}\left(\frac{\partial L}{\partial\left(\partial_{\mu}A_{\nu}\right)}\right) - \frac{\partial L}{\partial A_{\nu}} = 0,$$

$$\partial_{\mu}\left(\frac{\partial L}{\partial\left(\partial_{\mu}\overline{\Psi}\right)}\right) - \frac{\partial L}{\partial\overline{\Psi}} = 0,$$

$$\partial_{\mu}\left(\frac{\partial L}{\partial\left(\partial_{\mu}\Psi\right)}\right) - \frac{\partial L}{\partial\Psi} = 0.$$

As a result we get:

$$\partial_{\mu}F^{\mu\nu} = e\overline{\Psi}\gamma^{\nu}\Psi - \partial_{\mu}\left[\frac{e\kappa}{2m}\overline{\Psi}\sigma^{\mu\nu}\Psi + G^{\mu\nu}\right], \tag{8}$$

$$\left(i\overrightarrow{\widehat{\partial}} - m\right)\Psi =$$

$$= e\widehat{A}\Psi - \frac{i}{2}\left[\partial^{\nu}\left(K_{\sigma\nu}\gamma^{\sigma}\Psi\right) + K_{\sigma\nu}\gamma^{\sigma}\partial^{\nu}\Psi\right] + \frac{e\kappa}{4m}\sigma^{\mu\nu}F_{\mu\nu}\Psi, \tag{9}$$

$$\overline{\Psi}\left(i\overleftarrow{\widehat{\partial}} + m\right) =$$

$$= -\overline{\Psi}e\widehat{A} - \frac{i}{2}\left[\partial^{\nu}\left(\overline{\Psi}\gamma^{\sigma}K_{\sigma\nu}\right) + \left(\partial^{\nu}\overline{\Psi}\right)\gamma^{\sigma}K_{\sigma\nu}\right] - \overline{\Psi}\frac{e\kappa}{4m}\sigma^{\mu\nu}F_{\mu\nu}. \tag{10}$$

If equations (9) and (10) above is not limited to members of the second-order frequency of the radiation, they may be represented as:

$$\left(i\overrightarrow{\widehat{D}} - m\right)\Psi = 0, \quad \overline{\Psi}\left(i\overleftarrow{\widehat{D}} + m\right) = 0,$$

where $\overrightarrow{\widehat{D}}$ and $\overleftarrow{\widehat{D}}$ defined in (2) and (3). Anti-symmetric tensor $G^{\mu\nu}$ in (8) is:

$$G^{\mu\nu} = -\frac{\partial L^{(\alpha,\beta)}}{\partial\left(\partial_{\mu}A_{\nu}\right)} = \frac{4\pi}{m}\left[(\alpha+\beta)\left(F^{\mu}_{\rho}\widetilde{\Theta}^{\rho\nu} - F^{\nu}_{\rho}\widetilde{\Theta}^{\rho\mu}\right) - \beta\Theta^{\rho}_{\rho}F_{\mu\nu}\right], \tag{11}$$

where $\widetilde{\Theta}^{\rho\nu} = \frac{1}{2}\left(\Theta^{\rho\nu} + \Theta^{\nu\rho}\right)$. With the anti-symmetric tensor (11), the effective Lagrangian (7) can be represented as follows:

$$L^{(\alpha,\beta)} = -\frac{1}{4}F_{\mu\nu}G^{\mu\nu}.$$

If equations (9) and (10) to limit the contribution of the charge and magnetic moment, we obtain the well-known equation given, for example, [12].

To identify the physical meaning of tensor $G^{\mu\nu}$ lets use Gordon decomposition [16]. Current density j^{μ} of Dirac particles with the help of Gordon decomposition can be represented as follows:

$$j^{\mu} = e\overline{\Psi}\gamma^{\mu}\Psi = \frac{e}{2m}\overline{\Psi}i\overleftrightarrow{\partial}^{\mu}\Psi - \partial_{\nu}\left[\frac{e}{2m}\overline{\Psi}\sigma^{\nu\mu}\Psi\right],$$

where the notation

$$G_0^{\nu\mu} = \frac{e}{2m}\overline{\Psi}\sigma^{\nu\mu}\Psi, \quad j_e^{\mu} = \frac{e}{2m}\overline{\Psi}i\overleftrightarrow{\partial}^{\mu}\Psi.$$

The components of $G_0^{\mu\nu}$ tensor, which is called anti-symmetric dipole tensor, are static dipole moments of point-like particles. With the help of this tensor we can define the current

$$j_m^\mu = -\partial_\nu G_0^{\nu\mu}.$$

In the rest frame of the particle, we have the following relations:

$$m_0^i = \frac{i}{2}\varepsilon^{ijk}G_{0jk}, \quad d_0^i = G^{i0}.$$

Components 4-dimensional current can be defined by the dipole moments

$$\rho_0 = -\left(\overrightarrow{\partial}\overrightarrow{d}_0\right), \quad \overrightarrow{j}_0 = \partial_t\overrightarrow{d}_0 + \left[\overrightarrow{\partial}\overrightarrow{m}_0\right].$$

The Lagrangian of the interaction of electromagnetic fields with a charged particle with a static dipole moment is:

$$L_I = -j_e^\mu A_\mu - \frac{1}{2}G_0^{\nu\mu}F_{\nu\mu}. \tag{12}$$

By using Lagrangian (12), and the Lagrangian $L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ of the form based on the Euler-Lagrange equations, we get

$$\partial_\mu F^{\mu\nu} = j_e^\mu - \partial_\nu G_0^{\nu\mu}.$$

In relativistic electrodynamics is introduced tensor similar to induced dipole moments [17]. The current density and moments are expressed through $G^{\mu\nu}$ the following

$$j^\mu = -\partial_\nu G_0^{\nu\mu}, \quad d^\mu = G^{\mu\nu}U_\nu, \quad m^\mu = \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}G_{\nu\rho}U_\sigma. \tag{13}$$

Relations (13) satisfies the tensor form:

$$G^{\mu\nu} = (d^\mu U^\nu - U^\mu d^\nu) + \varepsilon^{\mu\nu\rho\sigma}m_\rho U_\sigma.$$

In the quantum description of the structural particles induced dipole moments pass to the operator form [18]:

$$\widehat{\overrightarrow{G}}^{\mu\nu} = -\frac{i}{2m}\left[\left(\widehat{d^\mu}\overrightarrow{\partial}^\nu - \widehat{d^\nu}\overrightarrow{\partial}^\mu\right) + \varepsilon^{\mu\nu\rho\sigma}\widehat{m}_\rho\overrightarrow{\partial}_\sigma\right],$$

$$\overleftarrow{\widehat{G}}^{\mu\nu} = \frac{i}{2m}\left[\left(\overleftarrow{\partial}^{\nu}\widehat{d}^{\mu} - \overleftarrow{\partial}^{\mu}\widehat{d}^{\nu}\right) + \varepsilon^{\mu\nu\rho\sigma}\overleftarrow{\partial}_{\sigma}\widehat{m}_{\rho}\right],$$

where $\widehat{d}^{\mu}$ and $\widehat{m}^{\mu}$ - operators of the induced dipole moments, which are dependent on the electromagnetic field tensor. If you require that the low-energy theorem for Compton scattering, then these operators can be defined as

$$\widehat{d}^{\mu} = 4\pi\alpha\widehat{F}^{\mu\nu}\gamma_{\nu}, \quad \widehat{m}^{\mu} = 4\pi\beta\widehat{\widetilde{F}}^{\mu\nu}\gamma_{\nu}.$$

Thus, the expression (11) is anti-symmetric tensor of the induced dipole moments of the nucleon. In this case, the interaction Lagrangian is defined as follow

$$L_I = -j_e^{\mu}A_{\mu} - \frac{1}{2}G_0^{\mu\nu}F_{\mu\nu} - \frac{1}{4}G^{\mu\nu}F_{\mu\nu},$$

which implies the Maxwell equation of the form:

$$\partial_{\nu}F^{\nu\mu} = j_e^{\mu} - \partial_{\nu}G_0^{\nu\mu} - \partial_{\nu}G^{\nu\mu}.$$

2 Covariant representation of the amplitude of Compton scattering on the nucleon, with input from the polarizabilities

We define the contribution of electric and magnetic polarizabilities of the amplitude of Compton scattering. To do this, use the method of Green's function [10]-[12]. We represent the differential equation (9), which will take into account the contributions of the polarizabilities in integral form:

$$\Psi(x) = \Psi^{(0)}(x) + \int S_F(x - x')\,V^{(\alpha,\beta)}(x')\,dx', \qquad (14)$$

where $V^{(\alpha,\beta)}(x') = -\frac{i}{2}\left[\partial^{\nu}\left(K_{\sigma\nu}(x')\gamma^{\sigma}\Psi(x')\right) + K_{\sigma\nu}(x')\gamma^{\sigma}\partial^{\nu}\Psi(x')\right].$

We define the matrix element S_{fi} of the scattering of photons on a nucleon. To do this, we turn (14), $\overline{\Psi}_{p_2}^{(r_2)}(x)$ when $t \to +\infty$ we use the relation

$$\int \overline{\Psi}_{p_2}^{(r_2)}(x)\,S_F(x - x')\,d^3x\,|_{t\to+\infty} = (-i)\,\overline{\Psi}_{p_2}^{(r_2)}(x'),$$

where

$$\overline{\Psi}_{p_2}^{(r_2)} = \frac{1}{(2\pi)^{\frac{3}{2}}}\sqrt{\frac{m}{E_2}}\,\overline{U}^{(r_2)}(\overrightarrow{p}_2)\,e^{ip_2x'}.$$

As a result, we get:

$$S_{fi} = (-i) \int \overline{\Psi}_{p_2}^{(r_2)} (x') V^{(\alpha,\beta)} (x') d^4x'. \tag{15}$$

Using the boundary conditions and the symmetry of the cross, the expression (15) can be represented as:

$$S_{fi} = \left(-\frac{1}{2}\right) \int \overline{\Psi}_{p_2}^{(r_2)} (x') \left\{\partial^{\nu} \left(K_{\sigma\nu}^{21} (x') \gamma^{\sigma} \Psi_{p_1}^{(r_1)} (x')\right) + \right.$$

$$\left. + K_{\sigma\nu}^{21} (x') \gamma^{\sigma} \partial^{\nu} \Psi_{p_1}^{(r_1)} (x')\right\} d^4x'. \tag{16}$$

Integrating by parts and using the definition of the electromagnetic field tensor in (16) we get:

$$S_{fi} = \frac{2\pi i}{m} \int \left[(\alpha + \beta) \left(F_{\sigma\mu}^{(2)} F_{(1)}^{\mu\nu} + F_{\sigma\mu}^{(1)} F_{(2)}^{\mu\nu}\right) \Theta_{(21)\nu}^{\sigma} + \right.$$

$$\left. + \beta \left(F_{\mu\nu}^{(2)} F_{(1)}^{\mu\nu}\right) \Theta_{(21)\rho}^{\rho}\right] d^4x'. \tag{17}$$

If we consider the wave functions of the nucleon and the photons in the initial and final states, the expression (17) takes the form:

$$S_{fi} = \frac{im\delta (k_1 + p_1 - k_2 - p_2)}{(2\pi)^2 \sqrt{4\omega_1\omega_2 E_1 E_2}} M, \tag{18}$$

where in M in the amplitude (18) is as follows:

$$M = \frac{2\pi}{m} \overline{U}^{(r_2)} (\vec{p}_2) (\alpha + \beta) \left\{ \left[\widehat{k}_2 e_{\mu}^{(\lambda_2)} - k_{2\mu} \widehat{e}^{(\lambda_2)}\right] \left[k_1^{\mu} \left(e^{(\lambda_1)} \mathcal{P}\right) - (k_1 \mathcal{P}) e^{(\lambda_1)\mu}\right] + \right.$$

$$\left. + \left[k_2^{\mu} \left(e^{(\lambda_2)} \mathcal{P}\right) - (k_2 \mathcal{P}) e^{(\lambda_2)\mu}\right] \left[\widehat{k}_1 e_{\mu}^{(\lambda_1)} - k_{1\mu} \widehat{e}^{(\lambda_1)}\right] \right\} +$$

$$+ m\beta \left[k_{2\mu} e_{\nu}^{(\lambda_2)} - k_{2\nu} e_{\mu}^{(\lambda_2)}\right] \left[k_1^{\mu} e^{(\lambda_1)\nu} - k_1^{\nu} e^{(\lambda_1)\mu}\right] U^{(r_1)} (\vec{p}_1). \tag{19}$$

We now define the amplitude (19) in the rest frame of the target and limit in M members are not higher than the second frequency radiation. In this case, we have [19]:

$$M = 4\pi\omega^2 \chi^{(r_2)+} \left[\alpha \left(\vec{e}^{(\lambda_2)} \vec{e}^{(\lambda_1)}\right) + \beta \left(\left[\vec{n}_2 \vec{e}^{(\lambda_2)}\right] \left[\vec{n}_1 \vec{e}^{(\lambda_1)}\right]\right)\right] \chi^{(r_1)}.$$

If the amplitude of M along with the contribution of the polarizabilities α and β take into account the contribution of the electric charge, then M can be represented as follows:

$$M = \chi^{(r_2)+}\left[\left(-\frac{e^2}{m}+4\pi\omega^2\alpha\right)\left(\overrightarrow{e}^{(\lambda_2)}\overrightarrow{e}^{(\lambda_1)}\right)+\right.$$

$$\left.+4\pi\omega^2\beta\left(\left[\overrightarrow{n}_2\overrightarrow{e}^{(\lambda_2)}\right]\left[\overrightarrow{n}_1\overrightarrow{e}^{(\lambda_1)}\right]\right)\right]\chi^{(r_1)}. \qquad (20)$$

Differential Compton cross section, for example at an angle $\theta = 0$, computed using (20) has the form [19]:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha_e}{m}\right)^2 - 2\frac{\alpha_e}{m}\left(\alpha+\beta\right)\omega^2,$$

where $\alpha_e = \frac{e^2}{4\pi}$.

3 Quasi-static polarizability of particles spin $\frac{1}{2}$ in QED

We find the quasi-static polarizability structureless fermions, which appear in the Compton scattering due to higher orders. In general, the AKP T forward ($\theta = 0$) and backward ($\theta = \pi$) up ω^2 can be written as:

$$T_{\lambda,\sigma}^{\lambda',\sigma'}\left(\theta=0\right) = 8\pi m_f\omega^2\left(\alpha_E+\beta_M\right)\delta_{\lambda,\lambda'}\delta_{\sigma,\sigma'},$$

$$T_{\lambda,\sigma}^{\lambda',\sigma'}\left(\theta=\pi\right) = 8\pi m_f\omega^2\left(\alpha_E-\beta_M\right)\lambda\delta_{-\lambda,\lambda'}\delta_{\sigma,-\sigma'}.$$

On the other hand it is possible to calculate the matrix elements, respectively, the amplitude of Compton scattering in the framework of QED, including next to the Born-order perturbation theory in the coupling constant α_{QED} (see, e.g., [20], [21]). In [22] developed a method of calculating the polarizabilities of fermions in the framework of quantum field models and theories by comparing the corresponding matrix elements. The outcome of this procedure in this case is the ratio:

$$\alpha_E^{q-s}+\beta_M^{q-s} = \frac{\alpha_{QED}^2}{3\pi m_f^3}\frac{11}{6}+\frac{8\alpha_{QED}^2}{3\pi m_f^3}\ln\frac{2\omega}{m_f}, \qquad (21)$$

$$\alpha_E^{q-s}-\beta_M^{q-s} = -\frac{\alpha_{QED}^2}{3\pi m_f^3}\frac{59}{6}+\frac{4\alpha_{QED}^2}{3\pi m_f^3}\ln\frac{2\omega}{\lambda}, \qquad (22)$$

where the parameter λ is an infinitely small mass of the photon.

As follows from (21) and (22) in addition to the quasi-static polarizability of the permanent members and contain non-analytic terms $\sim \ln\omega$, which differ in the Thomson limit $\omega \to 0$. This property was the reason that the work [23], [24] structure (21) and (22) were identified quasistatic polarizabilities. From (21) and (22) it is easy to find the electric (α_E^{q-s}) and magnetic (β_M^{q-s}) quasi-static polarizability and assess their contribution to the polarizability of the "Dirac" proton (point of zero fermion anomalous magnetic moment):

$$\alpha_E^{q-s} + \beta_M^{q-s} \approx -5,8 \cdot 10^{-7} Fm^3. \tag{23}$$

The experimental values [25]:

$$\alpha_E^{q-s} + \beta_M^{q-s} = (13,8 \pm 0,4) \cdot 10^{-4} Fm^3. \tag{24}$$

Numerical estimates are consistent with estimates of [26].

Conclusion

In the framework of the gauge-invariant approach we obtain the covariant equations of motion of a nucleon in the electromagnetic field, taking account of its electric and magnetic polarizabilities. Based on the decision of electrodynamic equations of motion of the nucleon obtained by the Green's function, it is shown that the developed covariant formalism of Lagrange interaction of low-energy photons with nucleons is consistent with the low-energy theorem of Compton scattering. Based on an original technique reproduced the known result for the combination of quasi-static polarizabilities $\alpha_E^{q-s}+\beta_M^{q-s}$ in QED framework and obtain a new expression for $\alpha_E^{q-s} - \beta_M^{q-s}$. The apparent advantage of method of defining "polarizabilities" referred to in Section 3, is its relative simplicity. This approach opens up more opportunities for the study of the internal structure of nucleons and can be applied in various quantum field theories and models.

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