Известия Гомельского государственного университета
имени Ф. Скорины, №6(69), 2011

PACS 12.15.Lk, 13.88.+e, 25.30.Bf

Electroweak radiative corrections for polarized Møller scattering at one loop and beyond

A. Alekseev, S. Barkanova, V. Zykunov

Мы представляем новый расчет однопетлевых электрослабых радиационных поправок к асимметрии меллеровского рассеяния, используя два подхода: точный, с использованием компьютерных программ FeynArts и FormCalc, и асимптотический, с применением приближений, сделанных "вручную". Также мы проводим точное сравнение результатов, полученных в различных схемах: перенормировке на массовой поверхности и связанной дифференциальной перенормировке. Мы обсуждаем часть двухпетлевых поправок, индуцированных квадратом однопетлевых диаграмм, и показываем значительную величину этой части, что указывает на необходимость уче-та полных двухпетлевых поправок для обеспечения необходимой точности предстоящих экспериментов.
Ключевые слова: поляризационное меллеровское рассеяние, нарушающая четность асимметрия, электрослабые радиационные поправки, схема перенормировки.

We perform the updated calculations of one-loop EWC for Møller scattering asymmetry using two different approaches: semi-automatic, precise, with FeynArts and FormCalc as base languages, and "by hand", with reasonable approximations. In addition, we provide a tuned comparison between the one-loop results obtained in two different renormalization schemes: on-shell and constrained differential renormalization. Also we discuss the two-loop EWC induced by squaring one-loop diagrams, and show that the significant size of this partial correction indicates a need for a complete study of the two-loop EWC in order to meet the precision goals of future experiments.
Keywords: polarized Moller scattering, parity-violating asymmetry, electroweak radiative corrections, renormalization scheme.

1. Introduction

Polarized electron-electron (Møller) scattering is a very clean process with a well-known kinematics and extremely suppressed backgrounds, and any inconsistency with the Standard Model will signal new physics. The next-generation experiment to study electron-electron scattering, MOLLER [1], planned at JLab following the 11 GeV upgrade, will offer a new level of sensitivity and measure the parity-violating asymmetry in the scattering of longitudinally polarized electrons off an unpolarized target to a precision of 0.73 ppb, and allow a determination of the weak mixing angle with an uncertainty of about 0.1%, a factor of five improvement over the measurement by E-158 [2, 3]. Obviously, before we can extract reliable information from the experimental data, it is necessary to take into account EWC. EWC to the parity-violating (PV) Møller scattering asymmetry were addressed in the literature earlier and were shown to be large [4–6]. A more detailed literature review can be found in [6], our first work on the topic. In [6], we calculated a full gauge-invariant set of the one-loop EWC and found the total correction to be close to -70%, with no significant theoretical uncertainty coming from the hadronic contributions to the vacuum polarization or other uncertain input parameters. Since it is possible that a much larger theoretical

uncertainty may come from two-loop corrections, we investigated the importance of two-loop contribution in [7], by comparing corrections calculated in two different renormalization schemes (RS) – on-shell (OS) and constrained differential renormalization (CDR) – and found a difference of about 11%. This means that the two-loop EWC may be larger than previously thought and cannot be dismissed, especially in the light of precision promised by MOLLER. We divide the two-loop EWC into two classes: the Q-part induced by quadratic one-loop amplitudes, and the T-part which includes the interference of Born and two-loop diagrams. In [8], we calculated the Q-part exactly and found that it can reach 4%. Here, we provide a brief review of our calculations done at the one-loop level [6], show details of the comparison between the corrections evaluated in the OS and CDR schemes [7], and outline some of our calculations of higher order corrections.

2. Born and one-loop corrections

The asymmetry between left/right longitudinally polarized electrons can be constructed in the following way:

$$A_{LR} = \frac{\sigma_{LL} + \sigma_{LR} - \sigma_{RL} - \sigma_{RR}}{\sigma_{LL} + \sigma_{LR} + \sigma_{RL} + \sigma_{RR}} = \frac{\sigma_{LL} - \sigma_{RR}}{\sigma_{LL} + 2\sigma_{LR} + \sigma_{RR}}, \tag{1}$$

enhancing the contributions induced by PV electroweak interactions. The notation $\sigma \equiv \equiv d\sigma/d\cos\theta$ stands for the differential cross section defined in the center of mass reference frame of incoming electrons. At the Born level (leading order (LO)), the asymmetry is

$$A_{LR} = \frac{s}{2m_W^2} \frac{y(1-y)}{1+y^4+(1-y)^4} \frac{1-4s_W^2}{s_W^2},\ y = -\frac{t}{s}, \tag{2}$$

where $s_W^2 \equiv \sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} \sim 0.24$. As one can see from Eq. 2, the asymmetry is highly sensitive to θ_W so any deviation from the SM value will signal new physics. Obviously, before we can extract reliable information from the experimental data, it is necessary to include EWC. The cross section including one-loop matrix elements is:

$$\sigma = \frac{\pi^3}{2s}|M_0 + M_1|^2 = \frac{\pi^3}{2s}\Big(M_0M_0^+ + 2\mathrm{Re}M_1M_0^+ + M_1M_1^+\Big) = \sigma_0 + \sigma_1 + \sigma_Q \tag{3}$$

where $\sigma_1 = \sigma_1^{\mathrm{BSE}} + \sigma_1^{\mathrm{Ver}} + \sigma_1^{\mathrm{Box}} \propto \alpha^3$ is an interference term between the Born and one-loop amplitudes (NLO), and the cross section $\sigma_Q \propto \alpha^4$ is a quadratic term of the same order as the two-loop contribution (NNLO). To make sure that our calculations at the one-loop level are error-free, we evaluate EWC using two different methods. The "by hand"method is to derive the compact analytic expressions for the leading one-loop correction manually using appropriate approximations for $\sqrt{s} < 30$ GeV and $\sqrt{s} > 500$ GeV [6]. The other, semi-automated, is to consider a full set of graphs with no approximations using computer-based algebra packages [9, 10] and [11]. To make sure that we calculate a gauge-invariant set of graphs, we use two sets of renormalization conditions (RC): the RC by Hollik (HRC) introduced in [12] for our “by hand” approach, and the RC proposed by Denner (DRC) in [13] for our semi-automated method. The infrared divergences (IR) are treated by the soft and hard-photon bremsstrahlung (see [6]). We choose our input parameters to be the fine structure constant ($\alpha = 1/137.03599$), the mass of the W-boson ($m_W = 80.398\,\mathrm{GeV}$) and the mass of the Z-boson ($m_Z = 91.1876\,\mathrm{GeV}$). A relative correction to the PV asymmetry is defined as $\delta_A^C = (A_{LR}^C - A_{LR}^0)/A_{LR}^0$, with the superscript in δ_A^C corresponding to the various contributions: “weak” indicates no IR-divergent graphs, and “QED” indicates only

IR divergent graphs treated by bremsstrahlung contribution. In order to see how our results compare to the literature [5], we compare the $\delta_A^{weak}([6]) = -0.2790$ for the $\sqrt{s} = 100\,\text{GeV}$ and $\delta_A^{weak}([5]) = -0.2787$ using the same input parameters as in [5] and obtain excellent agreement.

To establish if the NNLO contributions in a given RS are important (see [15]), we compare results in two RS: OS and CDR. Let us define the total correction to the unpolarized cross section as $\delta^{tot} = (\sigma^{tot} - \sigma^0)/\sigma^0$. In the low-energy area, the correction to the cross section is dominated by the QED contribution, and the difference between the two schemes is almost constant and rather small (~ 0.01), but it grows at $\sqrt{s} \geq m_Z$ as the weak correction becomes comparable to QED. As a result, the difference between the OS and CDR corrections to the PV asymmetry can reach as much as 10%, so contributions from two-loop corrections could become important.

3. Two-loops corrections: Q-part

The higher-order corrections ($\propto \alpha^4$) to the electroweak Born cross section can be divided into two classes, Q-part and T-part. The Q-part is induced by the quadratic one-loop amplitude ($\sim M_1 M_1^+$) (third term in Eq. 2) and the T-part is an interference term between the Born and two-loop amplitudes: $\sigma_T = \frac{\pi^3}{s}\text{Re}M_2 M_0^+ \propto \alpha^4$ (Fig. 1).

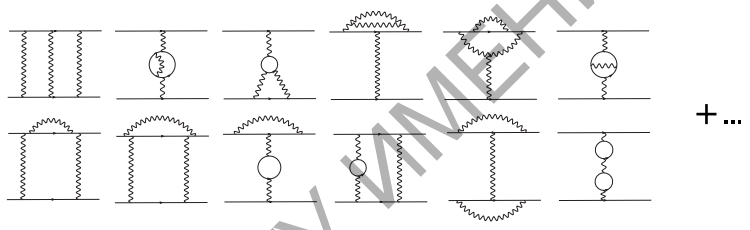

Figure 1 — Representative two-loop graphs for the Møller scattering.

The T-part still needs to be evaluated in the future, but we can provide some results for the Q-part in this paper. The cross section for the Q-part can be divided into two terms: $\sigma_Q = \sigma_Q^\lambda + \sigma_Q^f$. The former term, σ_Q^λ, is an IR-divergent and regularized part of the cross section and the latter term, $\sigma_Q^f = (\frac{\alpha}{\pi})^2 \delta_1^f \cdot \sigma_0$, is a finite contribution. The IR-divergent part has the following structure:

$$\sigma_Q^\lambda = \frac{\pi^3}{2s} M_1^{\lambda +}(M_1^\lambda + 2M_1^f) = \frac{1}{4}\Big(\frac{\alpha}{\pi}\Big)^2 \text{Re}\Big[\delta_1^{\lambda *}(\delta_1^\lambda + 2\delta_1^f)\Big] \cdot \sigma_0, \tag{4}$$

where $\delta_1^\lambda = 4 \ln \frac{\lambda}{\sqrt{s}} \Big(\ln \frac{tu}{m^2 s} - 1 + i\pi \Big)$. Since the Q-part contains terms of order $\propto \ln^2 \frac{\lambda}{\sqrt{s}}$ it deserves a special attention. To treat the IR divergences, we have to account not only for photon emission from one-loop diagrams but also include a complete treatment of the two-photon emission (Fig. 2).

A half of the bremsstrahlung contribution in Fig. 2(a) and 2(b) is responsible for the treatment of IR divergence in the Q-part and the other half for the T-part. We take the maximum energy of the emitted soft photon to be $\omega = 0.05 \cdot \sqrt{s}$. The bremsstrahlung cross section for Q-part is derived from Fig. 2(a) as:

$$\sigma_Q^\gamma = \frac{1}{2}\sigma^\gamma = \frac{1}{2}\Big(\frac{\alpha}{\pi}\Big)^2 \text{Re}[(-\delta_1^\lambda + R_1)^*(\delta_1^\lambda + \delta_1^f)] \cdot \sigma_0 \tag{5}$$

$$R_1 = -4 \ln \frac{\sqrt{s}}{2\omega} \Big(\ln \frac{tu}{m^2 s} - 1 + i\pi \Big) - \Big(\ln \frac{s}{m^2} - 1 \Big)^2 + 1 - \frac{\pi^3}{3} + \ln^2 \frac{u}{t}.$$

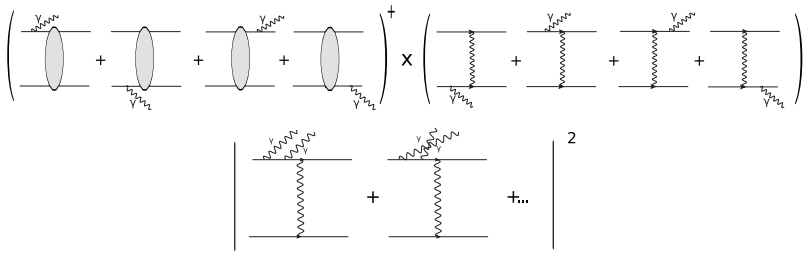

Figure 2 — Bremsstrahlung treatment of IR divergences in the Q-part. The top plot (a) represents interference between emission from one-loop (shaded bubble) and Born graphs. The bottom plot (b) is the two-photon emission amplitude squared.

Here, σ^{γ} is the total photon emission cross section and σ_Q^{γ} is the one-photon bremsstrahlung term from the Q-part. The two-photons emission for the Q-part ($\sigma_Q^{\gamma\gamma}$) is derived from Fig. 2(b):

$$\sigma_Q^{\gamma\gamma} = \frac{1}{2}\sigma^{\gamma\gamma} = \frac{1}{4}\left(\frac{\alpha}{\pi}\right)^2 \left(\left| -\delta_1^{\lambda} + R_1 \right|^2 - \frac{8}{3}\pi^2 \left| \ln\frac{tu}{m^2 s} - 1 + i\pi \right|^2 \right) \cdot \sigma_0. \tag{6}$$

Combining Eqs. 4, 5 and 6 gives the final result for $\sigma_Q^{\lambda} + \sigma_Q^{\gamma} + \sigma_Q^{\gamma\gamma}$ free from nonphysical parameters with the regularization parameter λ cancelled analytically. Detailed calculations can be found in [8].

As one can see from analisys, the correction induced by the Q-part ($\Delta_A = (A_{LR}^{1-loop+Q} - A_{LR}^{1-loop})/A_{LR}^{0}$) can reach as much as $\sim 4\%$ at $\theta = 90^{\circ}$. The energy dependence is nearly constant for $\sqrt{s} < m_Z$ but increases rapidly after weak interactions become comparable to QED.

4. Conclusion

With the one-loop EWC now under control, it is worth considering the two-loop EWC. One way to find some indication of NNLO EWC size is to compare results that are expressed in terms of quantities related to different RS, and our tuned comparison between the results obtained in the OS and CDR RS show a difference of about 11%. Although an argument can be made that the two-loop EWC are suppressed by a factor of α/π relative to the one-loop EWC, we see that they can no longer be dismissed, especially in the light of the 2% uncertainty to asymmetry promised by the MOLLER. At the MOLLER kinematic conditions, the part of the quadratic EWC considered here can increase the asymmetry up to $\sim 4\%$. For the high-energy region $\sqrt{s} \sim 2\,\text{TeV}$, a contribution from the quadratic EWC can reach +30%. It is impossible to say at this time if the Q-part will be enhanced or cancelled by other two-loop radiative corrections, but we suppose that the large size of the Q-part demands a detailed and consistent consideration of the T-part, which is the current task of our group.

References

1 The MOLLER Experiment. **M**easurement **O**f a **L**epton **L**epton **E**lectroweak **R**eaction. An Ultra-precise Measurement of the Weak Mixing Angle using Møller Scattering. An MIE proposal to The United States Department of Energy [Electronic resource] / J. Benesch [et al.] – The MOLLER Collaboration, 2011. – Mode of access:

http://hallaweb.jlab.org/12GeV/Moller/downloads/DOE_Proposal/DOE_Moller.pdf – Date of access: 12.09.2011.

2 Anthony, P. L. Observation of parity nonconservation in Møller scattering / SLAC E158 Collab.: P. L. Anthony [et al.] // Phys. Rev. Lett. – 2004. – Vol. 92, 181602 [hep-ex/0312035].

3 Anthony, P. L. Precision measurement of the weak mixing angle in Møller scattering / SLAC E158 Collab.: P. L. Anthony [et al.] // Phys. Rev. Lett. – 2005. – Vol. 95, 081601. [hep-ex/0504049].

4 Czarnecki, A. Electroweak radiative corrections to polarized Møller scattering asymmetries / A. Czarnecki, W. Marciano // Phys. Rev. D. – 1996. – Vol. 53. – P. 1066–1072.

5 Denner, A. Electroweak radiative corrections to polarized Møller scattering at high energies / A. Denner, S. Pozzorini // Eur. Phys. J. C. – 1999. – Vol. 7. – P. 185–195 [hep-ph/9807446].

6 Aleksejevs, A. Electroweak radiative corrections for polarized Moller scattering at future 11 GeV JLab experiment / A. Aleksejevs, S. Barkanova, A. Ilyichev, V. Zykunov // Phys. Rev. D – 2010. – Vol. 82, 093013, P. 1–16. arXiv:1008.3355 [hep-ph].

7 Aleksejevs, A. One-loop electroweak corrections for polarized Moller scattering at different renormalization schemes and conditions / A. Aleksejevs [et al.] // arXiv:1010.4185 [hep-ph] – 2010, P. 1–14.

8 Aleksejevs, A. Quadratic electroweak corrections for polarized Moller scattering / A. Aleksejevs [et al.] // arXiv:1110.1750 [hep-ph] – 2011, P. 1–16.

9 Hahn, T. Generating Feynman diagrams and amplitudes with FeynArts 3 / T. Hahn // Comput. Phys. Commun. – 2001. – Vol. 140. – P. 418–431. [hep-ph/0012260].

10 Hahn, T. Automatized one loop calculations in four-dimensions and D-dimensions / T. Hahn, M. Perez-Victoria // Comput. Phys. Commun. – 1999. – Vol. 118. – P. 153–165. [hep-ph/9807565].

11 Vermaseren, J. New features of FORM / J. Vermaseren // arXiv: math-ph/0010025 – 2000, P. 1–23.

12 Hollik, W. Radiative corrections in the standard model and their role for precision tests of the electroweak theory / W. Hollik // Fortschr. Phys. – 1990. – Vol. 38. – P. 165–260.

13 Denner, A. Techniques for the calculation of electroweak radiative corrections at the one-loop level and results for W-physics at LEP200 / A. Denner // Fortschr. Phys. – 1993. – Vol. 41. – P. 307–419.

14 del Aguila, F. Constraining differential renormalization in Abelian gauge theories / F. del Aguila [et al.] // Phys. Lett. B – 1998. – Vol. 419. – P. 263–271. [hep-th/9709067].

15 Hollik, W. Renormalization scheme dependence of electroweak radiative corrections / W. Hollik, H.-J. Timme // Z. Phys. C. – 1986. Vol. 33. – P. 125–133.

Memorial University,
Corner Brook, Canada
aaleksejevs@swgc.mun.ca

Acadia University, Wolfville, Canada
svetlana.barkanova@acadiau.ca

Belarussian State University of Transport,
Gomel, Belarus
vladimir.zykunov@cern.ch

Поступило 16.11.11