V. S. Zakrevskaya
Gomel, F. Skorina Gomel State University

FINITE GROUPS WITH PARTIALLY π-SUBNORMAL SUBGROUPS

All groups considered here are finite and G always denotes a finite group. Moreover, P is the set of all primes, $\pi \subseteq$ P and $\pi' =$ P $\setminus \pi$.

We say that a subgroup A of G is *strongly* $\mathfrak{U}$*-subnormal* [1] in G if either A is normal in G or $A_G \neq A^G$ and every chief factor of G between A_G and A^G is cyclic.

A subgroup A of G is called *π-subnormal* [2] in G if and only if it is $\mathfrak{F}$-subnormal in G in the sense of Kegel [3], where $\mathfrak{F}$ is the class of all π'-groups.

We say that a subgroup A of G is *partially π-subnormal* in G if $A = \langle L,T \rangle$, where L is a strongly $\mathfrak{U}$-subnormal subgroup and T is a π-subnormal subgroup of G.

Recall that if $M_n < M_{n-1} < \ldots < M_1 < M_0 = G$ (*), where M_i is a maximal subgroup of M_{i-1} for all $i = 1,\ldots,n$, then the chain (*) is said to be a *maximal chain of G of length n* and M_n ($n > 0$) is an *n-maximal subgroup* of G.

The relationship between n-maximal subgroups (where $n > 1$) of G and the structure of G was studied by many authors. One of the earliest results in this line research was obtained by Huppert in the article [4] who established the supersolubility of the group whose all second maximal subgroups are normal. In the same article Huppert proved also that if all 3-maximal subgroups of G are normal in G, then the commutator subgroup G^1 of G is a nilpotent group and the principal rank of G is at most 2. These two results were developed by many authors. Spencer studied [5] the groups G whose every n-maximal chain includes at least one proper subnormal subgroup of G. Mann proved [6] that if all n-maximal subgroups of a soluble group G are subnormal and $n \leq |\pi(G)| - 1$, then G is nilpotent; but if $n \leq |\pi(G)| + 1$, then G is φ-dispersive for some ordering φ of the set of all primes P. Finally, in the case $n \leq |\pi(G)|$ Mann described G completely.

Our main goal here is to obtain generalizations of some of these results on the base of the following

Theorem. *If in every maximal chain $M_3 < M_2 < M_1 < M_0 = G$ of G of length* 3, *one of M_3, M_2 and M_1 is partially π-subnormal in G, then G is π-soluble.*

Corollary 1. (Spencer [5]). *If in every maximal chain $M_3 < M_2 < M_1 < M_0 = G$ of G of length* 3, *one of M_3, M_2 and M_1 is subnormal in G, then G is soluble.*

Corollary 2. (Huppert [4]). *If every 3-maximal subgroup of G is normal in G, then G is soluble.*

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