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On radicals of products of finite soluble groups

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Dedicated to Professor Wolfgang Gaschütz on the occasion of his 80th birthday

In [1] V.S.Monakhov proved that $F(A) \cap F(B) \subseteq F(G)$ for every finite group $G = AB$ which is the product of two subgroups A and B. The result of Johnson [2] says that for every finite soluble group $G = AB$ and for every set of primes π, the maximal normal π-subgroups satisfy $O_\pi(A) \cap O_\pi(B) \subseteq O_\pi(G)$. B.Amberg and L.S.Kazarin in [3] showed that the result of Johnson cannot be extended to arbitrary finite groups and indicated conditions under which this problem has an affirmative solution.

Since classes of all nilpotent groups and all π-groups are Fitting classes, it is natural to look for Fitting classes $\mathfrak{F}$ such that $A_\mathfrak{F} \cap B_\mathfrak{F} \subseteq G_\mathfrak{F}$ for every group $G = AB$.

All groups considered are finite and soluble. The purpose of this paper is to prove the following theorem.

Theorem. *Let $\mathfrak{F}$ be a Fitting formation. Then the following statements are equivalent:*

1) if $G = AB$, then $A_\mathfrak{F} \cap B_\mathfrak{F} \subseteq G_\mathfrak{F}$;

2) $\mathfrak{F} = D_0(\cup_{i \in I} \mathfrak{S}_{\pi_i})$, where $\cup_{i\in I}\pi_i = \pi(\mathfrak{F})$ and $\pi_l \cap \pi_k = \varnothing$ for all $l \neq k$ in I.

Remark. From [4] and our theorem it follows that the collection of Fitting formations above mentioned coincides with the collection of subgroup-closed saturated formations $\mathfrak{F}$ such that the set of all $\mathfrak{F}$-subnormal subgroups is a lattice for every group.

We use definitions and results from [5,6]. Let $\mathfrak{X}$ be a class of groups. Following [6], we define $D_0\mathfrak{X} = (G : G = H_1 \times \ldots \times H_r$ with each $H_i \in \mathfrak{X})$. A group G is called a minimal non-$\mathfrak{X}$-group, if G is not in $\mathfrak{X}$ but all proper subgroups of G are in $\mathfrak{X}$. $\mathcal{M}(\mathfrak{X})$ denotes the class of all minimal non-$\mathfrak{X}$-groups. We remind that if $\mathfrak{F}$ is a Fitting class then each group has a unique maximal normal $\mathfrak{F}$-subgroup $G_\mathfrak{F}$, $\mathfrak{F}$-radical of G. The other used notations are: π is a set of primes; π' is the set of all primes different from $p \in \pi$; C_p is a group of order p; $GF(p)$ is the field with p elements; $\pi(G)$ is the set of primes dividing $|G|$; $\pi(\mathfrak{F}) = \cup\{\pi(G) : G \in \mathfrak{F}\}$; $F_p(G) = O_{p'p}(G)$ is the p-nilpotent radical of G; $[L]K$ is the semidirect product; $\mathfrak{S}_\pi$ is the class of soluble π-groups; $\mathfrak{F}^S$ is a maximal S-closed subclass of $\mathfrak{F}$.

Lemma 1. *Let $\{\mathfrak{X}_i : i \in I\}$ be a set of classes of groups. Then the following statements are satisfied:*

1) if $\mathfrak{X}_i$ is a local S-closed formation for every $i \in I$ and $\pi(\mathfrak{X}_l) \cap \pi(\mathfrak{X}_k) = \varnothing$, for all $l \neq k \in I$ then $D_0(\cup_{i\in I}\mathfrak{X}_i)$ is also a local S-closed formation;

2) if $\mathfrak{X}_i$ is a Fitting class for every $i \in I$ and $\pi(\mathfrak{X}_l) \cap \pi(\mathfrak{X}_k) = \varnothing$, for all $l \neq k \in I$ then $D_0(\cup_{i\in I}\mathfrak{X}_i)$ is also a Fitting class.

Proof follows directly from the definitions.

Lemma 2. *Let $\{\pi_i : i \in I\}$ be partition of non-empty set π of primes and $\mathfrak{F} = D_0(\cup_{i\in I}\mathfrak{S}_{\pi_i})$. Then $\mathfrak{F} = LF(f)$ is a local S-closed Fitting formation where $f(p) = \mathfrak{S}_{\pi_i}$ for $p \in \pi_i$, $f(p) = \varnothing$ for $p \in \pi'$.*

Proof. Since $\mathfrak{S}_{\pi_i}$ is an S-closed local Fitting formation for every $i \in I$ and $\pi_l \cap \pi_k = \varnothing$ for all $l \neq k \in I$ then, by lemma 1, $\mathfrak{F}$ is also a S-closed local Fitting formation.

Let f be a function such that $f(p) = \mathfrak{S}_{\pi_i}$ if $p \in \pi_i$, $f(p) = \varnothing$ for every $p \in \pi'$. Let $\mathfrak{F}^* = LF(f)$. We show that $\mathfrak{F} = \mathfrak{F}^*$. It is clear that $\mathfrak{F} \subseteq \mathfrak{F}^*$. Let G be a group of minimal order in $\mathfrak{F}^* \setminus \mathfrak{F}$. Since $\mathfrak{F}$ and $\mathfrak{F}^*$ are local formations, G is a primitive group. In this case $Soc(G) = N = C_G(N)$ is a p-group for some prime p and $G = [N]M$ where M is some maximal subgroup of G. Since $G \in \mathfrak{F}^*$, $G/C_G(N) = G/N \simeq M \in f(p) = \mathfrak{S}_{\pi_i}$, where $p \in \pi_i$. Hence, $G \in \mathfrak{S}_{\pi_i}$ for some $i \in I$. Since $\mathfrak{S}_{\pi_i} \subseteq \mathfrak{F}$ then $G \in \mathfrak{F}$, a contradiction. Therefore $\mathfrak{F}^* = \mathfrak{F}$. Lemma is proved.

Lemma 3. *Let $\mathfrak{F}$ be a class of groups. Then the following statements are satisfied:*
1) $\mathcal{M}(\mathfrak{F}) = \mathcal{M}(\mathfrak{F}^S)$;
2) if $\mathfrak{F}$ is a formation then $\mathfrak{F}^S$ is also a formation.

Proof follows directly from the definitions.

Lemma 4. *Let $\mathfrak{F}$ be a Fitting formation. Then $\pi(\mathfrak{F}) = \pi(\mathfrak{F}^S)$.*

Proof. It is clear that $\pi(\mathfrak{F}^S) \subseteq \pi(\mathfrak{F})$. Let $G \in \mathfrak{F}$ and $p \in \pi(G)$. Then by lemma 1.7 in [6, p. 565] $C_p \in S_n N_o S_n(G) \subseteq \mathfrak{F}$. Therefore $\pi(\mathfrak{F}) \subseteq \pi(\mathfrak{F}^S)$. Lemma is proved.

O.J.Schmidt in 1924 for the first time studied the minimal non-$\mathfrak{N}$-groups. Now these groups are called Schmidt groups. Following [7], we say that formation $\mathfrak{F}$ has the Shemetkov property if every minimal non-$\mathfrak{F}$-group is either a Schmidt group or a cyclic group of prime order.

Lemma 5 [8]. *Let $\mathfrak{F}$ be an S-closed formation with the Shemetkov property. Then $\mathfrak{F}$ is local.*

Lemma 6 [9]. *A local S-closed formation $\mathfrak{F}$ is a formation with the Shemetkov property if and only if $\mathfrak{F} = LF(f)$ and f satisfies the following conditions:*
1) $f(p) = \mathfrak{S}_{\pi(f(p))}$ for each $p \in \pi(\mathfrak{F})$;
2) $f(p) = \varnothing$ for each $p \notin \pi(\mathfrak{F})$;
3) $f(p) = \mathfrak{N}_p f(p)$ for each prime p.

Lemma 7. *Let $\mathfrak{F}$ be a Fitting formation, satisfying condition 2) of the theorem. If $\mathfrak{F}^S$ is a formation with the Shemetkov property then $\mathfrak{F}^S = D_0(\cup_{i\in I}\mathfrak{S}_{\pi_i})$ where $\cup_{i\in I}\pi_i = \pi(\mathfrak{F})$ and $\pi_l \cap \pi_k = \varnothing$ for all $l \neq k$ in I.*

Proof. Let $\mathfrak{F}$ be a Fitting formation satisfying condition 2) of the theorem. Set $\mathfrak{X} = \mathfrak{F}^S$. Since $\mathfrak{X}$ is a formation with the Shemetkov property then, by lemma 6, $\mathfrak{X} = LF(f)$, where $f(p) = \mathfrak{S}_{\pi(f(p))}$ for $p \in \pi(\mathfrak{F})$, $f(p) = \varnothing$ for $p \notin \pi(\mathfrak{F})$, and $f(p) = \mathfrak{N}_p f(p)$ for every prime p.

Assume $\pi(\mathfrak{X}) = \{p\}$. Then it is clear that $\mathfrak{F} = \mathfrak{N}_p$ and the statement of lemma is fulfilled.

Let $|\pi(\mathfrak{X})| > 1$ and p, q be some distinct primes in $\pi(\mathfrak{X})$. Assume that $q \in \pi(f(p))$. Show that $p \in \pi(f(q))$. Let $p \notin \pi(f(q))$. By [6, B. 10.7], there exists an irreducible and faithful C_p-module V over $GF(q)$. Let $X = [V]C_p$ be the corresponding semidirect product. It is clear that $X \in \mathfrak{X} \subseteq \mathfrak{F}$ because $C_p \in f(q)$. Let W be an irreducible and faithful X-module over $GF(p)$. We consider the group $Z = [W]X$. We show that $Z \notin \mathfrak{F}$. Suppose that $Z \in \mathfrak{F}$. Since $\mathfrak{F}$ is an S_n-closed formation, $H = [W]V \in \mathfrak{F}$. It is clear that $H \in \mathfrak{N}^2 \cap \mathfrak{F}$. Since $\mathfrak{F}$ is a Fitting formation then by theorem 2.1 in [6, p. 784], $\mathfrak{N}^2 \cap \mathfrak{F}$ is an S-closed Fitting formation. Therefore $S(H) \subseteq \mathfrak{F}$. This means that $H \in \mathfrak{X} = \mathfrak{F}^S$. Since $C_Z(W) = C_H(W) = W$, then $W \subseteq F_p(H) \neq H$ and by lemma 4.5 in [5], we have $H/F_p(H) \in f(q)$. Hence $p \in \pi(f(q))$, a contradiction. Hence $Z \notin \mathfrak{F}$.

Clearly that $Z_{\mathfrak{F}} \subseteq H$. Furthermore, we note that $Z = XY$ is the product of subgroups $X = VC_p$ and $Y = WC_p$. Since $X \in \mathfrak{F}$ and $Y \in \mathfrak{F}$ it follows that $X_{\mathfrak{F}} = X$ and $Y_{\mathfrak{F}} = Y$. Then $C_p \subseteq X_{\mathfrak{F}} \cap Y_{\mathfrak{F}}$ and $C_p \not\subseteq Z_{\mathfrak{F}}$. Therefore $X_{\mathfrak{F}} \cap Y_{\mathfrak{F}} \not\subseteq Z_{\mathfrak{F}}$, a contradiction. Thus, if $q \in \pi(f(p))$ then $p \in \pi(f(q))$.

Moreover we show that if $q \in \pi(f(p))$ then $\pi(f(p)) = \pi(f(q))$. Let $r \neq q$ and $r \in \pi(f(p)) \setminus \pi(f(q))$. By [6, B. 10.7], there exists an irreducible and faithful C_p-module U over $GF(r)$. Let $R = [U]C_p$ be the corresponding semidirect product. Since $r \in \pi(f(p))$ then $p \in \pi(f(r))$. Clearly, $r \in \pi(\mathfrak{F})$. This implies that $R \in \mathfrak{X} \subseteq \mathfrak{F}$. Let L be an irreducible and faithful R-module over $GL(q)$. We consider $G = [L]R$. Repeating the above argument we obtain that $G \notin \mathfrak{F}$ and $G_{\mathfrak{F}} \subseteq LU$. Furthermore $G = MN$ is the product of subgroups $M = LC_p$ and $N = UC_p$. It is clear that $M \in \mathfrak{F}$ and $N \in \mathfrak{F}$. Then $C_p \subseteq M \cap N = M_{\mathfrak{F}} \cap N_{\mathfrak{F}}$, but $C_p \nsubseteq G_{\mathfrak{F}}$, a contradiction. Hence, $\pi(f(p)) \subseteq \pi(f(q))$. Thus, we proved the statement: if $q \in \pi(f(p))$ then $\pi(f(p)) = \pi(f(q))$. This implies that there is a partition $\{\pi_i : i \in I\}$ of $\pi(\mathfrak{F})$, such that $p, q \in \pi_i$ if and only if $\pi(f(p)) = \pi(f(q))$. Since $f(p) = \mathfrak{S}_{\pi(f(p))}$ for every $p \in \pi(\mathfrak{F})$ then, by lemma 2, it follows that $\mathfrak{X} = D_0(\cup_{i\in I}\mathfrak{S}_{\pi_i})$. Lemma is proved.

Lemma 8. *Let $\mathfrak{F}$ be an S_n-closed formation. Assume that $\pi(\mathfrak{F}) = \pi(\mathfrak{F}^S)$ and $\mathfrak{F}^S = D_0(\cup_{i\in I}\mathfrak{S}_{\pi_i})$ where $\cup_{i\in I}\pi_i = \pi(\mathfrak{F}^S)$ and $\pi_l \cap \pi_k = \varnothing$ for all $l \neq k$ in I. Then $\mathfrak{F} = \mathfrak{F}^S$.*

Proof. Obviously $\mathfrak{F}^S \subseteq \mathfrak{F}$. Let G be a group of minimal order in $\mathfrak{F} \setminus \mathfrak{F}^S$. Since $\mathfrak{F}$ and $\mathfrak{F}^S$ are formations, G has a unique minimal normal subgroup $N = G^{\mathfrak{F}^S}$. By lemma 2, $\mathfrak{F}^S$ is a local formation, therefore $\Phi(G) = 1$ and G is a primitive group. In this case $N = C_G(N) = Soc(G)$ and N is a p-group for some prime p. Let M be a maximal subgroup of G with $Core_G(M) = 1$. Then $G = NM$ and $N \cap M = 1$. Since $G/N \simeq M \in \mathfrak{F}^S$ then $M = M_1 \times \ldots \times M_t$ where $M_i \in \mathfrak{S}_{\pi_i}$, $i = 1, \ldots, t$.

Assume that $t > 1$. Since $G \in \mathfrak{F}$ and $\pi(\mathfrak{F}) = \pi(\mathfrak{F}^S)$ then $p \in \pi(\mathfrak{F}^S)$. So there exists $i_0 \in I$ such that $p \notin \pi_{i_0}$ and $M_{i_0} \neq 1$. Notice that $NM_{i_0} \lhd G$. Since $\mathfrak{F}$ is S_n-closed formation and $G \in \mathfrak{F}$ then $NM_{i_0} \in \mathfrak{F}$. This implies that $NM_{i_0} \in \mathfrak{F}^S$ because $|NM_{i_0}| < |G|$. Therefore by definition of $\mathfrak{F}^S$, we have $NM_{i_0} = N \times M_{i_0}$. This implies that $M_{i_0} \subseteq C_G(N) = N$, which is a contradiction.

Assume that $t = 1$. In this case $p \in \pi_i$ and $\pi(M) \subseteq \pi_j$ for some $i \neq j \in I$. Let $R \neq 1$ be a normal subgroup in M. Clearly $NR \lhd G$. We may apply the apgument used earlier in the proof and conclude that $R \subseteq C_G(N) = N$. This contradicts $\pi(R) \cap \pi(N) = \varnothing$. So we may conclude that M has no non-trivial normal subgroups. It follows that M is a group of prime order q, where $q \neq p$. From this fact and $G \in \mathfrak{F}$ we conclude that $S(G) \subseteq \mathfrak{F}$. Since $\pi(\mathfrak{F}) = \pi(\mathfrak{F}^S)$ then $G \in \mathfrak{F}^S$. This final contradiction shows that no such counterexample G exists. Hence $\mathfrak{F} = \mathfrak{F}^S$. Lemma is proved.

Lemma 9. *Let $\mathfrak{F}$ be an S-closed formation. If $G \in \mathcal{M}(\mathfrak{F})$, then $G^{\mathfrak{F}}$ is a primary group.*

Proof. Let $G \in \mathcal{M}(\mathfrak{F})$. Suppose that a group G is nilpotent. Since $\mathfrak{F} = D_0\mathfrak{F}$ and $G \in \mathcal{M}(\mathfrak{F})$ then G is primary.

Let G be a non-nilpotent group. In this case $F(G) \neq G$ and $\Phi(G)$ is a proper subgroup of $F(G)$. Then there is a Sylow p-subgroup P of $F(G)$ for some prime p such that $P \nsubseteq \Phi(G)$. Then $G = PM$, where M is some subgroup of G. Since $G \in \mathcal{M}(\mathfrak{F})$ then $M \in \mathfrak{F}$ and $G/P \in \mathfrak{F}$. Hence $G^{\mathfrak{F}}$ is a primary group. Lemma is proved.

Lemma 10. Let $\{\mathfrak{F}_i : i \in I\}$ be a set of Fitting formations such that $\pi(\mathfrak{F}_i) \cap \pi(\mathfrak{F}_j) = \varnothing$ for all $i \neq j$ in I. If every formation $\mathfrak{F}_i$ satisfies condition 1) of the theorem then $\mathfrak{F} = D_0(\cup_{i\in I}\mathfrak{F}_i)$ also satisfies this condition.

Proof. Let $\mathfrak{F} = D_0(\cup_{i\in I}\mathfrak{F}_i)$. By lemma 1, $\mathfrak{F}$ is a Fitting formation. Let group $G = AB$ be a product of two subgroups A and B. Note that $\pi(G) \cap (\cup_{i\in I}\pi_i) = \cup_{i\in I}(\pi(G) \cap \pi_i)$. Since $\pi(G)$ is a finite set then $\pi(G) = \cup_{k=1}^{t}(\pi(G) \cap \pi_{i_k})$, where $\{\pi(G) \cap \pi_{i_k} : k = 1, \ldots, t\}$ is a partition of $\pi(G)$. Now $G_{\mathfrak{F}} = G_{\mathfrak{F}_{i_1}} \times \ldots \times G_{\mathfrak{F}_{i_k}}$, $A_{\mathfrak{F}} = A_{\mathfrak{F}_{i_1}} \times \ldots \times A_{\mathfrak{F}_{i_k}}$, $B_{\mathfrak{F}} = B_{\mathfrak{F}_{i_1}} \times \ldots \times B_{\mathfrak{F}_{i_k}}$, because $\mathfrak{F} = D_0(\cup_{i\in I}\mathfrak{F}_i)$ and $\pi(\mathfrak{F}_l) \cap \pi(\mathfrak{F}_k) = \varnothing$ for all $l \neq k$ in I. It is clear that $A_{\mathfrak{F}} \cap B_{\mathfrak{F}} =$

$(A_{\mathfrak{F}_{i_1}} \cap B_{\mathfrak{F}_{i_1}}) \times \ldots \times (A_{\mathfrak{F}_{i_k}} \cap B_{\mathfrak{F}_{i_k}})$. Since $A_{\mathfrak{F}_{i_j}} \cap B_{\mathfrak{F}_{i_j}} \subseteq G_{\mathfrak{F}_{i_j}}$, $j = 1, \ldots, k$, it follows that $A_{\mathfrak{F}} \cap B_{\mathfrak{F}} \subseteq G_{\mathfrak{F}}$. Lemma is proved.

Proof of Theorem. First we show that 1) implies 2). Let $G \in \mathcal{M}(\mathfrak{F}^S)$. By lemma 3, $G \in \mathcal{M}(\mathfrak{F})$ and $G \notin \mathfrak{F}$.

Assume that G is a nilpotent group. It is obvious that G is a p-group for some prime p. Let P be a subgroup of prime order p of G. Assume that $P \neq G$. Then $P \in \mathfrak{F}$ because $G \in \mathcal{M}(\mathfrak{F})$. Since $\mathfrak{F}$ is a Fitting class, by lemma 1.8 in [6, p. 565], $\mathfrak{N}_p \subseteq S_n N_o(P) \subseteq \mathfrak{F}$. It implies that $G \in \mathfrak{F}$. This contradicts our assumption and shows that G is a group of prime order p for some prime $p \notin \pi(\mathfrak{F})$.

Assume that G is a non-nilpotent group. Since $\mathfrak{F} = N_0\mathfrak{F}$ and $G \in \mathcal{M}(\mathfrak{F})$ then G has a unique maximal normal subgroup M. It is clear that $M = G_{\mathfrak{F}}$. Furthermore $|G : M| = p$ for some prime p. Since G is non-nilpotent, there exists a maximal subgroup R such that $G = F(G)R$. Since $F(G) \subseteq M$ and $|G : M| = p$ then $p \in \pi(R)$. Let R_p be some Sylow p-subgroup of R. It is not difficult to see that $R_p \not\subseteq M$. Write $H = F(G)R_p$.

Assume that $H \neq G$. It is clear that $G = HR$. Since $G \in \mathcal{M}(\mathfrak{F})$ then $H_{\mathfrak{F}} = H$ and $R_{\mathfrak{F}} = R$. Then by 1) of our theorem it follows that $H_{\mathfrak{F}} \cap R_{\mathfrak{F}} \subseteq G_{\mathfrak{F}}$. But this contradicts $R_p \subseteq H_{\mathfrak{F}} \cap R_{\mathfrak{F}}$ and $R_p \not\subseteq H = G_{\mathfrak{F}}$.

Assume that $G = F(G)R_p$. Then by lemma 9, $F(G)$ is a q-group for some prime q. Since G is a non-nilpotent group then $q \neq p$ and there exists a subgroup T in G such that T is a Schmidt group.

Assume that $T \neq G$. Then $T \in \mathfrak{F} \cap \mathfrak{N}^2$. Applying lemma 2.3 and proposition 2.4 from [6, p. 785], we see that $\mathfrak{N}_q\mathfrak{N}_p \subseteq \mathfrak{F} \cap \mathfrak{N}^2 \subseteq \mathfrak{F}$. It implies that $G \in \mathfrak{F}$, a contradiction. Therefore $T = G$ and $\mathfrak{F}^S$ is a formation with the Shemetkov property. Applying lemma 7 and 8, we obtain statement 2) of the theorem.

The implication: 2) $\Longrightarrow$ 1) follows from lemma 10 and Johnson result [2].

Резюме. В классе конечных разрешимых групп доказано, что если $\mathfrak{F}$ — формация Фиттинга, то следующие условия эквивалентны: 1) если $G = AB$, то $A_{\mathfrak{F}} \cap B_{\mathfrak{F}} \subseteq G_{\mathfrak{F}}$; 2) $\mathfrak{F} = D_0(\cup_{i \in I} \mathfrak{S}_{\pi_i})$, где $\cup_{i \in I} \pi_i = \pi(\mathfrak{F})$ и $\pi_i \cap \pi_j = \varnothing$ для всех $i \neq j$ из I.

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