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Complex Eigenmodes in Planar Waveguides with Pseudochiral Omega Media

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Abstract

In this paper we search for the complex eigenmodes of a dielectric asymmetric pseudochiral waveguide. The propagating hybrid modes are classified in homogeneous surface modes, inhomogeneous surface modes and semileaky modes, the later corresponding to complex solutions of the modal equation. Power leakage will occur, whenever one of the two characteristic waves ceases to be internally reflected at the film–substrate interface. Homogeneous guided modes turn into semileaky modes when the pseudochiral parameter exceeds a certain transition value.

1. Introduction

The pseudochiral or omega medium is an artificial complex medium [1], which is obtained by doping a host isotropic medium with Ω–shaped conducting microstructures. Any external electric or magnetic field induces both electric and magnetic polarizations, which arise perpendicular to each other. The electromagnetic properties of these media suggest promising applications in the design of reciprocal devices and components for the microwave and millimeter wave regimes [2]–[4].

In this paper, we investigate the complex solutions of the modal equation of an asymmetric dielectric pseudochiral slab. The complex spectrum of an asymmetric pseudochiral slab waveguide, in which both the film and the substrate are pseudochiral media but with an isotropic superstrate, is explored. The complex eigenmodes radiate energy into the substrate, provided that the constitutive parameters are properly chosen. They cannot be found in common isotropic waveguides, where only leaky unguided modes may exist. Power leakage into the substrate occurs because one of the two constituent characteristic waves ceases to be totally internal reflected at the film–substrate interface. These complex waves are usually termed semileaky waves [5] or leaky guided modes [6], since one of the two characteristic waves still remains completely guided by the film layer.

For the spatial orientation considered in this paper only hybrid modes propagate in the waveguide. Each hybrid mode is a weighted combination of the two characteristic waves. Leakage will occur under the form of an exiting wave if one of the two composite characteristic waves ceases to be totally reflected. In this case, the longitudinal wavenumber becomes complex, as two real roots of the modal equation collide over the real axis and originate a pair of conjugate complex roots. Therefore, the complete discrete spectrum of this waveguide includes a set of complex solutions in addition to the real guided modes.

2. Problem Formulation

The asymmetric pseudochiral waveguide depicted in Fig. 1 will be analyzed. It is assumed uniform in the y–direction and made of spatially nondispersive lossless Ω–media. For a bianisotropic Ω–media the constitutive relations may be written as

$$\begin{array}{rcl}\mathcal{D} &=& \overline{\epsilon}\cdot\boldsymbol{E}+\overline{\xi}\cdot\mathcal{H}\\ \mathcal{B} &=& \overline{\zeta}\cdot\boldsymbol{E}+\overline{\mu}\cdot\mathcal{H}\end{array}, \tag{1}$$

where $\mathcal{H}=Z_0\boldsymbol{H}$, $\mathcal{D}=\boldsymbol{D}/\epsilon_0$ and $\mathcal{B}=Z_0\boldsymbol{B}/\mu_0$ are normalized field vectors. In this case, $\overline{\epsilon}$ and $\overline{\mu}$ are dimensionless relative dielectric permittivity and relative magnetic permeability tensors, and $\overline{\xi}$ and $\overline{\zeta}$ are dimensionless magnetoelectric coupling tensors. As the medium is assumed to be spatially nondispersive, these relations are local.

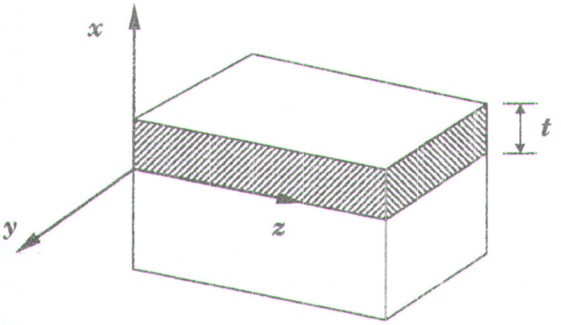

Fig. 1: Asymmetric pseudochiral slab waveguide: Both the film and the substrate are made of Ω–media and the upper medium is the air.

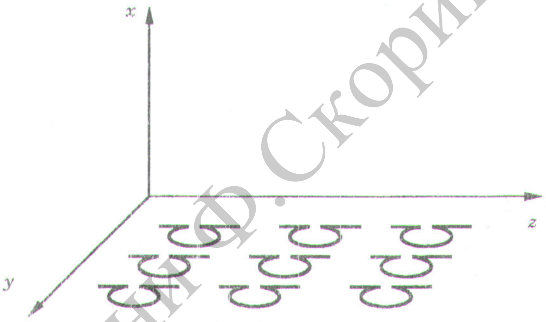

Fig. 2: Spatial orientation of the planar Ω–shaped perfectly conducting microstructures in the hosting isotropic medium.

We consider time-harmonic field variation of the form $\exp(j\omega t)$, and forward plane wave propagation of the form $\exp(-jkz)$, where k is the longitudinal wavenumber given by $k=(\beta-j\alpha)k_0$ with β beeing the normalized phase constant and α the normalized atenuation constant.

The Ω–shaped microstructures dopped in the isotropic host material are oriented as depicted in Fig. 2. The normal to the plane of the loops points in the x–direction while the stamps are aligned along z and the loops are oriented in the y–direction. Therefore tensors $\overline{\epsilon}$, $\overline{\mu}$, $\overline{\xi}$ and $\overline{\zeta}$ have the following dyadic representation [7]:

$$\begin{array}{rcl}\overline{\epsilon} &=& \epsilon_{xx}\hat{\boldsymbol{x}}\hat{\boldsymbol{x}}+\epsilon_{yy}\hat{\boldsymbol{y}}\hat{\boldsymbol{y}}+\epsilon_{zz}\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}\\ \overline{\mu} &=& \mu_{xx}\hat{\boldsymbol{x}}\hat{\boldsymbol{x}}+\mu_{yy}\hat{\boldsymbol{y}}\hat{\boldsymbol{y}}+\mu_{zz}\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}\\ \overline{\xi} &=& j\Omega\hat{\boldsymbol{z}}\hat{\boldsymbol{x}}\\ \overline{\zeta} &=& -j\Omega\hat{\boldsymbol{x}}\hat{\boldsymbol{z}}\end{array}, \tag{2}$$

where Ω is the dimensionless pseudochiral parameter here assumed to be the same in the two layers and positive. This parameter would be negative if the loops were oriented in the negative y–direction. Moreover, all the constitutive parameters are real since the media are assumed lossless.

With the spatial orientation depicted in Fig. 2, all the propagating modes in the waveguide are hybrid, since the two characteristic waves propagate coupled. When $\Omega=0$ the hybrid modes degenerate into the common TE and TM surface modes of the biaxial anisotropic case. The guided modes of a grounded pseudochiral slab in this configuration have been already addressed in the literature. Expressions for the transverse wavenumbers of the two characteristic waves, h_a and h_b in the film and q_a and q_b in the substrate, can be found in [7], and the derivation of the modal equation of the waveguide depicted in Fig. 1 can be found in [8].

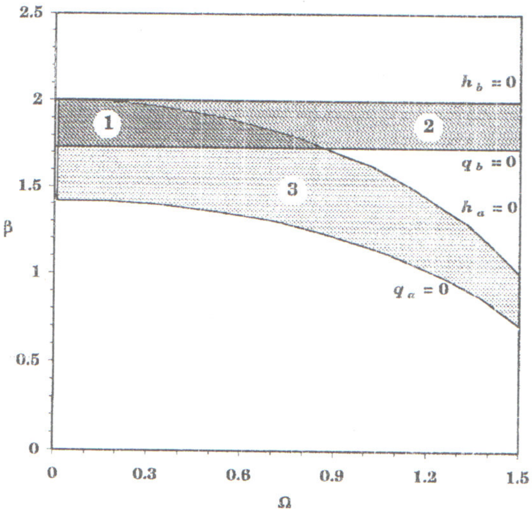

Fig. 3: The three ranges for the normalized phase constant β as a function of the pseudochiral parameter Ω, for the hybrid modes of the pseudochiral planar waveguide depicted in Fig. 1: (1) Homogeneous surface modes; (2) Inhomogeneous surface modes; (3) Semileaky modes.

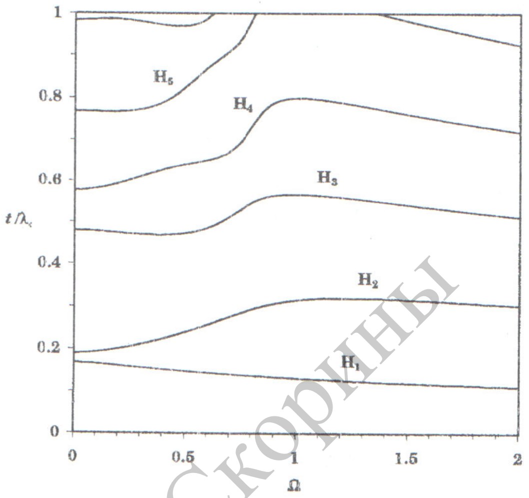

Fig. 4: Variation of the normalized phase constant β with Ω for the first propagating hybrid modes of an asymmetric slab waveguide with $t/\lambda = 0.5$. The thick line represents the semileaky mode while the curved dashed line corresponds to $h_a = 0$ and the horizontal to $q_b = 0$.

3. Numerical Results

For the sake of numerical simulation the following values for the dimensionless constitutive parameters were considered: $\epsilon^f_{xx} = \epsilon^s_{yy} = 2$, $\epsilon^f_{yy} = 4$, $\epsilon^f_{zz} = \epsilon^s_{zz} = 3$, $\epsilon^s_{xx} = 1$, $\mu^f_{xx} = \mu^s_{xx} = 1$, $\mu^f_{yy} = \mu^s_{zz} = 2$ and $\mu^f_{zz} = \mu^s_{yy} = 3$, where superscript s stands for *substrate* and f for *film*.

In Fig. 3 we show the three ranges for the normalized phase constant, as a function of the pseudochiral parameter Ω. Considering separately the contribution of the two characteristic waves propagating in the film, one has $\beta_b > \beta > \gamma_b$, where $\beta_b = \sqrt{\epsilon^f_{xx}\mu^f_{yy}}$ is such that $h_b = 0$ for $\beta = \beta_b$, while $\gamma_b = \sqrt{\epsilon^s_{xx}\mu^s_{yy}}$ is such that $q_b = 0$ for $\beta = \gamma_b$. For the other characteristic wave, one has $\beta_a > \beta > \gamma_a$, where $\beta_a = \sqrt{\epsilon^f_{yy}(\mu^f_{xx} - \Omega^2/\epsilon^f_{zz})}$ is such that $h_a = 0$ for $\beta = \beta_a$, while $\gamma_a = \sqrt{\epsilon^s_{yy}(\mu^s_{xx} - \Omega^2/\epsilon^s_{zz})}$ is such that $q_a = 0$ for $\beta = \gamma_a$.

From Fig. 3, one can easily see that both waves propagate with total reflection at $x = 0$ when $\beta > \gamma_b$. Within this range, region (1) corresponds to homogeneous surface modes, while region (2) corresponds to inhomogeneous surface modes.

If $\beta < \gamma_b$, *i.e.* in region (3), semileaky modes will occur since one of the two characteristic waves is not totally internally reflected from the film–substrate interface. These modes correspond to a coupled mode in which one of the characteristic waves is guided in the film while the other is radiating energy into the substrate, thereby forcing the longitudinal propagation constant to become complex.

In Fig. 4 the variation of the normalized phase constant β with the pseudochiral parameter Ω is depicted for the first propagating hybrid modes of an asymmetric slab waveguide with $t/\lambda = 0.5$. The descriptor H_p represents each hybrid mode, with the subscript p ($p \geq 1$) indicating the order of the mode when the modes are ordered after increasing cutoff frequencies. The curved dashed line corresponds to the asymptotic value $h_a = 0$ while the horizontal one corresponds to $q_b = 0$. The semileaky mode (thick line) starts propagating for Ω above some transition value Ω_t, after the collision of two leaky unguided modes.

Finally, in Fig. 5 we show the *root locus* in the complex plane of the normalized longitudinal wavenumber for the same semileaky mode of Fig. 4. The leakage starts occuring for Ω above the transition value Ω_t, roughly corresponding to $\beta = \gamma_b$.

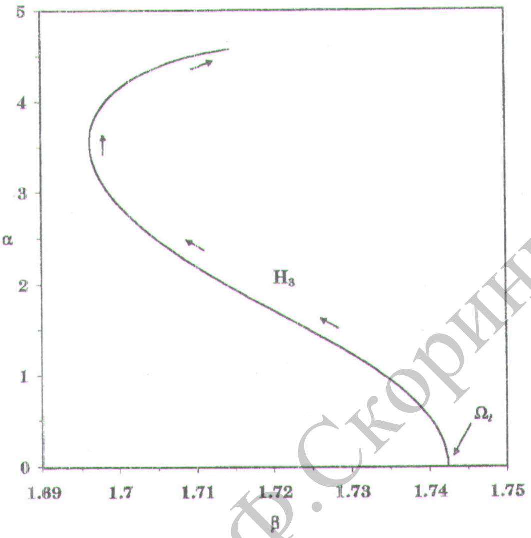

Fig. 5: Root locus in the complex plane of the normalized longitudinal wavenumber for the third hybrid mode, which becomes complex when Ω exceeds the transition value $\Omega_t = 0.71475$. Arrows indicate Ω increasing between Ω_t and 1.2.

4. Conclusions

The complex spectrum of an asymmetric pseudochiral slab waveguide was explored. It was shown that an asymmetric pseudochiral slab waveguide, where both the film and the substrate are made with pseudochiral Ω–media, can support semileaky modes radiating energy into the substrate, if the pseudochiral parameter Ω is greater than a certain transition value.

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