Affect of external electric field on optical properties of small metal particles

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In this communication we report analytical and numerical results of a study of interaction of electromagnetic waves with small ellipsoidal metal particles. We calculated in the electrostatic approximation the enhancement factor of the local field inside the particle and specified a domain of optical induced bistability (IOB) as a function of an amplitude and frequency of the electromagnetic wave. The results of numerical calculation for the small silver particles are presented graphically.

Let the electromagnetic wave falls on an metal particle in the form of rotational ellipsoid embedded in a dielectric host matrix The dielectric function (DF) of the particle depends on a frequency  $\omega$  and the local electric field E (inside the particle) and may be presented in the form [1]

$$\varepsilon(\omega, \vec{E}) = \varepsilon(\omega) + \chi(\omega) \, |\vec{E}|^2, \tag{1}$$

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where  $\chi(\omega)$  is the complex Kerr coefficient,  $\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega)$  is the linear part of DF and taken in the Drude form [2]

$$\varepsilon'(\omega) = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega^2 + \nu^2}, \quad \varepsilon''(\omega) = \frac{\nu}{\omega} \cdot \frac{\omega_p^2}{\omega^2 + \nu^2}.$$
(2)

Here  $\omega_p$  is the plasma frequency of electrons in the metal,  $\nu$  is their collision frequency,  $\varepsilon_{\infty}$  is some constant that may depend on the frequency  $u_{\mu}$  depending on a concrete metal.

In the electrostatic approximation (when the wave length of electromagnetic radiation is much larger than the typical size of the particle), the local field  $\vec{E}$  is uniform and in this case is parallel to  $\vec{E}_h$  at arbitrary dependence of  $\varepsilon_s(\omega, \vec{E})$ . It may be expressed in the form

$$\vec{E} = F \cdot \vec{E}_h, \quad F = \frac{\varepsilon_h}{\varepsilon_h \left(1 - L\right) + L \varepsilon_s \left(\omega, \vec{E}_s\right)},$$
(3)

where F is an enhancement factor, L is a depolarization factor along the field direction axis,  $\varepsilon_h$  is the dielectric function of the matrix [2]. For example, L = 1/3 corresponds to the spherical particle.

Relation (8) jointly with definitions (1),(1) allows one to find an equation for the local field E as a function of the applied field  $E_h$ , that specifies the IOB domain.

Here we consider the case of such small fields when the nonlinear term of the particle DF (1) may be ignored ( $\chi \rightarrow 0$ ). In this case (8) transforms to the expression

$$F_0 = \frac{\varepsilon_n}{L} \frac{1}{\left(\frac{1}{z_s^2} - \frac{1}{z^2 + \gamma^2}\right) + i\left(\frac{1}{\overline{z}_s^2} + \frac{\gamma}{z(z^2 + \gamma^2)}\right)}.$$
(4)

Here we introduced dimensionless frequencies:

$$z = \frac{\omega}{\omega_p}, \ \gamma = \frac{\nu}{\omega_p}, \ z_s = \frac{\omega_s}{\omega_p}, \ \omega_s = \omega_p \sqrt{\frac{L}{\varepsilon'_h (1 - L) + \varepsilon'_\infty L}}, \ \bar{z}_s = \sqrt{\frac{L}{\varepsilon''_h (1 - L) + \varepsilon''_\infty L}},$$

 $\omega_s$  is the resonant plasma frequency of a metal ellipsoid corresponding to direction of the electric field along the big semi-axis.

A magnitude of  $|F_0|^2$  considerably increases when the dimensionless frequency z approaches  $z_0 = (z_s^2 - \gamma^2)^{1/2} \approx z_s$  or the frequency of electromagnetic wave approaches to the surface plasmon frequency. It is intresting to note that the function  $|F_{0s}|^2$  has an extremum (maximum) in L in an interval (0,1). In a general case the extremum point may be found using the numerical cal



Figure 1:  $|F_0|^2$  versus depolarization factor L for the spherical silver particle  $\varepsilon'_{s\infty} = 4.5$ ,  $\varepsilon''_{s\infty} = 0.16$ ,  $\omega_p = 1.46 \cdot 10^{16} s^{-1}$ ,  $\nu = 1.68 \cdot 10^{14} s^{-1}$ ,  $\varepsilon_h = 2.25$ .

First curve for silver particles  $\varepsilon_{\infty} = 4.5 + i0.16$  at  $\varepsilon_h = 2.25$  has a maximum at  $L_0 = 0.189$ . If the imaginary part of the silver DF is set to be zero then the maximum moves to  $L_0 = 0.5$  (second curve). These curves are given in Fig.1.

Now we take into account the nonlinear part in the DF (1). In this case the local field must be found from a cubic equationin [3]. Introducing the following denotations  $|\chi| \left| \vec{E}_s \right|^2 \equiv X$ ;  $Y = \left| \frac{\varepsilon_h}{L} \right|^2 |\chi| \left| \vec{E}_h \right|^2$ , we obtain the cubic equation for X

$$X^3 + aX^2 + bX = Y, (5)$$

where

$$a = 2\left(\frac{\varepsilon'\chi' + \varepsilon''\chi''}{|\chi|}\right), \quad b = |\varepsilon|^2.$$

This equation determines a dependence of the "local field" X on the "applied field" Y, frequency  $\omega$ , and other parameters of the system that are hidden in parameters a and b.

In the case of the non-absorbing host medium ( $\varepsilon_h'' = 0$ ) and the metal inclusion with focusing non-absorbing nonlinear part of DF ( $\chi' > 0$  and  $\chi'' = 0$ ), equation (10) reduces to

$$X^{3} + 2\varepsilon' X^{2} + |\varepsilon|^{2} X = Y, \qquad (6)$$

where

$$\varepsilon'(z) = \frac{1}{z_s^2} - \frac{1}{z^2 + \gamma^2}, \quad \varepsilon''(z) = \varepsilon''_{\infty}(z) + \frac{\gamma}{z(z^2 + \gamma^2)}.$$

In this case the IOB emerges at conditions that was obtain in [4]. We numerically solved equation (11) for concrete particles at different polarization factors and using the experimental values of parameters of silver particles and the glass matrix. We obtain a range of amplitudes of the electric fields of wave and a range of frequencies when the IOB is possible. Some of these results are presented in Fig. 2-3.

possible. Some of these results are presented in Fig. 2-3. Figures 2a-3a depict dependencies of  $\chi \left| \vec{E_s} \right|^2$  on  $\chi \left| \vec{E_h} \right|^2$  at different depolarization coefficients L=1/3 (2a), L=1/5 (3a) at the frequency of electromagnetic field  $\omega = 0.2\omega_p$ . Figures 2b-3b show the IOB domains in the plane ( $\chi |E_h|^2$ , z) (shadow area) that are restricted by curves  $f_i$ 

$$f_i = -(2/9) \left[ Dx_i + ab/2 \right] \left( L/\varepsilon_h \right)^2,$$
$$D = a^2 - 3b.$$

where

The limiting values of the incident electric field are shown in Fig. 2b-3b by dash line at  $\omega = 0.2\omega_p$ . The bistability domain in the plane  $(\chi |E_h|^2, z)$  has a banana shape with ending points at low and high frequencies. Figure 2b shows the banana tip (beginning of bistability at low frequencies, point  $z_2$ ). Figure 1c shows the bistability domain near a cut point at high frequencies, point  $z_3$ .

We would like to note that similar problem have been treated in for a spherical inclusion [1]. In this paper, only one root  $z_2$  have been used. Probably, it was connected with character of approximations accepted in this paper.



Figure 2: IOB in the spherical silver particle  $\varepsilon'_{s\infty} = 4.5$ ,  $\varepsilon''_{s\infty} = 0.16$ ,  $\omega_p = 1.46 \cdot 10^{16} s^{-1}$ ,  $\nu = 1.68 \cdot 10^{14} s^{-1}$ ,  $\varepsilon_h = 2.25$ , L=0.333 (sphere), dependence  $\chi |E_s|^2$  on  $\chi |E_h|^2$  at z=0.2 (a), IOB domain in the plane ( $\chi |E_s|^2, z$ ) near  $z_3 = 0.318$  (b), IOB domain in the same plane near  $z_2 = 0.02$  (c).



Figure 3: The same dependencies as on Fig. 2 at L=0.2 (oblong spheroid  $z_2=0.16$ ,  $z_3=0.26$ )

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**Abstract.** In this communication we present the results of study of the local field in a ellipsoidal metal particle with a nonlinear (in the electric field) part of the dielectric function and specified the domains of the induced optical bistability (IOB) depending on the amplitude of the applied electric field and frequency of the electromagnetic wave.

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