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To the theory of ultrashort spatiotemporal solitons

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1. Introduction

Spontaneous symmetry breaking that results in formation of light patterns in transversely homogeneous nonlinear optical systems has attracted substantial research interest because they hold a promise of ultrafast all-optical switching and controlling light by light. Analysis of soliton interactions seems to be important for realizing all-optical nonlinear switching because solitons are expected to interact (attract, repel, etc.) as effective particles. These investigations began with radiation self-focusing in the form of self-trapping, or spatial solitons when diffractive spread of a propagating beam is compensated by its focusing with a nonlinear medium. Spatial solitons for example in a Kerr medium were found to be unstable. Temporal solitons in nonlinear optical fibers were shown to have high application potential [11]. Recently, the situation has changed. An idea of spatial-temporal solitons (light bullets) was suggested for a homogeneous medium with self-focusing nonlinearity and anomalous dispersion [1]. Although (2+1)-dimensional solitons of a pure Kerr medium are unstable and cannot be employed for soliton switching, recent experimental discoveries of stable (2+1)-dimensional solitons in different nonlinear media initiated the experimental study of three-dimensional interactions between solitary beams [2-5]. It was shown that the collapse suppression as a result of resonant parametric interaction is possible even in media with purely cubic (Kerr) nonlinearity [6]. Recently, two-dimensional spatial-temporal soliton solutions to the (2+1)-dimensional modified nonlinear Schrödinger equation, were derived. These solutions in the context of nonlinear optics constitute new dark light bullets which can be observed in real experiments since they are stable against the wave collapse. (3+1)-dimension optical spatial-temporal solitons called light bullets can be formed in nonlinear media under the influence of the powerful ultrashort laser pulses. There is no analytic description for such solitons. Dynamics of their formation and their properties are determined by numerical solutions of suitable nonlinear equations. When numerically simulating, soliton pulses of constant in space and time profile are usually considered. This is a very particular case of light bullets. Constant solitons may be compared with constant light beams in non-homogeneous waveguides [8]. But the constant beam in a waveguide is a very particular case permitting observation only at specific conditions. The general case is the light beam of periodically alternating transverse dimension. In the same way, light soliton must exist in general case in the form of an oscillating in space and time pulse. But numerical simulating of such a periodical soliton is very difficult to carry out. On the other hand, the spatial-temporal profile of light bullets is not sharp shaped. Numerical simulations of the profile of constant solitons show that they have a bell-like shape reminding the basic Gaussian function. Starting from this fact, we have tried to use the Gaussian functions when analyzing light bullet properties.

2. Light Bullets in Kerr Nonlinear Medium

The field envelope of a spatial-temporal light pulse obeys the following nonlinear equation [1,8]

$$\frac{\partial E}{\partial z} + \frac{i}{2k_0} \Delta_\perp E + \frac{i}{2} k_2 \frac{\partial^2 E}{\partial \eta^2} + i \frac{\Delta k^2}{2k_0} E = 0 , \tag{1}$$

where $E = A_0 \Psi(x, y, z, t)$. We will use the notation: k_0 represents a propagation constant of undisturbed medium; $\Delta_\perp$ stands for the transverse Laplacian; $k_2 = d^2k/d\omega^2$ denotes a medium dispersion; $\eta = t - z/u$, where u is the group velocity; Δk^2 designates a nonlinear addition to the constant of propagation. In the case of Kerr nonlinearity

$$\frac{\Delta k^2}{2k_0} = k_0 \Delta n = k_0 \beta_0 \mid E \mid^2 = k_0 \beta_0 A_0^2 \mid \Psi \mid^2 = \alpha \mid \Psi \mid^2 .$$

At numerically simulating the light bullet properties we will assume that the Kerr local nonlinearity is just the most suitable one for description of the almost instantaneous interaction of a light bullet with the medium. Other nonlinearities have finite time of response and cannot determine properties of squeezed, in space and time, light bullets. In is known that in the Kerr nonlinear media only 1-dimensional solitons are stable. Nevertheless, soliton squeezing process takes finite time. Estimation of the collapse time shows that because the light bullet is an ultrashort pulse there is not enough time for a soliton collapse to occur.

Let us introduce new variables in eq. (1):

$$x' = x\sqrt{2k_2},\ y' = y\sqrt{2k_0},\ \eta' = \eta\sqrt{2/k_2} , \tag{2}$$

then it takes on the form

$$\frac{\partial \Psi}{\partial z} + i \frac{\partial^2 \Psi}{\partial x'^2} + i \frac{\partial^2 \Psi}{\partial y'^2} + i \frac{\partial^2 \Psi}{\partial \eta'^2} + i\alpha \mid \Psi \mid^2 \Psi = 0 . \tag{3}$$

It is obvious that eq. (3) is symmetrical with respect to the coordinates x', y', η'. Taking this into account, we can use the spherical coordinate system in which $\rho = \sqrt{x'^2 + y'^2 + \eta^2}$. Let us consider only a radial dependence of a soliton when

$$\Delta = \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \rho^2 \frac{\partial}{\partial \rho} ,$$

then eq. (3) will assume the form

$$\frac{\partial \Psi}{\partial z} + i\Delta \Psi + i\alpha \mid \Psi \mid^2 \Psi = 0 . \tag{4}$$

Let us try to find a solution in the form

$$\Psi = exp(i\gamma_1 - \gamma_2 - \frac{\rho^2}{f\rho_0^2} + i \frac{\rho^2}{g\rho_0^2}) , \tag{5}$$

where $\gamma_1\gamma_2, f, g$ are some unknown functions of z. Because basic nonlinear effects are induced in the neighborhood of the maximum of the pulse we can obtain an approximate form

$$\mid \Psi \mid^2 = 1 - 2\gamma_2 - 2\frac{\rho^2}{f\rho_0^2} . \tag{6}$$

Substituting (5) and (6) into eq. (4), we arrive at the following system

$$\gamma_1' - 6\,\frac{1}{f\,\rho_0^2} + \alpha - 2\alpha\,\gamma_2 = 0\,, \tag{7}$$

$$\gamma_2' + 6\,\frac{1}{g\,\rho_0^2} = 0\,, \tag{8}$$

$$\frac{f'}{f} + 8\,\frac{1}{g\,\rho_0^2} = 0\,, \tag{9}$$

$$\frac{g'}{g} - 4\,\frac{1}{f^2\,\rho_0^2} + 4\,\frac{1}{g^2\,\rho_0^2} + 2\,\alpha\,\frac{1}{f} = 0\,, \tag{10}$$

where the prime stands for the derivative in z. Eqs. (7) and (8) determine the amplitude and phase z-dependence, whereas eqs. (9) and (10) prescribe a spacial-temporal form of the light bullet. After eliminating the $g(z)$, eqs. (9) and (10) give

$$2f'' = \frac{f'^2}{f} + \frac{64}{\rho_0^4}\,(\frac{1}{f} - \delta)\,, \tag{11}$$

where $\delta = \frac{1}{2}\,\rho_0^2\,\alpha$. Substitution $f'^2 = U(f)$ leads eq. (11) to

$$\frac{dU}{df} = \frac{1}{f}\,U(f) + \frac{64}{\rho_0^4}\,(\frac{1}{f} - \delta)\,. \tag{12}$$

This equation is well known in mathematics [12] and a solution to it is

$$U(f) = \frac{64}{\rho_0^4}\,(\,c\,f - 1 - \delta\,f\,\ln\,f\,)\,. \tag{13}$$

Thus, for the function $f(z)$ we will have the equation

$$\frac{df}{dz} = \pm\frac{8}{\rho_0^2}\,(c\,f - 1 - \delta\,f\,\ln f\,)^{1/2}\,. \tag{14}$$

To define the constant of integration we require that $f(z=0) = f_0$, and $f'(z=0) = 0$. As a result we obtain

$$\frac{df}{dz} = \pm\,\frac{8}{\rho_0^2\sqrt{f_0}}\,(\,f - f_0 - \delta\,f_0\,f\,\ln\frac{f}{f_0}\,)^{1/2}\,. \tag{15}$$

Unfortunately, there is no solution to eq. (15) in terms of known (elementary or transcendental) functions. Nevertheless, it enables us to describe some basic spatial-temporal peculiarities of the soliton pulse. In particular, from (15) it follows that there exists an inflection point where the equality

$$f(z) = f_c = f_0\,exp\,(\,\frac{1 - \delta\,f_0}{\delta\,f_0}\,)\,. \tag{16}$$

Consequently, the function $f(z)$ varies within finite limits. The variation limits are determined by a function

$$F(f) = f - f_0 - \delta\,f_0\,f\,\ln\frac{f}{f_0}\,, \tag{17}$$

which according to (15) is to be more than zero or equal to zero.

Approximate graphics of $F(f)$ are given at Fig. 1a,b,c:

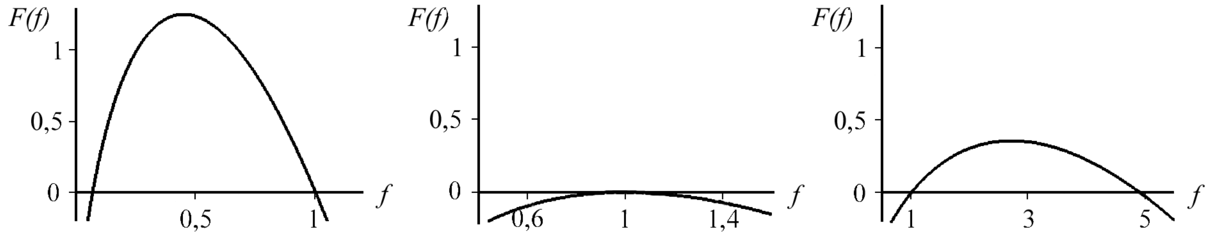

Figure 1: Function $F(f)$.

At Fig 1a $f_0 = 1, \delta = 5$; At Fig 1b $f_0 = 1, \delta = 1$; At Fig 1c $f_0 = 1, \delta = 0,5$.

Function $f(z)$ varies over the interval between the points at which $F(f) = 0$. The point of inflection $f(z) = f_c$ coincides with the point of the maximum of the function $F(f)$. It follows from the figures that if $f_c < f_0$ then $f(z)$ varies within the limits $f_{min.} \leq f \leq f_0$, providing that $\delta\ f_0 \geq 1$. This means that the spatial size of the light bullet ranges from from f_0 at $z = 0$ to $f_{min.}$ at $z > 0$, and then again increases to f_0 and so on. Under condition $f_c > f_0$ ($\delta\ f_0 < 1$) the spatial size ranges from f_0 at $z = 0$ to $f_{max.}$ and so on. If $f_c = f_0$ ($\delta\ f_0 = 1$) then $f(z) = const$. Just only in in this case, we have a constant soliton pulse – constant light bullet. From the figures and everything stated above it also follows that the minimal size of the soliton can tend to zero when increasing the amplitude of the pulse or the nonlinearity of the medium. This means that the soliton collapse is possible. But anyway, the constant pulse existing under condition $f_0 = f_c$ will be switched to oscillation mode with a varying parameter δ.

The total energy of the constant light bullet

$$W = \epsilon\, \pi^{3/2}\, k_0^{-3/2}\, \beta_0^{-3/2}\, A_0^{-1} \tag{18}$$

decreases with the amplitude A_0 increasing. It does not contradict with physics because its size

$$C_0 = \rho_0\, \sqrt{f_0} \ = \sqrt{\frac{1}{k_0 \beta_0}}\, A_0^{-1}\ . \tag{19}$$

decreases when A_0 increases. This reflects the fact that the narrower is such a soliton the less is its volume and the less energy is required for it to collapse.

For the oscillating light bullet the total energy is

$$W = \epsilon\ 2^{-3/2}\ (\pi f_0)^{3/2}\ \rho_0^3\ A_0^2\ , \tag{20}$$

where $W = \ const$ and

$$A(z) = A_0\ (\frac{f_0}{f})^{3/4}\ . \tag{21}$$

Consequently, for a soliton squeezing ($f < f_0$) amplitude obeys $A(z) > A_0$, and for a spreading ($f > f_0$) it obeys $A(z) < A_0$.

3. Conclusion

In this report we have analyzed relevant properties of light bullets in Kerr nonlinear medium. It is shown that the light bullets oscillate both in space and time . The type of nonlinearity and the collapse problem are discussed too. Our approach can be readily extended to analyzing the dynamics of light bullets in different nonlinear media.

Abstract. Gaussian functions are used to investigate properties of light bullets in Kerr nonlinear media. It is shown that the light bullets oscillate in space and time. The type of the nonlinearity and the collapse problem are discussed.

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