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Exotic bianisotropic media

O.V.Ivanov

Department of Quantum Electronics and
Optoelectronics, Ulyanovsk State University
ul. L.Tolstoy 42, 432700 Ulyanovsk, Russia
Tel: +7-8422-357-658; email: oliv@sv.uven.ru

1. Introduction

In spite of some restrictions and constrains, the range of generally bianisotropic media is very broad. Beside the well-known kinds of bianisotropic media in which there are two positive and two negative eigen waves for any direction of propagation of an electromagnetic wave, these restrictions do not forbid the existence of exotic bianisotropic media (EBM) with a different set of eigen waves.

There is no contradiction if we describe wave propagation in an infinite EBM. The first issue arise when the boundary problem is solved. In the classical solution of this problem, when electromagnetic wave is incident from usual medium, it is assumed that the amplitudes of two negative waves in the investigated medium are zero. In case of EMB, it may be more or less than two negative waves that results either in indefiniteness or absence of solution.

In the present report, we search for constitutive parameters which provide the existence of EBM. We consider anisotropic media which behave like an EBM, when the angle of incidence of light falls in a certain range. It is also investigated another problem which comes into notice if we consider an EBM slab. It emerges that EBM is neither transparent nor absorbing nor amplifying medium. The total reflected and transmitted energy periodically depends on the thickness of the slab.

2. Eigen wave of bianisotropic medium

Take a coordinate system so that the layer planes are normal to the z axis and the plane of incidence is the xz plane. The components of electromagnetic field $\mathbf{E}, \mathbf{D}, \mathbf{H}, \mathbf{B}$ are proportional to the factor $\exp[i(\omega t - k_x x)]$ and the Maxwell equations can be written as:

$$\nabla \times \mathbf{E} = -ik_0\mathbf{B}, \qquad \nabla \times \mathbf{H} = ik_0\mathbf{D}, \tag{1}$$

where $\nabla = (-ik_x, 0, \frac{\partial}{\partial z})$, k_0 represents the free space wavenumber. Substitute the constitutive relations for bianisotropic medium

$$\boldsymbol{D} = \epsilon \cdot \boldsymbol{E} + \alpha \cdot \boldsymbol{H}, \qquad \boldsymbol{B} = \mu \cdot \boldsymbol{H} + \beta \cdot \boldsymbol{E} \tag{2}$$

into equation (2). Elimination of normal to the boundaries components of the fields E_z, H_z leads to the following matrix differential equation for vector $\mathbf{g} = (E_x, -E_y, H_x, H_y)^T$ composed of the tangential field components:

$$\frac{d\mathbf{g}}{dz} = -ik_0 G(k_x, \epsilon, \mu, \alpha, \beta)\mathbf{g}. \tag{3}$$

Here 4×4 matrix G is determined by local properties of the medium and contains no differential operators. It is formed by the permittivity $\epsilon = \epsilon(z)$, the permeability $\mu = \mu(z)$, and the magneto-electric $\alpha = \alpha(z)$, $\beta = \beta(z)$ dyadics.

For a homogeneous medium G does not depend on coordinates, and the solution of equation (3) in the general case is a superposition of four direct and opposite eigen waves:

$$\mathbf{g} = \sum_{j=1}^{4} c_j \mathbf{g}_j \exp(-ik_{zj}z). \tag{4}$$

c_j are the amplitudes of eigen waves corresponding to the eigen vectors $\mathbf{g}_j$ of the matrix G. The eigen numbers $n_{zj} = k_{zj}/k_0$ are the solutions of the dispersion relation, which is a forth-order equation in n_z:

$$\det(G - n_z I) = 0, \tag{5}$$

where I is a unit matrix. Thus four eigen waves having different polarizations and refractive indices $n_j = \sqrt{n_x^2 + n_{zj}^2}$ can propagate in various directions.

3. Classification of media

There exist various kinds of classification of bianisotropic media. Some of them are based on the type eigen waves peculiar to a given medium (isotropic, gyrotropic, optically active, uniaxial, biaxial, etc.) [1]. Other classifications take into consideration eigen numbers of waves (transparent $\operatorname{Im} n_j = 0$, absorbing $\operatorname{Im} n_j < 0$, amplifying $\operatorname{Im} n_j > 0$). Although starting point is always a usual dielectric medium, which is widen by small additions of anisotropy, bianisotropy, absorption, or amplification. Therefore, it is always supposed that four eigen waves—two waves propagating in positive direction and other two waves propagating in opposite direction—exist in such medium as in a usual dielectric. However, actually it not always so. The signs of the four eigen waves can be various. It is possible to distinguish the following four cases:

1. There are two positive and two negative waves (usual medium)
2. All four waves are of the same direction
3. Three eigen waves are positive and one is negative
4. Two waves of the same polarization are positive and two waves of another polarization are negative

We name structures in which cases 2-4 are realized as exotic bianisotropic media.

Our further task is to clarify the conditions when such media exist and how to solve problems about propagation of waves in such structures.

It is complex enough to consider a generally bianisotropic medium; therefore, we consider some special cases in which basic features of EBM are manifested, but which it is much easier for analysis. It would be significant simplification to examine structures and schemes which do not required to consider various polarization effects. In such structure, p- and s-polarized waves are eigen, that allows to consider them independently from each other. Let us name such geometry pseudo-isotropic. Of course, the structures are not isotropic generally, since their constitutive dyadics can contain off-diagonal components and their characteristics depend on the direction of wave propagation.

Bianisotropic medium. Let us consider a medium with the following constitutive dyadics:

$$\begin{gathered} \varepsilon_{ij} = \varepsilon_{ii}\delta_{ij}, \mu_{ij} = \mu_{ii}\delta_{ij}; \\ \alpha_{ij} = 0, \beta_{ij} = 0 \quad \text{except for} \quad \alpha_{xy}, \alpha_{yx}, \beta_{xy}, \beta_{yx}. \end{gathered} \tag{6}$$

The matrix of constitutive parameters of such medium has the form:

$$T = \begin{pmatrix} \beta_{yx} & 0 & 0 & \mu_{yy} - n_x^2/\varepsilon_{zz} \\ 0 & -\beta_{xy} & \mu_{xx} & 0 \\ 0 & \varepsilon_{yy} - n_x^2/\mu_{zz} & -\alpha_{yx} & 0 \\ \varepsilon_{xx} & 0 & 0 & \alpha_{xy} \end{pmatrix}. \qquad (7)$$

This matrix is of block type; therefore, propagation of p- and s-polarized waves can be separated. The matrices describing separately the indicated waves represented by vectors (E_x, H_y) and $(H_x, -E_y)$, respectively, can be written as:

$$T_p = \begin{pmatrix} \beta_{yx} & \mu_{yy} - n_x^2/\varepsilon_{zz} \\ \varepsilon_{xx} & \alpha_{xy} \end{pmatrix}, \qquad T_s = \begin{pmatrix} -\alpha_{yx} & \varepsilon_{yy} - n_x^2/\mu_{zz} \\ \mu_{xx} & -\beta_{xy} \end{pmatrix}. \qquad (8)$$

Thus, the matrices of constitutive parameters has the following form:

$$T = \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \qquad (9)$$

Eigenvalues of this matrix are:

$$n_z = (a + d)/2 \pm \sqrt{bc + (a - d)^2/4}. \qquad (10)$$

The condition of exotic propagation is the presence of two eigenvalues of the same sign; that is true when $ad > bc$ (in case where all magnitudes are real quantities). After substitution of both polarizations into this inequality, we have:

$$\begin{array}{ll} \alpha_{xy}\beta_{yx} > \varepsilon_{xx}(\mu_{yy} - n_x^2/\varepsilon_{zz}) & \text{(p-polarization)}, \\ \alpha_{yx}\beta_{xy} > \mu_{xx}(\varepsilon_{yy} - n_x^2/\mu_{zz}) & \text{(s-polarization)}. \end{array} \qquad (11)$$

If the wave is normally incident on the structure ($n_x = 0$), then the magnitudes of magnetoelectric dyadics components should be rather large, of order of unit; therefore, it is not absolutely clear whether such mediums exist. In work [2] is shown that effective parameters of a stratified periodic medium in resonant case look like (7) and can exceed unit; therefore, probably, just such medium can be referred to the class of EBM.

In case of oblique incidence, inequality (11) can be true near the critical angle of total reflection, when the radicand in (10) is close to zero. Exceeding the critical angle may suppress the investigated effect.

Anisotropic medium. Not only bianisotropic structures can manifest the exotic property, but also simple anisotropic structures in some geometries and in special intervals of incidence angle. Let us consider a uniaxial anisotropic structure which axis is directed along the y. The dielectric and magnetic dyadics of such structure has the form:

$$\varepsilon = \begin{pmatrix} \varepsilon_{xx} & 0 & \varepsilon_{xz} \\ 0 & \varepsilon_{yy} & 0 \\ \varepsilon_{zx} & 0 & \varepsilon_{zz} \end{pmatrix}, \qquad \mu = \begin{pmatrix} \mu_{xx} & 0 & \mu_{xz} \\ 0 & \mu_{yy} & 0 \\ \mu_{zx} & 0 & \mu_{zz} \end{pmatrix}. \qquad (12)$$

P- and s-polarized waves are also independent for such structure. The corresponding matrices of constitutive parameters for these waves are:

$$T_p = \begin{pmatrix} -n_x\varepsilon_{zx}/\varepsilon_{zz} & \mu_{yy} - n_x^2/\varepsilon_{zz} \\ \varepsilon_{xx} & -n_x\varepsilon_{xz}/\varepsilon_{zz} \end{pmatrix}, \qquad T_s = \begin{pmatrix} -n_x\mu_{zx}/\mu_{zz} & \varepsilon_{yy} - n_x^2/\mu_{zz} \\ \mu_{xx} & -n_x\mu_{xz}/\mu_{zz} \end{pmatrix}. \qquad (13)$$

In case of transparent medium, the condition of existence of two waves of the same sign and absence of total reflection has the form:

$$n_x^2(\varepsilon''_{xz})^2/\varepsilon_{zz}^2 < \varepsilon_{xx}(\mu_{yy} - n_x^2/\varepsilon_{zz}) < n_x^2((\varepsilon'_{xz})^2 + (\varepsilon''_{xz})^2)/\varepsilon_{zz}^2, \qquad \text{(p-polarization).} \qquad (14)$$

For s-polarization the result can be obtained by replacing ε by μ. It follows from the last expression that, no matter how small is off-diagonal components of the dyadics, there is a range of angles near to the critical angle, where both eigen waves propagate in one direction. The less the off-diagonal component, the more narrow this range.

4. Discussion

What signify the fact that two waves of one polarization propagates in a medium in the same direction? It does not contradict known conceptions about propagation of radiation in media. However, there is a problem if we try to solve the problem about transmission and reflection of an electromagnetic wave at the boundary of an exotic medium. Usually the amplitude of backward wave in a medium, into which the wave is transmitted, is equaled to zero. In case of exotic medium, backward waves can be non-existent, then the problem has no solution, or there are two backward waves, then the solution is indeterminate. The indicated problem is somehow similar to the problem of selection of solution in an amplifying medium near the critical angle [3], where there are two waves—attenuating and amplifying in depth—and it is not correct to select only attenuating one, as it is made in a transparent medium. Several methods can be used to approach the solution of the indicated problem:

1. To consider a layer of substance, when the wave is transmitted into a usual medium.
2. To use the Green function formalism.
3. To use principles of radiation and additional physical criteria [4].
4. To consider reflection from a continuously inhomogeneous exotic medium which is gradually changes into a usual medium.

We considered a layer of exotic substance. The obtained result was inconsistent with assumption that the medium was transparent. The reflection and transmission coefficients periodically depended on the thickness of the slab and sometimes exceeded unit.

Abstract. Beside the well-known kinds of bianisotropic media in which two positive and two negative electromagnetic eigen waves can propagate in any given direction, theoretically may exist exotic bianisotropic media with a different set of eigen waves. Problems concerning such media arise if we consider reflection and transmission of electromagnetic wave at the boundary of usual and exotic media.

References

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