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ПРОХОЖДЕНИЕ ПЛОСКИХ ЭЛЕКТРОМАГНИТНЫХ ВОЛН ЧЕРЕЗ МНОГОСЛОЙНУЮ БИИЗОТРОПНУЮ СТРУКТУРУ

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TRANSMISSION OF PLANE ELECTROMAGNETIC WAVES TROUGH A MULTILAYER BIISOTROPIC STRUCTURE

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Задача о нормальном падении плоской электромагнитной волны на систему биизотропных слоёв в общем случае решена методом многократных отражений и матричным методом. Полученные формулы применены для численного расчёта периодических структур биизотропных слоёв. Обнаружены периодически повторяющиеся пики и провалы модуля коэффициента прохождения с изменением толщины биизотропных слоёв.

Ключевые слова: *материальные уравнения, биизотропная среда, слой, коэффициенты прохождения и отражения, периодическая структура.*

The problem of normal incidence of a plane electromagnetic wave on a system of biisotropic layers is solved in general case using the multiple reflection method as well as the matrix method. The resulting formulas are applied in order to numerically calculate periodic structures of biisotropic layers. Periodically repeating peaks and troughs for the transmission coefficient modulus with the changing thickness of biisotropic layers are found.

Keywords: *constitutive equations, biisotropic medium, layer, transmission and reflection coefficients, periodic structure.*

Introduction

The problem of electromagnetic wave propagation in biisotropic media [1-6] takes an important place in modern electrodynamics. The constitutive relations of such media

$$\vec{D} = \varepsilon\vec{E} + (\chi + i\alpha)\vec{H};$$
$$\vec{B} = (\chi - i\alpha)\vec{E} + \mu\vec{H}$$

lead to the fact that only circularly polarized waves with refractive indices $n_{\pm} = \sqrt{\varepsilon\mu - \chi^2} \pm \alpha$ can propagate in such media. It should be noted that in literature, since the mid-90s of the last century to the present, there is a discussion about "recognizable existence" of biisotropic media [7]–[13]. Consideration of the problem of reflection and refraction of electromagnetic waves at planar and spherical boundaries can help to find and investigate effects that allow to explicitly demonstrate how to recognize the biisotropic media [14]–[18]. The problem of reflection and transmission with two flat boundaries was considered in [19]. This paper is devoted to an analogous problem with many flat boundaries, namely to the problem of normal incidence of an electromagnetic circularly polarized wave on a structure of planar biisotropic layers.

1 The problem of normal incidence on a structure of biisotropic layers

Suppose that the area $z \le 0$ is filled with biisotropic medium 1 with parameters $\varepsilon_1, \mu_1, \alpha_1, \chi_1$, the area $\sum_{i=2}^{p-1} d_i < z \le \sum_{i=2}^{p} d_i$ where $3 \le p \le N-1$ (N is the whole number of layers, d_i is the thickness of the i-th layer) is filled with biisotropic medium i with parameters $\varepsilon_p, \mu_p, \alpha_p, \chi_p$, and the area $z > \sum_{i=2}^{N-1} d_i$ is filled with biisotropic medium with parameters $\varepsilon_N, \mu_N, \alpha_N, \chi_N$. Assuming the incident electromagnetic wave is circularly polarized we can write expressions for its electric and magnetic fields as

$$\vec{E}_{\nu}^{\uparrow 1} = \left(\vec{i} + i\nu\vec{j}\right) E_{\nu}^{\uparrow 1} \exp\left(ik_{\nu}^{1} z - i\omega t\right);$$
$$\vec{H}_{\nu}^{\uparrow 1} = -b_{\nu}^{1}\vec{E}_{\nu}^{\uparrow 1}, \qquad (1.1)$$

where $\vec{i}, \vec{j}$ are unit vectors directed along the axes Ox, Oy. Therefore, the wave propagating in the *p-th* medium (except $p = N$ where $E_{-\nu}^{\downarrow N} = 0$) is a superposition of two following waves (Figure 1.1):

$$\vec{E}_{\nu}^{\uparrow p} = \left(\vec{i} + i\nu\vec{j}\right) E_{\nu}^{\uparrow p} \exp\left(ik_{\nu}^{p}\left(z - \sum_{s=2}^{p-1} d_p\right) - i\omega t\right);$$
$$\vec{H}_{\nu}^{\uparrow p} = -b_{\nu}^{p}\vec{E}_{\nu}^{\uparrow p};$$
$$\vec{E}_{-\nu}^{\downarrow p} = \left(\vec{i} + i\nu\vec{j}\right) E_{-\nu}^{\downarrow p} \exp\left(-ik_{-\nu}^{p}\left(z - \sum_{s=2}^{p-1} d_p\right) - i\omega t\right);$$
$$\vec{H}_{-\nu}^{\downarrow p} = -b_{-\nu}^{p}\vec{E}_{-\nu}^{\downarrow p}; \qquad (1.2)$$
$$1 \le p \le N.$$

Proportionality coefficients between the electric and magnetic fields b_σ^p and wave numbers k_σ^p ($\sigma = \pm\nu$) have the following form

$$b_\sigma^p = \left(\chi_p + i\sigma\sqrt{\varepsilon_p\mu_p - \chi_p^{\,2}}\right)\frac{1}{\mu_p};$$

$$k_\sigma^p = \left(\sqrt{\varepsilon_p\mu_p - \chi_p^{\,2}} + \sigma\alpha_p\right)\frac{2\pi}{\lambda}. \qquad (1.3)$$

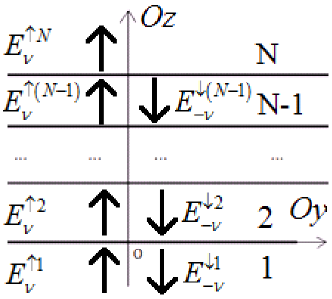

Figure 1.1 – Wave propagation through biisotropic layers

It is clear from (1.1)–(1.3) that in medium N there is no wave incident on the structure from $z \to +\infty$, so the amplitude coefficients $\vec{E}_\nu^{\uparrow N}$ and $\vec{E}_{-\nu}^{\downarrow 1}$ determine the transmission coefficients $T_\nu^{1N} = E_\nu^{\uparrow N} / E_\nu^{\uparrow 1}$ and reflection coefficients $R_\nu^{1N} = E_{-\nu}^{\downarrow 1} / E_\nu^{\uparrow 1}$ for the problem. In the same way we can define the transmission and reflection coefficients for the case when there is only p ($p < N$) layers: T_σ^{1p}, R_σ^{1p} for the incident wave polarization σ ($\sigma = \pm\nu$) and analogous coefficients for the wave incident in medium p from $z \to +\infty$: T_σ^{p1}, R_σ^{p1}.

2 The multiple reflection method

We first consider the problem at $N = 3$, that is, find the transmission and reflection coefficients for the wave propagation from biisotropic medium 1 to medium 3 through medium 2 in the absence of all the other layers. A similar problem with equal parameters of media 1 and 3 was solved in [19]. We carry out analogous discussion for the case of arbitrary parameters of media 1 and 3. Let

$$\tau_\sigma^{ps} = \left(b_\sigma^p - b_{-\sigma}^p\right) / \left(b_\sigma^s - b_{-s}^p\right)$$

and

$$\rho_\nu^{ps} = \left(b_\nu^p - b_\nu^s\right) / \left(b_\nu^s - b_{-\nu}^p\right)$$

be the transmission and reflection coefficients, respectively, for the interface of the adjacent semi-infinite *p-th* and *s-th* media for the incident wave polarization σ ($\sigma = \pm\nu$) [15], and $\eta_\sigma^p = \exp(ik_\sigma^p d_p)$ be the coefficient which takes into account the phase change during the propagation in the layer p of the thickness d_p. Let the subscript to the left indicate the serial number of transmitted (reflected) wave [19], then:

$${}_0E_{-\nu}^{\downarrow 1} = E_\nu^{\uparrow 1}\rho_\nu^{12}; \quad {}_1E_\nu^{\uparrow 3} = E_\nu^{\uparrow 1}\tau_\nu^{12}\eta_\nu^2\tau_\nu^{23};$$
$${}_1E_{-\nu}^{\downarrow 1} = E_\nu^{\uparrow 1}\tau_\nu^{12}\eta_\nu^2\rho_\nu^{23}\eta_{-\nu}^2\tau_{-\nu}^{21}. \qquad (2.1)$$

Introducing the denotation $q_\nu = \rho_\nu^{23}\rho_{-\nu}^{21}\eta_\nu^2\eta_{-\nu}^2$, it is easy to see that

$${}_pE_\nu^{\uparrow 3} = {}_1E_\nu^{\uparrow 3}\cdot q_\nu^{p-1}; \quad {}_pE_{-\nu}^{\downarrow 1} = {}_1E_{-\nu}^{\downarrow 1}\cdot q_\nu^{p-1}.$$

Using these formulae, and (2.1), one can find $E_\nu^{\uparrow 3}$ and $E_{-\nu}^{\downarrow 1}$ as

$$E_\nu^{\uparrow 3} = \sum_{s=1}^{\infty} {}_sE_\nu^{\uparrow 1} = {}_1E_\nu^{\uparrow 3}\sum_{s=1}^{\infty} q_\nu^{s-1} = {}_1E_\nu^{\uparrow 3}\frac{1}{1-q_\nu};$$

$$E_\nu^{\uparrow 3} = E_n^{\uparrow 1}\tau_\nu^{12}\eta_\nu^2\tau_\nu^{23}\frac{1}{1-q_\nu} = E_\nu^{\uparrow 1}\cdot T_\nu^{13}. \qquad (2.2)$$

$$E_{-\nu}^{\downarrow 1} = {}_0E_{-\nu}^{\downarrow 1} + {}_1E_{-\nu}^{\downarrow 1}\sum_{s=1}^{\infty} q_\nu^{s-1} = {}_0E_{-\nu}^{\downarrow 1} + \frac{{}_1E_{-\nu}^{\downarrow 1}}{1-q_\nu};$$

$$E_{-\nu}^{\downarrow 1} = E_\nu^{\uparrow 1}\rho_\nu^{12} + \frac{E_\nu^{\uparrow 1}\tau_\nu^{12}\eta_\nu^2\rho_\nu^{23}\eta_{-\nu}^2\tau_{-\nu}^{21}}{1-q_\nu} = E_\nu^{\uparrow 1}\cdot R_\nu^{13}. \qquad (2.3)$$

$$R_\nu^{13} = \rho_\nu^{12} + \frac{\tau_\nu^{12}\eta_\nu^2\rho_\nu^{23}\eta_{-\nu}^2\tau_{-\nu}^{21}}{1-\rho_\nu^{23}\rho_{-\nu}^{21}\eta_\nu^2\eta_{-\nu}^2} =$$
$$= \frac{\rho_\nu^{12}\left[1-\left(\rho_\nu^{12}\rho_{-\nu}^{21} - \tau_\nu^{12}\tau_{-\nu}^{21}\right)\frac{\rho_\nu^{21}}{\rho_\nu^{12}}\frac{\rho_\nu^{23}}{\rho_\nu^{21}}\eta_\nu^2\eta_{-\nu}^2\right]}{1-\rho_\nu^{23}\rho_{-\nu}^{21}\eta_\nu^2\eta_{-\nu}^2}. \qquad (2.4)$$

By using similar arguments and taking into account the identity $\left(\rho_\nu^{12}\rho_{-\nu}^{21} - \tau_\nu^{12}\tau_{-\nu}^{21}\right)\frac{\rho_\nu^{21}}{\rho_\nu^{12}} = 1$ [19] one can obtain ($\sigma = \pm\nu$)

$$T_\sigma^{13} = \eta_\sigma^2\frac{\tau_\sigma^{12}\tau_\sigma^{23}}{1-\rho_\sigma^{23}\rho_{-\sigma}^{21}\eta_\sigma^2\eta_{-\sigma}^2};$$

$$R_\sigma^{13} = \frac{\rho_\sigma^{12}\left[1-\eta_\sigma^2\eta_{-\sigma}^2\left(\rho_\sigma^{23}/\rho_\sigma^{21}\right)\right]}{1-\rho_\sigma^{23}\rho_{-\sigma}^{21}\eta_\sigma^2\eta_{-\sigma}^2};$$

$$T_\sigma^{31} = \eta_\sigma^2\frac{\tau_\sigma^{32}\tau_\sigma^{21}}{1-\rho_\sigma^{21}\rho_{-\sigma}^{23}\eta_\sigma^2\eta_{-\sigma}^2}; \qquad (2.5)$$

$$R_\sigma^{31} = \frac{\rho_\sigma^{32}\left[1-\eta_\sigma^2\eta_{-\sigma}^2\left(\rho_\sigma^{21}/\rho_\sigma^{23}\right)\right]}{1-\rho_\sigma^{21}\rho_{-\sigma}^{23}\eta_\sigma^2\eta_{-\sigma}^2}.$$

Let us turn to the case $N = 4$. The following identities for the transmission and reflection coefficients τ_σ^{13}, ρ_σ^{13}, τ_σ^{31}, ρ_σ^{31} can be proved for the case of electromagnetic wave transmission trough the planar interface between media 1 and 3 when medium 2 is absent [15]:

$$\begin{cases}\tau_\sigma^{13}\rho_\sigma^{31} + \rho_\sigma^{13}\tau_{-\sigma}^{13} = 0;\\ \tau_\sigma^{13}\tau_\sigma^{31} + \rho_\sigma^{13}\rho_{-\sigma}^{13} = 1,\end{cases}$$
$$\begin{cases}\tau_\sigma^{31}\rho_\sigma^{13} + \rho_\sigma^{31}\tau_{-\sigma}^{31} = 0;\\ \tau_\sigma^{31}\tau_\sigma^{13} + \rho_\sigma^{31}\rho_{-\sigma}^{31} = 1.\end{cases} \qquad (2.6)$$

Similarly one can prove that the coefficients T_σ^{13}, R_σ^{13}, T_σ^{31}, R_σ^{31} (2.5) (medium 2 is present) satisfy the same identities:

$$\begin{cases} T_\sigma^{13} R_\sigma^{31} + R_\sigma^{13} T_{-\sigma}^{13} = 0; \\ T_\sigma^{13} T_\sigma^{31} + R_\sigma^{13} R_{-\sigma}^{13} = 1, \end{cases} \quad \begin{cases} T_\sigma^{31} R_\sigma^{13} + R_\sigma^{31} T_{-\sigma}^{31} = 0; \\ T_\sigma^{31} T_\sigma^{13} + R_\sigma^{31} R_{-\sigma}^{31} = 1. \end{cases} \qquad (2.7)$$

This means that we can effectively remove the layer between media 1 and 3,with corresponding changes to the coefficients:

$$\tau_\sigma^{13} \to T_\sigma^{13};\ \rho_\sigma^{13} \to R_\sigma^{13};\ \tau_\sigma^{31} \to T_\sigma^{31};\quad \rho_\sigma^{31} \to R_\sigma^{31}.$$

Therefore, the problem for $N = 4$ can be reduced to the problem for $N = 3$ by replacing the section between the first and third media with the boundary plane containing the transmission and reflection coefficients T_σ^{13}, R_σ^{13}, T_σ^{31}, R_σ^{31} in the corresponding directions. As a result, the following formulas for the coefficients T_σ^{14}, R_σ^{14}, T_σ^{41}, R_σ^{41} are obtained:

$$T_\sigma^{14} = \eta_\sigma^3 \frac{T_\sigma^{13} \tau_\sigma^{34}}{1 - \rho_\sigma^{34} R_{-\sigma}^{31} \eta_\sigma^3 \eta_{-\sigma}^3};$$
$$R_\sigma^{14} = \frac{R_\sigma^{13}\left[1 - \eta_\sigma^3 \eta_{-\sigma}^3 \left(\rho_\sigma^{34} / R_\sigma^{31}\right)\right]}{1 - \rho_\sigma^{34} R_{-\sigma}^{31} \eta_\sigma^3 \eta_{-\sigma}^3};$$
$$T_\sigma^{41} = \eta_\sigma^3 \frac{\tau_\sigma^{43} T_\sigma^{31}}{1 - R_\sigma^{31} \rho_{-\sigma}^{34} \eta_\sigma^3 \eta_{-\sigma}^3};$$
$$R_\sigma^{41} = \frac{\rho_\sigma^{43}\left[1 - \eta_\sigma^3 \eta_{-\sigma}^3 \left(R_\sigma^{31} / \rho_\sigma^{34}\right)\right]}{1 - R_\sigma^{31} \rho_{-\sigma}^{34} \eta_\sigma^3 \eta_{-\sigma}^3}. \qquad (2.8)$$

Coefficients T_σ^{14}, R_σ^{14}, T_σ^{41}, R_σ^{41} (2.8) also satisfy the identities of the type (2.6)–(2.7). Summarizing these arguments, one can obtain recurrence formulas and using them step-by-step find all of the transmission and reflection coefficients in both directions ($3 \le p \le N-1$):

$$T_\nu^{1(p+1)} = \eta_\nu^p \frac{T_\nu^{1p} \tau_\nu^{p(p+1)}}{1 - \rho_\nu^{p(p+1)} R_{-\nu}^{p1} \eta_\nu^p \eta_{-\nu}^p};$$
$$R_\sigma^{1(p+1)} = \frac{R_\sigma^{1p}\left[1 - \eta_\sigma^p \eta_{-\sigma}^p \left(\rho_\sigma^{p(p+1)} / R_\sigma^{p1}\right)\right]}{1 - \rho_\sigma^{p(p+1)} R_{-\sigma}^{p1} \eta_\sigma^p \eta_{-\sigma}^p};$$
$$T_\sigma^{(p+1)1} = \eta_\sigma^p \frac{\tau_\sigma^{(p+1)p} T_\sigma^{p1}}{1 - R_\sigma^{p1} \rho_{-\sigma}^{p(p+1)} \eta_\sigma^p \eta_{-\sigma}^p};$$
$$R_\sigma^{(p+1)1} = \frac{\rho_\sigma^{(p+1)p}\left[1 - \eta_\sigma^p \eta_{-\sigma}^p \left(R_\sigma^{p1} / \rho_\sigma^{p(p+1)}\right)\right]}{1 - R_\sigma^{p1} \rho_{-\sigma}^{p(p+1)} \eta_\sigma^p \eta_{-\sigma}^p}. \qquad (2.9)$$

All of the above coefficients satisfy the identities of the type (2.6)–(2.7).

3 The matrix method

Now consider the problem of the normal incidence of an electromagnetic wave on the structure of biisotropic layers in the matrix method [19]. Introducing the notations

$$\mathrm{E}_p = \begin{pmatrix} E_\nu^{\uparrow p} \\ E_{-\nu}^{\downarrow p} \end{pmatrix},\ 1 \le p \le N-1;$$
$$\mathrm{E}_N = \begin{pmatrix} E_\nu^{\uparrow N} \\ 0 \end{pmatrix}; \qquad (3.1)$$
$$\mathrm{M}_p = \begin{pmatrix} 1 & 1 \\ -b_\nu^p & -b_{-\nu}^p \end{pmatrix};\ \ 1 \le p \le N;$$
$$\Phi_p = \begin{pmatrix} \exp\left(i\nu k_\nu^p d_p\right) & 0 \\ 0 & \exp\left(-i\nu k_{-\nu}^p d_p\right) \end{pmatrix}; \qquad (3.2)$$
$$2 \le p \le N-1,$$

it is easy to show that boundary conditions of continuity of the tangential components of vectors $\vec{E}$ and $\vec{H}$ can be represented as the following system of matrix equations (assuming that $\Phi_1 = 1$):

$$\mathrm{M}_{p+1}\mathrm{E}_{p+1} = \mathrm{M}_p \Phi_p \mathrm{E}_p,\quad p = 1, 2, ..., N-1. \qquad (3.3)$$

Expressing E_p through E_{p+1} one can find expression E_1 through E_N:

$$\mathrm{E}_1 = \left[\prod_{p=1}^{N-1}\left(\Phi_p^{-1} \mathrm{M}_p^{-1} \mathrm{M}_{p+1}\right)\right]\mathrm{E}_N = \Lambda \mathrm{E}_N, \qquad (3.4)$$

where Λ, introduced in (3.4), is a 2x2 matrix with the elements Λ_{ij}, that is,

$$\begin{pmatrix} E_\nu^{\uparrow 1} \\ E_{-\nu}^{\downarrow 1} \end{pmatrix} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \begin{pmatrix} E_\nu^{\uparrow N} \\ 0 \end{pmatrix}. \qquad (3.5)$$

Representing (3.5) in the form of equations for the unknown coefficients $E_\nu^{\uparrow N}$, $E_{-\nu}^{\downarrow 1}$ one can obtain expressions for T_ν, R_ν.

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \Lambda_{11} & 0 \\ \Lambda_{21} & -1 \end{pmatrix} \begin{pmatrix} T_\nu \\ R_\nu \end{pmatrix};$$
$$\begin{pmatrix} T_\nu \\ R_\nu \end{pmatrix} = \frac{1}{\Lambda_{11}} \begin{bmatrix} 1 & 0 \\ \Lambda_{21} & -\Lambda_{11} \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \qquad (3.6)$$

It is easy to see that the coefficients T_ν, R_ν (3.6) satisfy identity (2.6).

4 A numerical example: a periodic structure of layers

Suppose that the area $z \ge 0$ has a periodic structure: layers of biisotropic media 2 of thickness d_2 imbedded in medium 1 with distance d_1 between them. Let us investigate the dependence of the modulus of the transmission coefficient (the incident wave has the right circular polarization) on distance d_1. Figure 4.1 shows plots for the following media parameters:

$$\varepsilon_1 = 1.3,\ \mu_1 = 1.2,\ \chi_1 = 0.2,\ \alpha_1 = 0.1;$$
$$\varepsilon_2 = 3.2,\ \mu_2 = 1.3,\ \chi_2 = 0.4,\ \alpha_2 = 0.4;\ d_2 = 1.2.$$

(Numerical results obtained using the method of multiple reflections and the matrix method, are identical).

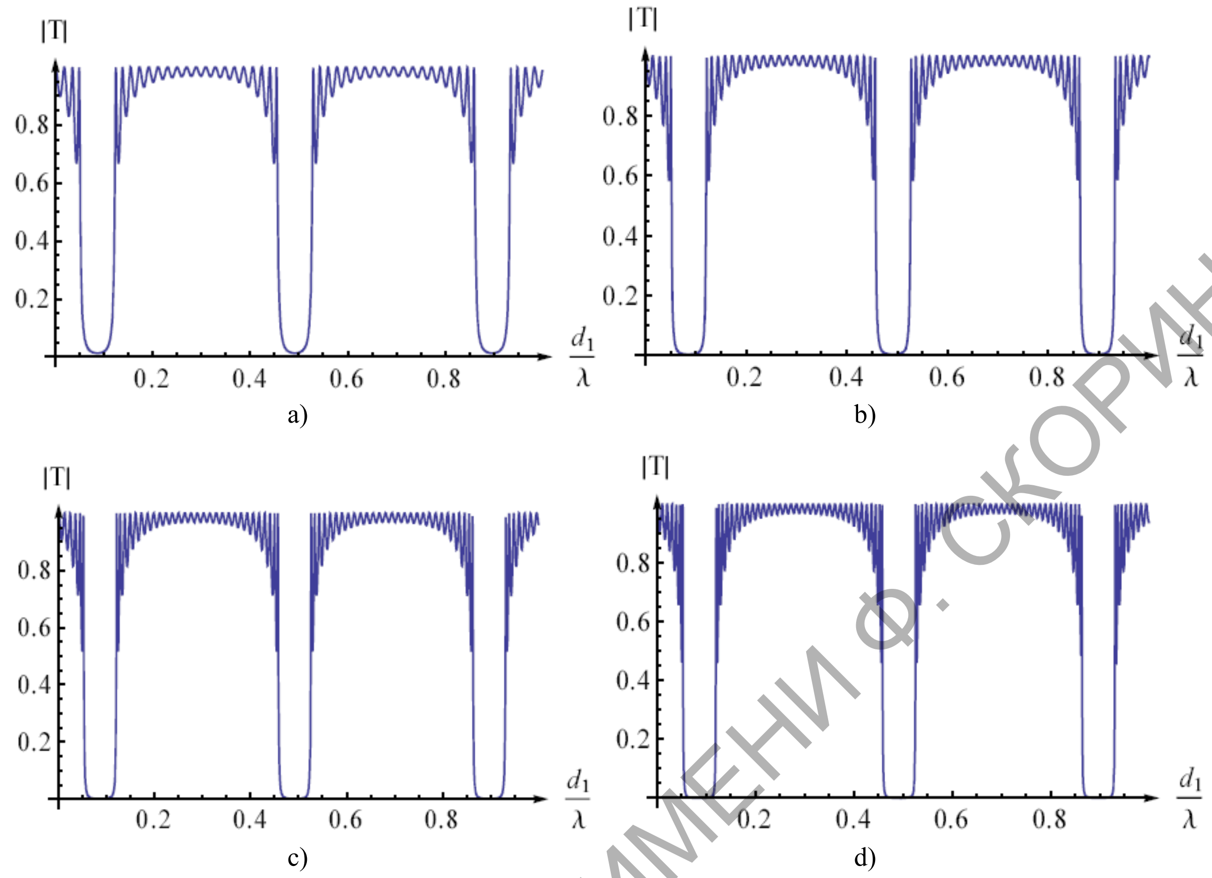

Figure 4.1 – The dependence of the transmission coefficient on distance d_1.
Number of layers 2: a) 20, b) 25, c) 30, d) 35

Graphs represent sequences of symmetric “allowed” zones, in which the transmission coefficient varies around unity, divided by “forbidden” zones in which the transmission coefficient is close to 0. Increasing number of layers leads to the fact that the minimum value of the transmission coefficient in the troughs approaches 0. The number of maximums in an “allowed” zone equals the number of layers of media 2 minus one. The value of the transmission coefficient at each maximum is equal to 1, that is, there is a complete penetration.

Conclusion

Thus, the problem of transmission and reflection of electromagnetic waves for the multilayer biisotropic structure is solved by two different methods: the method of multiple reflections and the matrix method. Both methods yield the results which coincide with each other with a very high degree of accuracy. At the same time these methods have different advantages. The advantage of the multiple reflections method is comparatively small numerical values, which increases the stability of the numerical solutions. This also significantly accelerates the calculation of periodic structures by re-using pre-calculated transmission and reflection coefficients. The advantage of the matrix method is the ease of implementation and a significant speed increase compared to the method of multiple reflections in the case of non-periodic layered medium structure.

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