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Some notes on minimal subgroups of finite groups

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1. Introduction

All the groups in this paper will be finite.

Let G be a group. A minimal subgroup of G is a subgroup of prime order. For a group of even order, it is also helpful to consider cyclic subgroups of order 4. There has been a considerable interest in studying the group structure under the assumption that minimal subgroups and cyclic subgroups of order 4 are well-situated in G (see [8, p. 435], [3,4,6,9-11,15-18]).

It is natural to limit the hypothesis for fewer minimal subgroups and cyclic subgroups of order 4. In 2001, L. A. Shemetkov (see [12]) introduced concepts of a Q-central element and a $Q\mathfrak{F}$-central element. Later, O. L. Shemetkova [14] proved that if all elements of prime order and all Q_8-elements of order 4 are Q-central in G, then G is nilpotent. In this paper we describe the structure of a group assuming that minimal subgroups and Q_8-elements of order 4 of some normal subgroups are $Q\mathfrak{U}$-central or Q-central. We will use the following concepts.

Definition 1.1 (see [14]). An element $x \in G$ is called a Q_8-element if there exists a section A/B of G such that $xB \in A/B$, $A/B \simeq Q_8$ (the quaternion group of order 8) and the order of x is equal to the order of xB in A/B.

Definition 1.2 (see [12]). (1) An element x of a group G is called Q-*central* if there exists a central chief factor A/B of G such that $x \in A \setminus B$.

(2) An element x of a group G is called $Q\mathfrak{F}$-*central* if there exists a $\mathfrak{F}$-central chief factor A/B of G such that $x \in A \setminus B$.

(3) An element x of a group G is called Qf-*central* (f is a local satellite) if there exists a f-central chief factor A/B of G such that $x \in A \setminus B$.

By definition, we consider the identity 1 as a Q-central element and $Q\mathfrak{F}$-central element. We denote by $QZ(G)$ and $QZ_{\mathfrak{F}}(G)$ the set of all Q-central elements and the set of all $Q\mathfrak{F}$-central elements of G respectively. Obviously, $QZ(G)$ contains the hypercenter $Z_\infty(G)$ of G, and $QZ_{\mathfrak{F}}(G)$ contains the $\mathfrak{F}$-hypercenter $Z_\infty^{\mathfrak{F}}(G)$ of G. If $\mathfrak{F} = LF(f)$ is a saturated formation with an local integrated satellite f, we use a denotation $QZ_f(G)$ instead of $QZ_{\mathfrak{F}}(G)$.

Definition 1.3 (see [6]). Let G be a group and $\mathfrak{U}$ be the class of supersoluble groups. We say that a subgroup H of G is $\mathfrak{U}$-*supplemented* in G if there exists a subgroup K of G such that $G = HK$ and $(H \cap K)H_G/H_G$ is contained in the $\mathfrak{U}$-hypercenter $Z_\infty^{\mathfrak{U}}(G/H_G)$ of G/H_G.

We say that $x \in G$ is $\mathfrak{U}$-supplemented in G if $\langle x\rangle$ is $\mathfrak{U}$-supplemented in G.

It is clear that normal, c-normal, c-supplemented and complemented subgroups are $\mathfrak{U}$-supplemented, but the converse is not true. For example, Let Z be a group of order 5, and $G = [Z]Aut(Z)$. Then $Aut(Z)$ is a cyclic subgroup of order 2^2. Let L be a subgroup of order 2 in $Aut(Z)$. Then L is not normal, c-normal, complemented and c-supplemented in G, but it is $\mathfrak{U}$-supplemented in G, since G is supersoluble.

Recall that for a class $\mathfrak{F}$ of groups, a chief factor H/K of a group G is called $\mathfrak{F}$-central (see [21], p. 127 or [7], Definition 2.4.2) if $[H/K](G/C_G(H/K)) \in \mathfrak{F}$. The symbol $Z_\infty^{\mathfrak{F}}(G)$

denotes the $\mathfrak{F}$-hypercenter of a group G, that is, the product of all normal subgroups H of G whose G-chief factors are $\mathfrak{F}$-central. A subgroup H of G is said to be $\mathfrak{F}$-hypercentral in G if $H \leq Z_\infty^{\mathfrak{F}}(G)$. A class $\mathfrak{F}$ of groups is called a formation provide that $\mathfrak{F}$ contains all of homomorphic images of its groups and if G/M and G/N are in $\mathfrak{F}$, then $G/M \cap N$ is in $\mathfrak{F}$. Obviously, every group G has a smallest normal subgroup N such that G/N is in a non-empty formation $\mathfrak{F}$. This uniquely determined normal subgroup of G is called the $\mathfrak{F}$-residual of G and denoted by $G^{\mathfrak{F}}$. A formation $\mathfrak{F}$ is saturated if $G/\Phi(G) \in \mathfrak{F}$ always implies that G belongs to $\mathfrak{F}$. We use $\mathfrak{N}$ and $\mathfrak{U}$ to denote the formation of all the nilpotent and supersoluble groups, respectively. For the formation $\mathfrak{N}$ of all nilpotent groups, $Z_\infty^{\mathfrak{N}}(G)$ is usually denoted by $Z_\infty(G)$. A group G is a called a Schmidt group, if $G \notin \mathfrak{N}$ and $M \in \mathfrak{N}$ for any proper subgroup M of G.

Let $\mathbb{P}$ be the set of prime numbers. A local satellite (see [22]) is a function f defined on $\mathbb{P}$ such that $f(p)$ is a (possibly empty) formation. A chief factor H/K of a group G is called f-central in G if $G/C_G(H/K) \in f(p)$ for all primes p dividing $|H/K|$. A non-empty formation $\mathfrak{F}$ is saturated if and only if there exists a local satellite f such that $\mathfrak{F}$ is the class of all groups with f-central chief factors. We write $\mathfrak{F} = LF(f)$ and say that f is a local satellite of $\mathfrak{F}$. A local satellite f of a formation $\mathfrak{F} = LF(f)$ is called: 1) semi-integrated if, for each prime p, a formation $f(p)$ either is contained in $\mathfrak{F}$ or coincides with the class $\mathfrak{E}$ of all groups; 2) integrated if $f(p)$ is contained in $\mathfrak{F}$ for each prime p; 3) full if $\mathfrak{N}_p f(p) = f(p)$ for each prime p; 4) semi-canonical if f is full and semi-integrated; 5) canonical if f is full and integrated.

For notations and terminologies not given in this paper, the reader is referred to [5,7,21].

2. Preliminaries

Lemma 2.1 ([14], Lemma 1). *Let G be a group and H a normal subgroup of G. If $x \in H$ is Q-central in G, then x is Q-central in H.*

Lemma 2.2 ([13], Theorem 3.1). *Let p be a prime, and $\mathfrak{F} = LF(f)$ a saturated formation, where f is a semi-canonical local satellite such that $f(q) = \mathfrak{E}$ for every prime $q \neq p$. Let H be a normal subgroup of a group G. Assume that every element of order p and every element of order 4 (if $p = 2$) is Qf-central in G. Then every G-chief factor of H is f-central in G.*

Lemma 2.3 ([7], Corollary 3.2.9). *If $\mathfrak{F} \neq \varnothing$ is a saturated formation, then for any group G we have $[G^{\mathfrak{F}}, Z_\infty^{\mathfrak{F}}(G)] = 1$.*

Lemma 2.4 ([1], Lemma 3.2). *Let $G = AB$, where B is a maximal subgroup of G and $A = \langle x \rangle$ is a cyclic 2-subgroup of G. Then $x \in QZ(G)$.*

Lemma 2.5 ([7], Corollary 3.2.7). *Let $\mathfrak{F} \neq \varnothing$ be a saturated formation, and G a group. If H is a $\mathfrak{F}$-hypercentral normal subgroup of G, then $G/C_G(H) \in \mathfrak{F}$.*

Lemma 2.6 ([15], Lemma 2.12). *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{U}$, and G a group with a normal subgroup E such that $G/E \in \mathfrak{F}$. If E is cyclic, then $G \in \mathfrak{F}$.*

We say that a normal subgroup R of G is p-hypercentral in G for a prime p, if every G-chief pd-factor of R is central in G (a chief pd-factor is a chief factor whose the order is divisible by p).

Lemma 2.7 ([14], Lemma 3). *Let G be a group and $R \trianglelefteq G$. If R is non p-hypercentral in G, then G has a p-closed Schmidt pd-subgroup S such that:*

(1) *a Sylow p-subgroup S_p of S is contained in R,*

(2) $QZ(G) \cap S_p \subseteq \Phi(S_p)$.

Lemma 2.8 ([14], Lemma 4). *Let S be a Schmidt group with a normal and non-abelian Sylow 2-subgroup P. Let $|Z(P)| = 2$. If x is an element of order 4 in S, then $x \in L \leq S$ and $L \simeq Q_8$.*

Lemma 2.9 ([10], Lemma 2.3). *For any group G we have $C_G(F^*(G)) \leq F^*(G)$. If $F^*(G)$ is soluble, then $F^*(G) = F(G)$.*

Lemma 2.10. *Let G be a group, and H a normal subgroup of G. Let $\mathfrak{F}$ be a saturated formation. If $F^*(H) \subseteq Z_\infty^{\mathfrak{F}}(G)$, then $H \subseteq Z_\infty^{\mathfrak{F}}(G)$.*

Proof. Let $C = C_G(F^*(H))$, then $G/C \in \mathfrak{F}$ by Lemma 2.5. Since $HC/C \trianglelefteq G/C \in \mathfrak{F}$, HC/C is $\mathfrak{F}$-hypercentral in G/C. By the G-isomorphism $HC/C \simeq H/H \cap C$, we have that $H/H \cap C$ is $\mathfrak{F}$-hypercentral in $G/H \cap C$. By Lemma 2.9, $H \cap C \subseteq F^*(H)$. It follows that $H/F^*(H)$ is $\mathfrak{F}$-hypercentral in $G/F^*(H)$ and hence $H \subseteq Z_\infty^{\mathfrak{F}}(G)$. □

Lemma 2.11. *Let G be a group, and x its element of order 2^n. Then $x \in QZ(G)$ if and only if $x \in QZ_{\mathfrak{U}}(G)$.*

Proof. We need only to consider the case $x \in QZ_{\mathfrak{U}}(G)$. Then there exists a G-chief factor H/K such that $|H/K| = p$ and $x \in H \setminus K$. So $p = 2$. It follows that $H/K \subseteq Z(G/K)$ and consequently $x \in QZ(G)$. □

Lemma 2.12. *Let G be a group of odd order, and x its element of order p. If x is $\mathfrak{U}$-supplemented in G, then $x \in QZ_{\mathfrak{U}}(G)$.*

Proof. By hypothesis, there exists a subgroup T in G such that $\langle x\rangle T = G$ and $(\langle x\rangle \cap T)\langle x\rangle_G/\langle x\rangle_G \subseteq Z_\infty^{\mathfrak{U}}(G/\langle x\rangle_G)$.

If $\langle x\rangle_G = \langle x\rangle$. Then $x \in QZ_{\mathfrak{U}}(G)$. Assume that $\langle x\rangle_G = 1$. Then $\langle x\rangle \cap T \subseteq Z_\infty^{\mathfrak{U}}(G)$. If $T = G$, then $x \in Z_\infty^{\mathfrak{U}}(G) \subseteq QZ_{\mathfrak{U}}(G)$. If $T \neq G$, then $|G : T| = p$. □

Lemma 2.13. *Let G be a group, and x its 2-element. If x is $\mathfrak{U}$-supplemented in G, then $x \in QZ(G)$.*

Proof. By hypothesis, there exists a subgroup T in G such that $\langle x\rangle T = G$ and $(\langle x\rangle \cap T)\langle x\rangle_G/\langle x\rangle_G \subseteq Z_\infty^{\mathfrak{U}}(G/\langle x\rangle_G)$.

If $T \neq G$, then $x \in QZ(G)$ by Lemma 2.4. If $T = G$, then $\langle x\rangle/\langle x\rangle_G \subseteq Z_\infty^{\mathfrak{U}}(G/\langle x\rangle_G)$. From Lemma 2.11, we have $\langle x\rangle/\langle x\rangle_G \subseteq QZ(G/\langle x\rangle_G)$. It follows that $x \in QZ(G)$. □

3. Main Results

Lemma 3.1. *Let G be a group, and H a normal subgroup of G. If all elements of order 2 in H and all Q_8-elements of order 4 in H are Q-central in G, then H is 2-hypercertral in G.*

Proof. Suppose that H is non 2-hypercertral in G. By Lemma 2.7, G has a 2-closed Schmidt 2d-subgroup $S = [S_2]S_q$ such that: (1) S_2 is a Sylow 2-subgroup of S and $S_2 \subseteq H$, (2) $QZ(G) \cap S_2 \subseteq \Phi(S_2)$.

If S_2 is abelian, then S_2 is an elementary abelian 2-group. So $S_2 = S_2 \cap QZ(G) \subseteq \Phi(S_2) = 1$, a contradiction.

If S_2 is non-abelian, then $\Phi(S_2) = Z(S_2)$ is an elementary abelian 2-group. Let Z_0 be a subgroup of index 2 in $Z(S_2)$, and xZ_0 be an element of order 4 in S_2/Z_0. By Lemma 2.8, xZ_0 is contained in a subgroup which is isomorphic to Q_8. Since x is of order 4, x is a Q_8-element. By the hypothesis, x is Q-central in G, therefore $x \in \Phi(S_2)$, a contradiction. □

Theorem 3.2. *Let G be a finite group, and H a normal subgroup of G. If all elements of prime order in H and all Q_8-elements of order 4 in H are $Q\mathfrak{U}$-central in G, then $H \subseteq Z_\infty^{\mathfrak{U}}(G)$.*

Proof. From Lemma 2.11 and Lemma 3.1 we have that H is 2-hypercentral in G. Therefore H is 2-nilpotent. Let H_0 be a normal 2-complement of H. Then $H_0 \trianglelefteq G$ and

$H/H_0 \subseteq Z_\infty(G/H_0) \subseteq Z^{\mathfrak{U}}_\infty(G/H_0)$. From Lemma 2.2 we have $H_0 \subseteq Z^{\mathfrak{U}}_\infty(G)$. Thus $H \subseteq Z^{\mathfrak{U}}_\infty(G)$. □

Using Lemma 3.1, Lemma 2.12 and Lemma 2.13 we obtain the following.

Corollary 3.2.1. *Let G be a finite group, and H a normal subgroup of G. If all elements of prime order in H and all Q_8-elements of order 4 in H are $\mathfrak{U}$-supplemented in G, then $H \subseteq Z^{\mathfrak{U}}_\infty(G)$.*

Corollary 3.2.2. *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{U}$. If $G \notin \mathfrak{F}$, then there exists an element $x \in G^{\mathfrak{F}}$ such that x is not $Q\mathfrak{U}$-central in G and either $|\langle x\rangle|$ is a prime or x is a Q_8-element of order 4.*

Proof. Apply Theorem 3.2 and Lemma 2.6. □

Corollary 3.2.3 (see [6]). *Let $\mathfrak{F}$ be a S-closed saturated formation containing all supersoluble groups. Suppose that G is a group with a normal subgroup E such that $G/E \in \mathfrak{F}$. If all cyclic subgroups of E of prime order and order 4 are $\mathfrak{U}$-supplemented in G, then $G \in \mathfrak{F}$.*

Corollary 3.2.4 (see [2]). *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{U}$ and G be a group. If all minimal subgroups and all cyclic subgroups of order 4 of $G^{\mathfrak{F}}$ are c-normal in G, then $G \in \mathfrak{F}$.*

Corollary 3.2.5 (see [16]). *If all cyclic subgroups of a group G with prime order and order 4 are c-normal in G, then G is supersoluble.*

Corollary 3.2.6 (see [3]). *Let G be a group with a normal subgroup N such that G/N is supersoluble. If every element of prime order and order 4 of N is c-supplemented in G, then G is supersoluble.*

Corollary 3.2.7 (see [19]). *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{U}$. Assume that G is a group with a normal subgroup N such that $G/N \in \mathfrak{F}$. If every element of prime order and order 4 of N is c-supplemented in G, then $G \in \mathfrak{F}$.*

Corollary 3.2.8. *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{U}$, and G a group. Then $G \in \mathfrak{F}$ if and only if there exists a normal soluble subgroup H in G such that $G/H \in \mathfrak{F}$ and all cyclic subgroups of $F(H)$ of prime order and order 4 are $Q\mathfrak{U}$-central in G.*

Proof. If $G \in \mathfrak{F}$, then the hypotheses is true with $H = 1$.

For the sufficiency part, applying Theorem 3.2, Lemma 2.5 and Lemma 2.6, Corollary holds. □

Remark 1. Corollary 3.2.8 is not true if we omit the solubility of H. Set $G = H \times K$, where $H = SL(2,5)$ and $K \in \mathfrak{U}$. Then $|F(H)| = 2$ and $G/H \simeq K \in \mathfrak{U}$. $F(H) \subseteq QZ_{\mathfrak{U}}(G)$, but $G \notin \mathfrak{U}$.

Corollary 3.2.9 (see [9]). *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{U}$, and let G be a group. Then $G \in \mathfrak{F}$ if and only if there exists a normal soluble subgroup H in G such that $G/H \in \mathfrak{F}$ and all cyclic subgroups of $F(H)$ of prime order and order 4 are c-normal in G.*

Corollary 3.2.10. *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{U}$. Assume G is a group with a normal subgroup N such that $G/N \in \mathfrak{F}$. If every element of prime order and Q_8-element order 4 of $F^*(N)$ is $Q\mathfrak{U}$-central in G, then $G \in \mathfrak{F}$.*

Proof. By Theorem 3.2, $F^*(N) \subseteq Z^{\mathfrak{U}}_\infty(G) \subseteq Z^{\mathfrak{F}}_\infty(G)$. Then apply Lemma 2.10. □

Corollary 3.2.11 (see [19]). *Suppose G is a group with a normal subgroup N such that G/N is supersoluble. If every element of prime order and order 4 of $F^*(N)$ is c-supplemented in G, then G is supersoluble.*

Corollary 3.2.12 (see [20]). *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{U}$. Assume G is a group with a normal subgroup N such that $G/N \in \mathfrak{F}$. If every element of prime order and order 4 of $F^*(N)$ is c-supplemented in G, then $G \in \mathfrak{F}$.*

Theorem 3.3. *Let G be a finite group, and H be a normal subgroup of G. If all elements of prime order in H and all Q_8-elements of order 4 in H are Q-central in G, then $H \subseteq Z_\infty(G)$.*

Proof. From Lemma 3.1 we have that H is 2-hypercentral in G. So H is 2-nilpotent. Let H_0 be a normal 2-complement of H. Then $H_0 \trianglelefteq G$ and $H/H_0 \subseteq Z_\infty(G/H_0)$. By Lemma 2.2, $H_0 \subseteq Z_\infty(G)$, therefore $H \subseteq Z_\infty(G)$. □

Corollary 3.3.1. *Let G be a group with a normal subgroup N such that $G/N \in \mathfrak{N}$. If every element of prime order and every Q_8-element of order 4 in $F^*(N)$ is Q-central in G, then $G \in \mathfrak{N}$.*

Proof. By Theorem 3.3, $F^*(N) \subseteq Z_\infty(G)$. By Lemma 2.10, $N \leq Z_\infty(G)$. Consequently $G \in \mathfrak{N}$. □

Corollary 3.3.2 (see [18]). *Suppose that N is a normal subgroup of a group G such that G/N is nilpotent. Suppose that every element of order 4 of $F^*(N)$ is c-normal in G. Then G is nilpotent if and only if every element of prime order of $F^*(N)$ is contained in the hypercenter $Z_\infty(G)$ of G.*

Corollary 3.3.3 (see [19]). *Suppose that N is a normal subgroup of a group G such that G/N is nilpotent. Suppose every element of order 4 of $F^*(N)$ is c-supplemented in G. Then G is nilpotent if and only if every element of prime order of $F^*(N)$ is contained in the hypercenter $Z_\infty(G)$ of G.*

Theorem 3.4. *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{N}$. Suppose that every Q_8-element of order 4 in $F^*(G^{\mathfrak{F}})$ is Q-central in G. If $G \notin \mathfrak{F}$, then there exists an element of prime order in $F^*(G^{\mathfrak{F}}) \setminus Z^{\mathfrak{F}}_\infty(G)$.*

Proof. Suppose that the set of elements of prime order in $F^*(G^{\mathfrak{F}}) \setminus Z^{\mathfrak{F}}_\infty(G)$ is empty. Let x be an arbitrary element of prime order of $F^*(G^{\mathfrak{F}})$. Then, by Lemma 2.3, $x \in Z^{\mathfrak{F}}_\infty(G) \cap G^{\mathfrak{F}} \leq Z(G^{\mathfrak{F}}) \leq Z_\infty(G^{\mathfrak{F}})$. Since every Q_8-element of order 4 of $F^*(G^{\mathfrak{F}})$ is Q-central in G, it is Q-central in $G^{\mathfrak{F}}$ by Lemma 2.1. Applying Corollary 3.3.1 for $G^{\mathfrak{F}}$, we have $G^{\mathfrak{F}} \in \mathfrak{N}$. So $F^*(G^{\mathfrak{F}}) = F(G^{\mathfrak{F}}) = G^{\mathfrak{F}}$. In particular, $G^{\mathfrak{F}}$ has a normal Hall $2'$-subgroup $G^{\mathfrak{F}}_{2'}$. Using Lemma 2.2, we have $G^{\mathfrak{F}}_{2'} \leq Z^{mathfrakF}_\infty(G^{\mathfrak{F}})$.

Suppose that $G^{\mathfrak{F}}$ is non 2-hypercentral in G. By Lemma 2.7, G has a 2-closed Schmidt $2d$-subgroup $S = [S_2]S_q$ (q is a prime) such that: (1) S_2 is a Sylow 2-subgroup of S and $S_2 \subseteq G^{\mathfrak{F}}$, (2) $QZ(G) \cap S_2 \subseteq \Phi(S_2)$. Clearly, S_2 is non-abelian. Then $\Phi(S_2) = Z(S_2)$ is an elementary abelian 2-group. Let Z_0 be a subgroup of index 2 in $Z(S_2)$ and xZ_0 be an element of order 4 in S_2/Z_0. By Lemma 2.8, xZ_0 is contained in a subgroup which is isomorphic to Q_8. Since x is order of 4, x is a Q_8-element. Sinse x is Q-central in G, we have $x \in \Phi(S_2)$, a contradiction. □

Corollary 3.4.1 ([19]). *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{N}$. Suppose that G is a group such that every element of order 4 of $G^{\mathfrak{F}}$ is c-supplemented in G. Then $G \in \mathfrak{F}$ if and only if every element of prime order of $G^{\mathfrak{F}}$ lies in $Z^{\mathfrak{F}}_\infty(G)$.*

Corollary 3.4.2 (see [20]). *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{N}$. Suppose that G is a group such that every element of order 4 of $F^*(G^{\mathfrak{F}})$ is c-supplemented in G. Then $G \in \mathfrak{F}$ if and only if every element of prime order of $F^*(G^{\mathfrak{F}})$ lies in $Z^{\mathfrak{F}}_\infty(G)$.*

Abstract. In this paper we describe the structure of a finite group assuming that minimal subgroups and Q_8-elements of order 4 of some normal subgroups are $Q\mathfrak{U}$-central or Q-central.

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