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РЕЗОНАНСНАЯ СТРУКТУРА СЕЧЕНИЙ РАССЕЯНИЯ И ЭКСТИНКЦИИ В ПРОБЛЕМЕ МИ ДЛЯ БИИЗОТРОПНОГО ШАРА

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RESONANCE STRUCTURE OF THE SCATTERING AND EXTINCTION CROSS SECTIONS IN THE MIE PROBLEM FOR BIISOTROPIC SPHERE

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На основе точного решения задачи Ми рассчитаны сечения экстинкции и рассеяния плоской монохроматической циркулярно поляризованной волны на биизотропном шаре, помещённом в биизотропную среду. Зависимости сечений от отношения радиуса шара к длине волны проанализированы численно. Обнаружена резонансная структура сечений рассеяния и экстинкции с узкими и очень узкими пиками, которые накладываются на медленно осциллирующую дифракционную зависимость.

Ключевые слова: *биизотропная среда, материальные уравнения, теория Ми, сферические электромагнитные волны, сечение рассеяния, сечение экстинкции, резонансная структура.*

The extinction and scattering cross sections for the case of the plane monochromatic circularly polarized electromagnetic wave scattered by a biisotropic sphere embedded in a biisotropic medium are calculated on the basis of the exact solution of the Mie problem. Scattering and extinction cross section dependences on the ratio of the sphere radius to the wavelength are analyzed numerically. The resonance structure of the scattering and extinction cross sections is revealed. These cross sections have narrow and very narrow peaks which are superimposed upon slowly oscillating diffraction background.

Keywords: *biisotropic medium, constitutive relations, Mie theory, spherical electromagnetic waves, scattering cross section, extinction cross section, resonance structure.*

Introduction

This paper is devoted to the study of the extinction cross section behavior in the problem of electromagnetic plane monochromatic wave scattering by a biisotropic homogeneous sphere of radius R imbedded in another biisotropic medium. The study of biisotropic media whose electromagnetic properties are described by the constitutive relations of the form

$$\vec{D} = \varepsilon \vec{E} + (\chi + i\alpha)\vec{H}; \quad \vec{B} = (\chi - i\alpha)\vec{E} + \mu\vec{H}$$

[1]–[5], is important due to the fact that remaining isotropic these media, nevertheless, may have electromagnetic properties that are significantly different from the properties of simple isotropic media like isotropic dielectrics.

1 Analytical solution of the Mie problem

At first we discuss briefly the results of the corresponding scattering problem solving and present analytical expressions for the coefficients of scattered and internal fields [6]. Solving boundary problems with spherical symmetry it is convenient to use the theory of spherical vectors $\vec{Y}^{L}_{JM}(\vec{n}_r)$ [7] and define the spherical waves $\vec{F}^{(z)}_{J\nu M}(k\,|\,\vec{r})$ [8]–[12]. Taking into account the spherical wave expansion of a plane monochromatic circularly polarized wave which is incident on the biisotropic sphere [6], [10], [12]:

$$\vec{E}^{in}_{\nu}(\vec{r}) = \sum_{J=1}^{\infty} E_J \vec{F}^{(j)}_{J\nu\nu}(k_\nu\,|\,\vec{r});$$

$$\vec{H}^{in}_{\nu}(\vec{r}) = -\sum_{J=1}^{\infty} E_J b_\nu \vec{F}^{(j)}_{J\nu\nu}(k_\nu\,|\,\vec{r}),$$

one should use analogous expansions for the scattered and internal fields [6], [10], [12]:

$$\vec{E}^{sct}_{\nu}(\vec{r}) = -\sum_{J=1}^{\infty} E_J \sum_{\sigma=\pm1} f^{J}_{\sigma\nu} \vec{F}^{(h^1)}_{J\sigma\nu}(k_\sigma\,|\,\vec{r}),$$

$$\vec{E}^{prt}_{\nu}(\vec{r}) = \sum_{J=1}^{\infty} E_J \sum_{\sigma=\pm1} g^{J}_{\sigma\nu} \vec{F}^{(z)}_{J\sigma\nu}(k^1_\sigma\,|\,\vec{r}).$$

$$\vec{H}^{sct}_{\nu}(\vec{r}) = \sum_{J=1}^{\infty} E_J \sum_{\sigma=\pm1} f^{J}_{\sigma\nu} b_\sigma \vec{F}^{(h^1)}_{J\sigma\nu}(k_\sigma\,|\,\vec{r}),$$

$$\vec{H}^{prt}_{\nu}(\vec{r}) = -\sum_{J=1}^{\infty} E_J \sum_{\sigma=\pm1} g^{J}_{\sigma\nu} b^1_\sigma \vec{F}^{(z)}_{J\sigma\nu}(k^1_\sigma\,|\,\vec{r}). \quad (1.1)$$

$$E_J = E_0\sqrt{2\pi(2J+1)}\,i^J;$$

$$b_\sigma = \left(\chi + i\sigma\sqrt{\varepsilon\mu - \chi^2}\right)/\mu;$$

$$k_\nu = \left(\sqrt{\varepsilon\mu - \chi^2} + \nu\alpha\right)\omega/c.$$

The continuity of the tangential components of the electric and magnetic fields at the interface between two media yields a system of algebraic equations for the expansion coefficients from which one can determine the coefficients $f_{\sigma\nu}^J$ and $g_{\sigma\nu}^J$ of the scattered and internal fields. The solution of this system can be written as:

$$f_{\sigma\nu}^J = \frac{k_\sigma}{k_\nu}\frac{\Delta_{\sigma\nu}^f}{\Delta};$$
$$g_{\sigma\nu}^J = \frac{k_\sigma^1}{k_\nu}\frac{\Delta_{\sigma\nu}^g}{\Delta}, \qquad (1.2)$$

where

$$\begin{aligned}\Delta = &\left[b_{+1}b_{-1} + b_{+1}^1 b_{-1}^1\right]\Pi\left(\hat{h}_+\hat{h}_-\right)\Pi\left(\hat{z}_+\hat{z}_-\right) + \\ &+\left[b_{+1}b_{+1}^1 + b_{-1}b_{-1}^1\right]W\left(\hat{h}_+\hat{z}_+\right)W\left(\hat{h}_-\hat{z}_-\right) - \\ &-\left[b_{+1}b_{-1}^1 + b_{-1}b_{+1}^1\right]\Pi\left(\hat{h}_-\hat{z}_+\right)\Pi\left(\hat{h}_+\hat{z}_-\right);\end{aligned} \qquad (1.3)$$

$$\hat{j}_\nu = \hat{j}_J\left(k_\nu R\right);\ \hat{h}_\sigma = \hat{h}_J^{(1)}\left(k_\sigma R\right);\ \hat{z}_\sigma = \hat{j}_J\left(k_\sigma^1 R\right),$$

$$\begin{aligned}\Delta_{\sigma\nu}^f = &\left[b_\nu b_{-\sigma} + b_{+1}^1 b_{-1}^1\right]\times \\ &\times\Pi\left(\hat{z}_+\hat{z}_-\right)\left(\hat{j}_\nu\hat{h}'_{-\sigma} + \nu\sigma\hat{j}'_\nu\hat{h}_{-\sigma}\right) + \\ &+\left[b_{-\sigma}b_{-\sigma}^1 + b_\nu b_\sigma^1\right]W\left(\hat{h}_{-\sigma}\hat{z}_{-\sigma}\right)\left(\hat{j}_\nu\hat{z}'_\sigma - \nu\sigma\hat{j}'_\nu\hat{z}_\sigma\right) - \\ &-\left[b_{-\sigma}b_\sigma^1 + b_\nu b_{-\sigma}^1\right]\Pi\left(\hat{h}_{-\sigma}\hat{z}_\sigma\right)\left(\hat{j}_\nu\hat{z}'_{-\sigma} + \nu\sigma\hat{j}'_\nu\hat{z}_{-\sigma}\right),\end{aligned} \qquad (1.4)$$

$$\begin{aligned}\Delta_{\sigma\nu}^g = &\left[b_\nu b_{-\sigma}^1 + b_+ b_-\right]\times \\ &\times\Pi\left(\hat{h}_+\hat{h}_-\right)\left(\hat{j}_\nu\hat{z}'_{-\sigma} + \nu\sigma\hat{j}'_\nu\hat{z}_{-\sigma}\right) - \\ &-\left[b_\nu b_\sigma + b_{-\sigma}b_{-\sigma}^1\right]W\left(\hat{h}_{-\sigma}\hat{z}_{-\sigma}\right)\left(\hat{j}_\nu\hat{h}'_\sigma - \nu\sigma\hat{j}'_\nu\hat{h}_\sigma\right) - \\ &-\left[b_\nu b_{-\sigma} + b_\sigma b_{-\sigma}^1\right]\Pi\left(\hat{h}_\sigma\hat{z}_{-\sigma}\right)\left(\hat{j}_\nu\hat{h}'_{-\sigma} + \nu\sigma\hat{j}'_\nu\hat{h}_{-\sigma}\right).\end{aligned} \qquad (1.5)$$

Writing the determinants we use the denotation

$$W\left(y_1 y_2\right) = y_1 y_2' - y_1' y_2,$$
$$\Pi\left(y_1 y_2\right) = y_1 y_2' + y_1' y_2.$$

Formulae (1.2)–(1.5) altogether constitute the solution of the coefficients determination problem for the scattered and internal field expansions.

2 Absorption, scattering and extinction cross sections

Now let us turn to the calculation of the scattering, absorption, and extinction cross sections. The Pointing vector for the field outside the scattering particle can be represented as the sum of three terms:

$$\vec{S} = \mathrm{Re}\left[\vec{E}, \vec{H}^*\right] = \vec{S}^{in} + \vec{S}^{sct} + \vec{S}^{ext}, \qquad (2.1)$$
$$\vec{S}^{ext} = \mathrm{Re}\left\{\left[\vec{E}^{in}, \vec{H}^{sct*}\right] + \left[\vec{E}^{sct}, \vec{H}^{in*}\right]\right\};$$
$$\vec{S}^{sct} = \mathrm{Re}\left[\vec{E}^{sct}, \vec{H}^{sct*}\right];$$
$$\vec{S}^{in} = \mathrm{Re}\left[\vec{E}^{in}, \vec{H}^{in*}\right],$$

where $\vec{S}^{in}$ is the Pointing vector of the incident wave, $\vec{S}^{sct}$ is the Pointing vector of the scattered field, $\vec{S}^{ext}$ is the term due to the interaction between the incident and scattered waves. Therefore, the energy fluxes trough a sphere of radius $r > R$ $\left(\Phi^{abs} = -\int \vec{S}d\vec{\sigma}\right)$ may be written as the sum of three terms: $\Phi^{abs} = \Phi^{in} - \Phi^{sct} + \Phi^{ext}$,
where

$$\Phi^{sct} = \int \vec{S}^{sct} d\vec{\sigma},$$
$$\Phi^{ext} = -\int \vec{S}^{ext} d\vec{\sigma},$$
$$\Phi^{in} = -\int \vec{S}^{in} d\vec{\sigma}. \qquad (2.2)$$

For the nonabsorbing surrounding medium $\Phi^{in} = 0$, consequently $\Phi^{ext} = \Phi^{abs} + \Phi^{sct}$, which means that the extinction of the electromagnetic wave is due to the absorption and scattering. Calculating the fluxes Φ^{sct} and Φ^{ext} (2.2) and dividing them by the intensity of the incident wave $\mathrm{I}^{in} = \left|\vec{S}^{in}\right|$, one can obtain the scattering cross section and the extinction cross section. The explicit form of the Pointing vector of the incident wave (2.1) is

$$\vec{S}^{in} = \frac{\sqrt{\varepsilon\mu - \chi^2}}{\mu}E_0^2\vec{k};$$
$$\mathrm{I}^{in} = \left|\vec{S}^{in}\right| = \frac{\sqrt{\varepsilon\mu - \chi^2}}{\mu}E_0^2. \qquad (2.3)$$

Using formulae (2.1) and (2.2) one can easily calculate the scattered energy flux:

$$\begin{aligned}\Phi^{sct} &= \int \mathrm{Re}\left[\vec{E}^{sct}, \vec{H}^{sct*}\right]\vec{n}\,d\sigma = \\ &= \int \mathrm{Re}\left\{\vec{E}^{sct}\left[\vec{H}^{sct*}, \vec{n}\right]d\sigma\right\} = \\ &= \mathrm{Re}\int \vec{H}^{sct*}\left[\vec{n}, \vec{E}^{sct}\right]r^2 d\Omega.\end{aligned}$$

Considering the scattered field in the far field zone ($k_\sigma r >> 1$) for the scattered flux we get the following expression:

$$\begin{aligned}\Phi^{sct} = \mathrm{Re}\Bigg\{&E_0\sum_{J=1}^{\infty}\sqrt{\gamma_J 2\pi}\,i\left(-E_0\right)\times \\ &\times\sum_{J'=1}^{\infty}\sqrt{\gamma'_J 2\pi}\sum_{\sigma=\pm1} f_{\sigma\nu}^J f_{\sigma\nu'}^{J'*} b_\sigma^* \times \\ &\times\int \frac{e^{-ik_\sigma r}}{k_\sigma r}\frac{e^{ik_\sigma r}}{k_\sigma r}\left\{\vec{Y}_{J\nu}^{(0)*} - \sigma\vec{Y}_{J\nu}^{(1)*}\right\}\left\{\vec{Y}_{J'\nu'}^{(1)} - \sigma\vec{Y}_{J'\nu'}^{(0)}\right\}r^2 d\Omega\Bigg\} = \\ &= \mathrm{Re}\left\{-i2\pi E_0^2\sum_{J=1}^{\infty}\sqrt{\gamma_J 2\pi}\times\right.\end{aligned} \qquad (2.4)$$

$$\times\sum_{J'=1}^{\infty}\sqrt{\gamma_{J'}2\pi}\sum_{\sigma=\pm1} f_{\sigma\nu}^{J} f_{\sigma\nu'}^{J'*} b_{\sigma}^{*}\frac{1}{k_{\sigma}^{2}}\times$$
$$\times\int\left[-\sigma\vec{Y}_{J\nu}^{(1)*}\vec{Y}_{J'\nu'}^{(1)}-\sigma\vec{Y}_{J\nu}^{(0)*}\vec{Y}_{J'\nu'}^{(0)}\right]r^{2}d\Omega\Big\}=$$
$$=\mathrm{Re}\left\{\frac{4\pi}{\mu}E_0^2\sum_{J=1}^{\infty}\gamma_J\sum_{\sigma=\pm1}\left|f_{\sigma\nu}^{J}\right|^2\frac{1}{k_\sigma^2}\left(i\sigma\chi+\sqrt{\varepsilon\mu-\chi^2}\right)\right\}=$$
$$=4\pi\frac{\sqrt{\varepsilon\mu-\chi^2}}{\mu}E_0^2\sum_{J=1}^{\infty}\gamma_J\sum_{\sigma=\pm1}\left|f_{\sigma\nu}^{J}\right|^2\frac{1}{k_\sigma^2},$$
$$\gamma_J=2J+1.$$

Dividing the scattered flux (2.4) by the incident irradiance (2.3) for the scattering cross section we obtain

$$\sigma_{\nu}^{sct}=\frac{\Phi^{sct}}{\mathrm{I}^{in}}=4\pi\sum_{J=1}^{\infty}\left(2J+1\right)\sum_{\sigma=\pm1}\left|f_{\sigma\nu}^{J}\right|^2\frac{1}{k_\sigma^2}. \quad (2.5)$$

Similarly, one can calculate the extinction cross section starting with the flux Φ^{ext}:

$$\Phi^{ext}=-\mathrm{Re}\int\left\{\vec{E}^{in}\left[\vec{H}^{sct*},\vec{n}\right]+\vec{E}^{sct}\left[\vec{H}^{in*},\vec{n}\right]\right\}d\sigma=$$
$$=\mathrm{Re}\int\left\{\vec{E}^{in*}\left[\vec{n},\vec{H}^{sct}\right]-\vec{H}^{in*}\left[\vec{n},\vec{E}^{sct}\right]\right\}r^2d\Omega.$$

Taking into account the expansions for the vectors $\vec{E}$ and $\vec{H}$ one can find the following analytical expression for the flux Φ^{ext}:

$$\Phi^{ext}=\mathrm{Re}\left[E_0^2\frac{2\pi}{\mu}\sum_{J=1}^{\infty}\sqrt{\gamma_J}\left(-i\right)^J\sum_{J'=1}^{\infty}\sqrt{2J'+1}i^{J'}\times\right.$$
$$\times\sum_{\sigma=\pm1}f_{\sigma\nu'}^{J'}\left(\sigma+\nu\right)i^2\frac{\sqrt{\varepsilon\mu-\chi^2}}{k_\sigma k_\nu}\times$$
$$\times\int\left(i\sigma\hat{j}_J\left(k_\nu r\right)\hat{h}_{J'}^{(1)\prime}\left(k_\sigma r\right)\vec{Y}_{J\nu}^{(0)*}\left(\vec{n}_r\right)\vec{Y}_{J'\nu'}^{(0)}\left(\vec{n}_r\right)-\right.$$
$$\left.\left.-i\nu\hat{j}'_J\left(k_\nu r\right)\hat{h}_{J'}^{(1)}\left(k_\sigma r\right)\vec{Y}_{J\nu}^{(1)*}\left(\vec{n}_r\right)\vec{Y}_{J'\nu'}^{(1)}\left(\vec{n}_r\right)\right)d\Omega\right]=$$
$$=\mathrm{Re}\left[-\frac{2\pi}{\mu}E_0^2\sum_{J=1}^{\infty}\gamma_J\sum_{\sigma=\pm1}f_{\sigma\nu}^{J}\frac{1}{k_\sigma k_\nu}i\sqrt{\varepsilon\mu-\chi^2}\left(\sigma+\nu\right)\times\right.$$
$$\left.\times\left[\sigma\hat{j}_J\left(k_\nu r\right)\hat{h}_J^{(1)\prime}\left(k_\sigma r\right)-\nu\hat{j}'_J\left(k_\nu r\right)\hat{h}_{J'}^{(1)}\left(k_\sigma r\right)\right]\right]=$$
$$=\frac{2\pi}{\mu}E_0^2\sqrt{\varepsilon\mu-\chi^2}\sum_{J=1}^{\infty}\sqrt{\gamma_J}\sum_{\sigma=\pm1}\mathrm{Re}\left[f_{\sigma\nu}^{J}\frac{1}{k_\sigma k_\nu}\left(1+\sigma\nu\right)\right].$$

Finally, for the extinction cross section we obtain

$$\sigma_{\nu}^{ext}=\frac{\Phi^{ext}}{\mathrm{I}^{n}}=$$
$$=2\pi\sum_{J=1}^{\infty}\left(2J+1\right)\sum_{\sigma=\pm1}\mathrm{Re}\left[f_{\sigma\nu}^{J}\frac{1+\sigma\nu}{k_\sigma k_\nu}\right]. \quad (2.6)$$

The absorption cross section $\sigma^{abs}=\sigma^{ext}-\sigma^{sct}$.

3 Results of numerical calculations

The results obtained for the scattering cross section (2.5) and extinction cross section (2.6) have been studied by numerical calculations performed in the Mathematica system. It should be noted that describing electromagnetic processes it is convenient to use the efficiency factor of extinction (scattering), which is defined as the cross section divided by the particle radius squared (or by πR^2). The dependence of this factor on the parameter R/λ was calculated, the frequency dispersion was not taken into account, and this corresponds to the efficiency factor dependence on the radius at the fixed frequency of the incident radiation. In order to emphasize the effects associated with the parameters α and χ we used their overestimated values.

For small values of the ratio R/λ the efficiency factor increases initially from zero to some maximum, and then shows slow oscillations about some mean value. Such oscillations are usually referred to as the interference structure (interference between incident and scattered waves). Figure 3.1 shows interference behaviour of the efficiency factor in the case when the incident wave is left circularly polarized. For the right circular polarization of the incident wave the refractive index for which ($n_\nu=\sqrt{\mu\varepsilon-\chi^2}+\nu\alpha$) is higher than for the left at the same values of the media parameters, the interference structure is changed drastically by the additional resonance structure. Consideration of a smaller area allows specifying the structure. The figure clearly shows that the additional peaks can be of two types – narrow and very narrow.

Structures of this kind in the cross sections of the atomic and nuclear processes are of particular interest. In our opinion they require special attention in case of electromagnetic scattering as well. It should be also noted that numerical calculations show the same behaviour of the scattering cross section. Moreover, in the case under consideration (non-absorbing media) the scattering cross section coincides with the extinction cross section with high accuracy.

Conclusion

In this paper the scattering and extinction cross sections of the plane circularly polarized electromagnetic wave on a biisotropic sphere embedded in another biisotropic environment are calculated on the basis of the exact solution of the Mie problem. The efficiency factor of extinction (and scattering) calculated numerically shows the interference structure on which the additional resonance structure of narrow and very narrow peaks is imposed. In our opinion the explanation of the existence of such narrow peaks is that the behaviour of individual partial terms of the cross sections has a pronounced resonant character due to the existence of the poles of the amplitudes $f_{\sigma\nu}^{J}$ (2) near the real R/λ-axis.

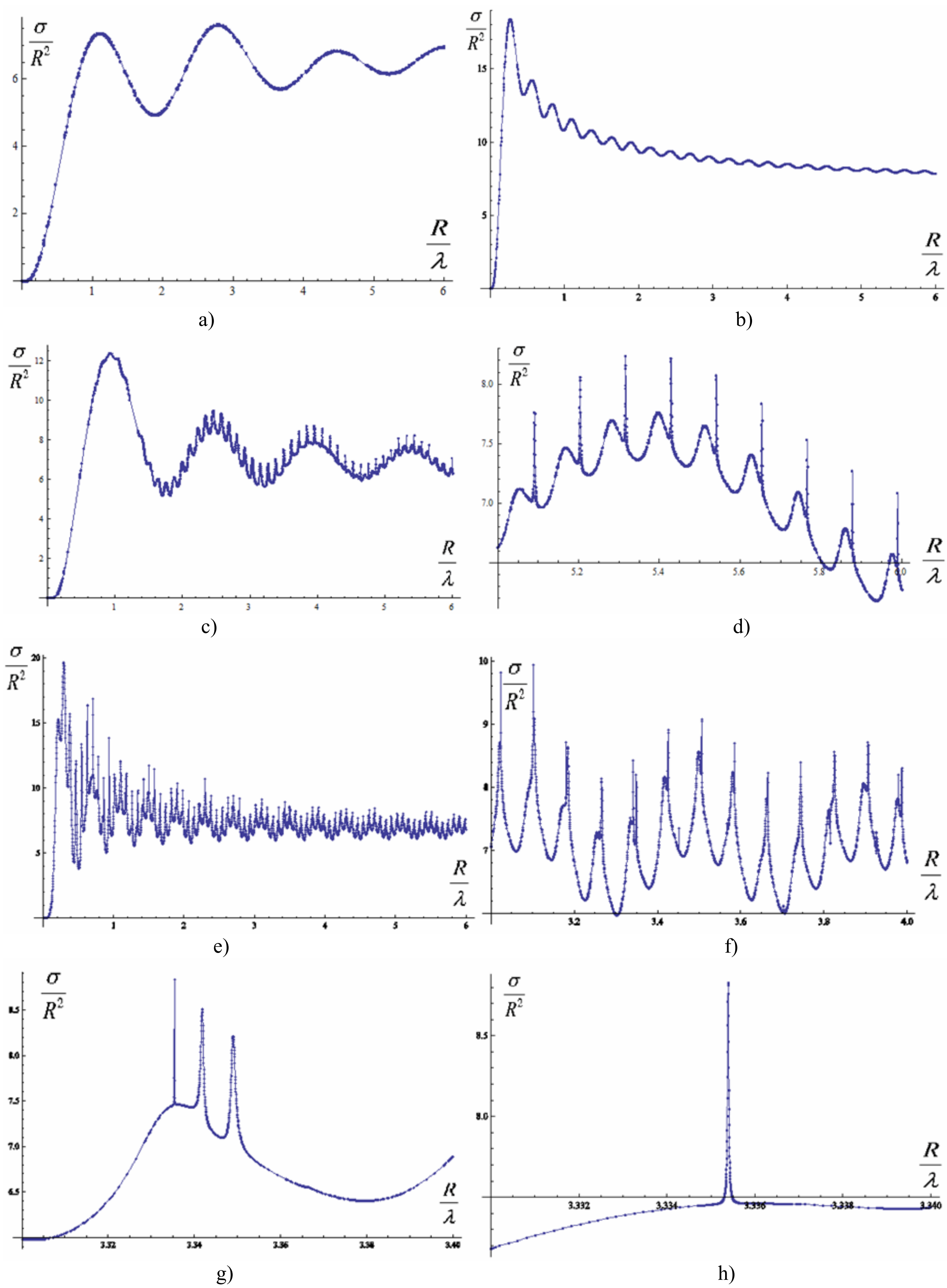

Figure 3.1 – The extinction cross section of a circularly polarized incident wave for the next parameters:
a) $\nu=-1;\ \chi=0.1;\ \varepsilon=1.1;\ \mu=1;\ \alpha=0.1$ and $\chi_1=0.1;\ \varepsilon_1=1.5;\ \mu_1=1.1;\ \alpha_1=0.4$;
b) $\nu=-1;\ \chi=0.1;\ \varepsilon=1.1;\ \mu=1;\ \alpha=0.1$ and $\chi_1=0.1,\ \varepsilon_1=1.1;\ \mu_1=1;\ \alpha_1=2$;
c), d) $\nu=1;\ \chi=0.1;\ \varepsilon=1.1;\ \mu=1;\ \alpha=0.1$ and $\chi_1=0.1;\ \varepsilon_1=1.3;\ \mu_1=1.1;\ \alpha_1=0.3$;
e), f), g), h) $\nu=1;\ \chi=0.1;\ \varepsilon=1.1;\ \mu=1;\ \alpha=0.1$ and $\chi_1=0.1;\ \varepsilon_1=2;\ \mu_1=2;\ \alpha_1=0.4$

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