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SELECTIVE AMPLIFICATION OF ELECTROMAGNETIC WAVES IN A MEDIUM WITH A ROTATING UNIAXIAL STRUCTURE

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The propagation of light in a medium with a rotating spiral structure was studied in [1]. Such a structure can be formed in a nongyrotropic crystal, exhibiting cubic nonlinearity, under the action of two circularly polarized electromagnetic waves with different frequencies. It is shown in this work that the Kerr action of these waves on the optical properties of a medium with natural gyrotropy can lead to the formation of a spatially uniform structure. The problem of propagation of electromagnetic waves in a medium with a rotating uniaxial structure is solved. It is shown that selective intensification of a circularly polarized electromagnetic wave, whose frequency and direction of rotation of polarization are identical to that of the induced uniaxial structures, is possible.

We shall study two strong circularly polarized electromagnetic waves with electric field intensities

$$\underline{\mathscr{E}}_{1,2} = \mathscr{E}_0 \mathbf{n}_{\pm} \exp\{i[K_{1,2}(\Omega_{1,2})z - \Omega_{1,2}t]\}, \tag{1}$$

which propagate collinearly to one another in a naturally gyrotropic crystal, exhibiting cubic nonlinearity. The waves (1) have the same amplitude $\mathscr{E}_0$, different frequencies Ω_1 and Ω_2, and opposite directions of circular polarization, fixed by the unit vectors $\mathbf{n}_{\pm} = (\mathbf{a} \pm i\mathbf{b})/\sqrt{2}$. The propagation of these waves in a naturally gyrotropic crystal is characterized by the wave numbers [2]

$$K_{1,2}(\Omega_{1,2}) = \frac{\Omega_{1,2}}{c}\left(\sqrt{\varepsilon_0(\Omega_{1,2})} \pm \alpha(\Omega_{1,2})\right), \tag{2}$$

where $\varepsilon_0(\Omega)$ is the dielectric constant and $\alpha(\Omega)$ is the molecular optical activity of the crystal at the frequency Ω. By choosing aoppropriate frequencies Ω_1 and Ω_2 it is possible to make the wave numbers $K_1(\Omega_1)$ and $K_2(\Omega_2)$ (2) equal. Neglecting the frequency dispersion of the parameters ε and α the condition $K_1(\Omega_1) = K_2(\Omega_2)$ has the form

$$(\Omega_2 - \Omega_1)\sqrt{\varepsilon_0} = \alpha(\Omega_1 + \Omega_2). \tag{3}$$

When the condition (3) holds, the field $\mathscr{E} = \mathscr{E}_1 + \mathscr{E}_2$, arising as a result of the superposition of the waves (1), can be represented in the form

$$\underline{\mathscr{E}} = \sqrt{2}\,\mathscr{E}_0 \exp[i(Kz - \overline{\Omega}t)]\,U(t)\,\mathbf{a}. \tag{4}$$

Here $U(t) = \exp(-\Delta\Omega t \mathbf{c}^{\times})$ is the matrix of rotation by an angle $\varphi = -\Delta\Omega t$ around the z axis (the unit vector $\mathbf{c}$); $\Delta\Omega = (\Omega_2 - \Omega_1)/2$; $\overline{\Omega} = (\Omega_1 + \Omega_2)/2$; $K = K_1(\Omega_1) = K_2(\Omega_2)$; $\mathbf{c}^{\times}$ is a completely antisymmetric tensor, dual to the vector $\mathbf{c}$; the unit vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are orthogonal and form a right-handed triad.

The optical properties manifested by the crystal with respect to a weak electromagnetic

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signal with frequency $\omega_0 \ll \bar{\Omega}$, propagating in the field of the strong waves (1), can be described with the help of the dielectric constant tensor

$$\varepsilon(t) = U(t)\,\varepsilon U^{-1}(t), \tag{5}$$

averaged over a period of the high-frequency field (4). Here $\varepsilon = \varepsilon_0 - 2\Delta\varepsilon \mathbf{a}.\mathbf{a}$ is the instantaneous dielectric constant tensor, taking into account the Kerr interaction of the field (4); $\Delta\varepsilon = -2\theta\mathscr{E}_0^2$; θ is the electrooptical coefficient; and the point between the vectors indicates a direct (dyadic) product. Thus, owing to the Kerr effect, a rotating uniaxial structure, characterized by the dielectric constant tensor (5), can appear in the crystal. The intensity of the electric field of a weak probing electromagnetic signal propagating in such a medium along the z axis satisfies the wave equation

$$\operatorname{rot}\operatorname{rot}\mathbf{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varepsilon(t)\mathbf{E} = 0, \tag{6}$$

whose solution must be sought in the form [1]:

$$\mathbf{E} = A\mathbf{n}_+ \exp[ikz - i(\omega - \Delta\Omega)t] + B\mathbf{n}_- \exp[ikz - i(\omega + \Delta\Omega)t]. \tag{7}$$

Substituting (7) into (6), we obtain a system of two algebraic linear equations

$$\begin{aligned} &\left(k^2 - \frac{\omega_2^2}{c^2}\bar{\varepsilon}(\omega_2)\right)A + \frac{\omega_2^2}{c^2}\Delta\varepsilon(\omega_1)B = 0, \\ &\frac{\omega_1^2}{c^2}\Delta\varepsilon(\omega_2)A + \left(k^2 - \frac{\omega_1^2}{c^2}\bar{\varepsilon}(\omega_1)\right)B = 0, \end{aligned} \tag{8}$$

where $\omega_{1,2} = \omega \pm \Delta\Omega$; $\bar{\varepsilon} = \varepsilon_0 - \Delta\varepsilon$. The nonzero solutions of the system of equations (8) determine the wave numbers:

$$k_{1,2}(\omega) = \left\{\frac{1}{2}\left[\frac{\omega_2^2}{c^2}\bar{\varepsilon}(\omega_2) + \frac{\omega_1^2}{c^2}\bar{\varepsilon}(\omega_1)\right] \pm \left[\frac{1}{4}\left(\frac{\omega_2^2}{c^2}\bar{\varepsilon}(\omega_2) - \frac{\omega_1^2}{c^2}\bar{\varepsilon}(\omega_1)\right)^2 + \frac{\omega_1^2\omega_2^2}{c^4}\Delta\varepsilon(\omega_1)\Delta\varepsilon(\omega_2)\right]^{1/2}\right\}^{1/2}, \quad k_{3,4} = -k_{2,1} \tag{9}$$

and the ellipticity

$$\xi_i(\omega) = \frac{A}{B} = \frac{\Delta\varepsilon(\omega_1)\,\omega_2^2}{\bar{\varepsilon}(\omega_2)\,\omega_2^2 - k_i^2(\omega)\,c^2} \tag{10}$$

of the characteristic modes of the electromagnetic field.

The values of the parameter ω, characterizing the frequencies of these modes (7), must be found from the boundary conditions. The solution of the boundary-value problem shows that the probing circularly polarized electromagnetic signal with frequency $\omega_0 \approx \Delta\Omega$ and with the field vectors rotating opposite to the direction of rotation of the vector $\mathbf{a}(t) = U(t)\mathbf{a}$ of the induced anisotropy does not interact with the strong waves (1) and only undergoes reflection from the crystal boundaries. In the case when the field vectors of the incident probing electromagnetic wave with circular polarization

$$\mathbf{E}_e = E_0\mathbf{n}_- \exp\left[i\left(\frac{\omega_0}{c}z - \omega_0 t\right)\right] \tag{11}$$

rotate in the same direction as the vector $\mathbf{a}(t)$, two characteristic electromagnetic modes are excited in the medium:

$$\mathbf{E} = \sum_{i=1}^{2} A_i\{\mathbf{n}_- e^{-i\omega_0 t} + \xi_i(\omega_0 - \Delta\Omega)\,\mathbf{n}_+ e^{-i(\omega_0 - 2\Delta\Omega)t}\}\exp(ik_i(\omega_0 - \Delta\Omega)z). \tag{12}$$

The transmitted and reflected electromagnetic waves

$$\mathbf{E}_t = E_{0t}\mathbf{n}_- \exp\left[i\left(\frac{\omega_0}{c}z - \omega_0 t\right)\right], \quad \mathbf{E}_r = E_{0r}\mathbf{n}_- \exp\left[-i\,\frac{2\Delta\Omega - \omega_0}{c}z - i(2\Delta\Omega - \omega_0)t\right] \tag{13}$$

are also circularly polarized.

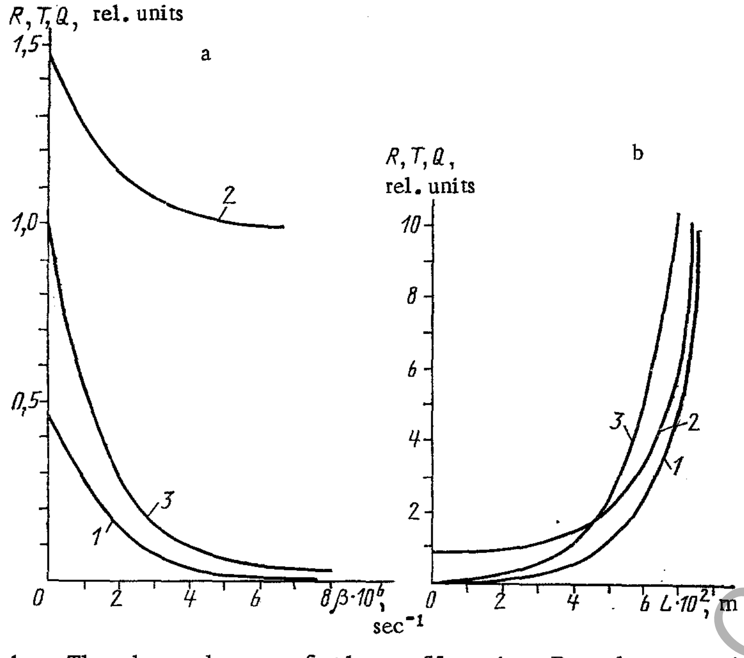

Fig. 1. The dependence of the reflection R and transmission T coefficients and the power of the sources Q on the frequency detuning β and the thickness of the crystal L: a) 1 – R(β), 2) T(β), 3) Q(β) for L = 0.036 m; b) 1 – R(L), 2) T(L), 3) Q(L) for β = 0.

We determine the amplitudes of the waves (12) and (13) from the continuity of the tangential components of the vectors **E** and **H** at the boundaries of the crystal:

$$A_1 = \frac{\xi_2(\omega_0 - \Delta\Omega) E_0}{\Delta},$$
$$A_2 = -\frac{\xi_1(\omega_0 - \Delta\Omega) E_0 \exp\{i\Delta k(\beta) L\}}{\Delta}, \tag{14}$$
$$E^*_{0r} = \frac{\xi_2(\beta)\xi_1(\beta) E_0 [1 - \exp(i\Delta k(\beta) L)]}{\Delta},$$
$$E_{0t} = \frac{[\xi_2(\beta) - \xi_1(\beta)] E_0 \exp\{i[k_1(\beta) - \omega_0/c]L\}}{\Delta},$$

where $\Delta = \xi_2(\beta) - \xi_1(\beta) \exp[i\Delta k(\beta)L]$; L is the thickness of the crystal; $\Delta k(\beta) = k_1(\beta) - k_2(\beta)$; the asterisk indicates complex conjugations; and $\beta = \omega_0 - \Delta\Omega$.

Near the resonance frequency $\Delta\Omega$ the following approximate expressions are valid for the wave numbers (9)

$$k_{1,2}(\beta) \approx \left[\bar{\varepsilon}\,\frac{\beta^2 + \Delta\Omega^2}{c^2} \pm \left(4\bar{\varepsilon}^2 \frac{\beta^2\Delta\Omega^2}{c^4} + \frac{\Delta\Omega^4}{c^4}\,\Delta\varepsilon^2\right)^{1/2}\right]^{1/2}$$

and the ellipticities (10)

$$\xi_{1,2}(\beta) \approx \frac{2\beta\Delta\Omega\bar{\varepsilon} \mp [4\bar{\varepsilon}^2(\beta^2\Delta\Omega^2 + \Delta\Omega^4\Delta\varepsilon^2)]^{1/2}}{\Delta\Omega^2\Delta\varepsilon},$$

taking which into account, together with (14), we find the coefficient of reflection R and transmission T of the electromagnetic wave (11):

$$R = \frac{\Delta\varepsilon^2\Delta\Omega^2 \sin^2[\Delta k(\beta) L/2]}{4\bar{\varepsilon}^2\beta^2 + \Delta\varepsilon^2\Delta\Omega^2\cos^2[\Delta k(\beta) L/2]}, \tag{15}$$

$$T = \frac{\Delta\varepsilon^2\Delta\Omega^2 + 4\bar{\varepsilon}^2\beta^2}{4\bar{\varepsilon}^2\beta^2 + \Delta\varepsilon^2\Delta\Omega^2\cos^2[\Delta k(\beta) L/2]}. \tag{16}$$

The relations (15) and (16) show that energy exchange between the strong waves (1), forming in the medium a rotating uniaxial structure, and the weak electromagnetic signal (11) is possible. In the process selective intensification of the circularly polarized probe signal (11), whose frequency and direction of rotation of the polarization vector are identical to those of the induced uniaxial structure, can occur. In this case the power of the sources of the electromagnetic field is determined by the expression

$$Q = \frac{2\Delta\varepsilon^2\Delta\Omega^2 \sin^2[\Delta k(\beta) L/2]}{4\bar{\varepsilon}^2\beta^2 + \Delta\varepsilon^2\Delta\Omega^2\cos^2[\Delta k(\beta) L/2]}.$$

For $\bar{\Omega} \sim 10^{15}$ sec^{-1}, $\alpha \sim 10^{-3}$ [3, 4], $\theta \sim 10^{-19}$ m^2/V^2 [5] and intensities of the interacting waves ~10^{14} W/m^2 [4, 5], we have $\Delta\varepsilon \sim 10^{-2}$, $\Delta\Omega \sim 10^{12}$ sec^{-1}. Graphs of the dependence of the reflection R and transmission T coefficients as well as of the power of the sources Q on the frequency detuning β and the thickness of the crystal L for the above-indicated parameters are presented in Fig. 1.

The results show that the intensification of electromagnetic waves in a medium with a rotating uniaxial structure, examined above, can be employed to reverse the wavefront in the microwave and far-IR regions.

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