AN OPEN QUEUEING NETWORK WITH PARTLY NON-ACTIVE CUSTOMERS.

J. Bojarovich*, **Yu. Malinkovsky**
Gomel State University named after Skorina
Gomel, Belarus
* juls1982@list.ru

An open queueing network with partly non-active customers is considered. Non-active customers are in a system queue and do not get service. Customers may turn from non-active condition into condition, when they may get their service and vice versa. The form of stationary distribution and criterion of stationary distribution existence are obtained.

Keywords: partly non-active customers, reversibility, ergodicity, traffic equations, stationary distribution.

1. INTRODUCTION

Nowadays queueing networks with partly non-active customers become actual to a marked degree. Non-active customers are in a system queue and do not get service. Results for queueing networks with partly non-active customers may be used for epidemic progress analyzing and prognosis.

In paper [1] G. Tsitsiashvili and M. Osipova observed an open queueing network with non-active customers and established the form of stationary distribution.

This paper generalizes results for network from [1]. We consider network, where customers may partly loose their capacity for service. Customers may turn from non-active condition into condition, when they may get their service and vice versa. We researched the form of stationary distribution and established the criterion of stationary distribution existence.

2. SYSTEM DESCRIPTION

Consider isolated system. Positive and negative customers arrive at the system according to a Poisson processes at rates λ and λ^- accordingly. When arriving at system positive customer queues up and needs service. A negative customer induces a positive customer, if any, to leave the system immediately. A negative customer does not produce any action if there are no positive customers at the system. There is input Poisson flow of non-active customers at rate θ. Non-active customers queues up and can not get service. There are three input Poisson flows of signals at rates ν, ψ and φ. When arriving at the system the signal at rate ν induces an ordinary customer, if any, to become non-active. When arriving at the system the signal at rate φ induces a non-active customer, if any, to become an ordinary. A signal at rate ψ induces a non-active customer, if any, to leave the system immediately. Signals do not need service. The service time has exponential distribution with the parameter μ.

Let $n(t)$ and $n'(t)$ are numbers of ordinary and non-active customers at the system at time t accordingly. $X(t) = (n(t), n'(t))$ is a continuous-time Marcov chain with a space of states $X = \{(n, n'),\ n, n' = 0, 1, \ldots\}$.

Under the condition of stationary distribution existence stationary probabilities $\pi(n, n')$ satisfy global balance equations:

1) $n = 0,\ n' = 0$:

$$(\lambda + \theta)\pi(n, n') = (\mu + \lambda^-)\pi(n + 1, n') + \psi\pi(n, n' + 1)\ ;$$

2) $n > 0,\ n' = 0$:

$$(\lambda + \mu + \lambda^- + \nu + \theta)\pi(n, n') = \lambda\pi(n - 1, n') + (\mu + \lambda^-)\pi(n + 1, n') +$$

$$+\varphi\pi(n - 1, n' + 1) + \psi\pi(n, n' + 1)\ ;$$

3) $n = 0,\ n' > 0$:

$$(\lambda + \varphi + \theta + \psi)\pi(n, n') = (\mu + \lambda^-)\pi(n + 1, n') + \nu\pi(n + 1, n' - 1) +$$

$$+\theta\pi(n, n' - 1) + \psi\pi(n, n' + 1)\ ;$$

4) $n > 0,\ n' > 0$:

$$(\lambda + \mu + \lambda^- + \nu + \varphi + \theta + \psi)\pi(n, n') = \lambda\pi(n - 1, n') + (\mu + \lambda^-)\pi(n + 1, n') +$$

$$+\nu\pi(n + 1, n' - 1) + \varphi\pi(n - 1, n' + 1) + \theta\pi(n, n' - 1) + \psi\pi(n, n' + 1)\ ;$$

Lemma. *Condition of reversibility is:*

$$\lambda\nu\psi = \theta(\mu + \lambda^-)\varphi.$$

Theorem 1. *If inequalities*

$$\lambda < \mu + \lambda^-,\quad \theta < \psi,$$

hold, hence Marcov process $X(t)$ is ergodic. And under the condition of reversibility $X(t)$ has stationary distribution:

$$\pi(n, n') = \Big(1 - \frac{\lambda}{\mu + \lambda^-}\Big)\Big(1 - \frac{\theta}{\psi}\Big)\Big(\frac{\lambda}{\mu + \lambda^-}\Big)^n\Big(\frac{\theta}{\psi}\Big)^{n'}.$$

3. NETWORK DESCRIPTION

Consider open queueing network with set of systems $J = \{1, 2, \dots, N\}$. Positive and negative customers arrive at the network according to a Poisson processes at rates λ_i and λ_i^- accordingly, $i \in J$. Non-active customers arrive at the network according to a Poisson processes at rates θ_i, $i \in J$. Non-active customers queue up to corresponding system and can not get service. There are input Poisson flows of signals at rates ν_i, φ_i и ψ_i, $i \in J$. When arriving at the system $i \in J$ the signal at rate ν_i induces an ordinary customer at system, if any, to become non-active. When arriving at the system $i \in J$ the signal at rate φ_i induces an non-active customer, if any, to become an ordinary. A signal at rate ψ_i induces an non-active customer, if any, to leave the system $i \in J$ immediately. Signals do not need service. Service times are independent exponentially distributed random values with parameters μ_i, $i \in J$.

After finishing of service process at system $i \in J$ customer is routed to system $j \in J$ with the probability $p_{i,j}$ as a positive customer, with the probability $p_{i,j}^-$ as a negative customer, with the probability $q_{i,j}$ as a non-active customer and with the probability $p_{i,0}$ is removed from network ($\sum_{j=1}^{N}(p_{i,j} + p_{i,j}^- + q_{i,j}) + p_{i,0} = 1$), $i \in J$. Let $p_{i,i} = p_{i,i}^- = q_{i,i} = 0$, $i \in J$.

Let $n_i(t), n_i'(t)$ are numbers of ordinary and non-active customers at system $i \in J$ at time t. Consider $X(t) = \Big((n_1(t), n_1'(t)), \dots, (n_N(t), n_N'(t))\Big)$. $X(t)$ is a continuous-time Markov chain.

A traffic equations system is:

$$\varepsilon_i = \lambda_i + \sum_{j=1}^{N} \frac{\varepsilon_j}{\varepsilon_j^- + \mu_j} p_{j,i},$$

$$\varepsilon_i^- = \lambda_i^- + \sum_{j=1}^{N} \frac{\varepsilon_j}{\varepsilon_j^- + \mu_j} p_{j,i}^-,$$

$$\omega_i = \theta_i + \sum_{j=1}^{N} \frac{\varepsilon_j}{\varepsilon_j^- + \mu_j} q_{j,i}, \quad i \in J.$$

One can prove that under certain conditions traffic equations system has unique non-trivial solution.

Under the condition of stationary distribution existence stationary probabilities $\pi(n, n')$ ($n = (n_1, \dots, n_N), n' = (n_1', \dots, n_N')$) satisfy global balance equations:

$$\pi(n, n')\Big[\sum_{i \in J} \Big(\lambda_i + \mu_i I_{n_i \neq 0} + \lambda_i^- I_{n_i \neq 0} + \nu_i I_{n_i \neq 0} +$$

$$+\varphi_i I_{n_i' \neq 0} + \theta_i + \psi_i I_{n_i' \neq 0}\Big)\Big] =$$

$$= \sum_{i \in J} \Big[\pi(n - e_i, n')\lambda_i I_{n_i \neq 0} + \pi(n + e_i, n')\mu_i p_{i,0} + \pi(n + e_i, n')\lambda_i^- +$$

$$+\pi(n+e_i,n'-e_i)\nu_i I_{n_i'\neq 0}+\pi(n-e_i,n'+e_i)\varphi_i I_{n_i\neq 0}+$$
$$+\pi(n,n'-e_i)\theta_i I_{n_i'\neq 0}+\pi(n,n'+e_i)\psi_i+$$
$$+\sum_{j\in J}\Big(\pi(n+e_i-e_j,n')\mu_i p_{i,j} I_{n_j\neq 0}+\pi(n+e_i+e_j,n')\mu_i p^-_{i,j}+$$
$$+\pi(n+e_i,n'-e_j)\mu_i q_{i,j} I_{n_j'\neq 0}\Big)\Big].$$

Theorem 2. *Under conditions of ergodicity:*

$$\varepsilon_i<\mu_i+\varepsilon_i^-,$$

$$\omega_i<\psi_i,\ i=1,\ldots,N,$$

under conditions of reversibility:

$$\frac{\varepsilon_i}{\varepsilon_i^-+\mu_i}\frac{\nu_i}{\varphi_i}=\frac{\omega_i}{\psi_i},\ i\in J,$$

$X(t)$ has stationary distribution:

$$\pi(n,n')=\pi_1(n_1,n_1')\pi_2(n_2,n_2')\ldots\pi_N(n_N,n_N'),$$

where

$$\pi_i(n_i,n_i')=\Big(1-\frac{\varepsilon_i}{\mu_i+\varepsilon_i^-}\Big)\Big(1-\frac{\omega_i}{\psi_i}\Big)\Big(\frac{\varepsilon_i}{\mu_i+\varepsilon_i^-}\Big)^{n_i}\Big(\frac{\omega_i}{\psi_i}\Big)^{n_i'},$$

here ε_i, ε_i^-, ω_i, $i\in J$ – is a solution of a traffic equations system.

REFERENCES

1. *Tsitsiashvili G., Osipova M.* Queueing models with different schemes of customers transformations // Queues: flows, systems, networks (BWWQT - 2007): proceedings of the Int. Conf. "Mathematical Methods for Increasing Efficiency of Information Telecommunication Networks". 2007. № 9. P. 128–133.
2. *Кравченко С. В.* Открытые сети с информационными сигналами // Известия Гомельского государственного университета им. Ф. Скорины. 2002. V. 15. № 6. P. 175–178.
3. *Малинковский Ю. В.* Критерий представимости стационарного распределения состояний открытой марковской сети обслуживания с несколькими классами заявок в форме произведения // Автоматика и телемеханика. 2002. № 4. P. 75–83.
4. *Якубович О. В.* Стационарное распределение сети массового обслуживания с различными типами отрицательных и положительных заявок и ограничением на время ожидания // Известия Гомельского государственного университета им. Ф. Скорины. 2007. V. 45. № 6. P. 198–202.