STABILIZATION OF DYNAMICAL SYSTEMS WITH THE HELP OF OPTIMIZATION METHODS

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Abstract: Schemes of realization of bounded stabilizing feedbacks based on optimal control methods and linear programming are under consideration. Both approaches use the principle of correction of current solutions in real–time mode. The methods elaborated are used for robust stabilization and stabilization under additional conditions on transients, such as degree of stability, degree of oscillation, degree of overcontrol, monotonicity of transients. The result are illustrated by nontrivial examples. *Copyright ©1998 IFAC*

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1. INTRODUCTION

Stabilization problem is one of the important problems of the control theory (Aizerman, 1958; Malkin, 1966). With the growth of dimensions of stabilized systems and the rise of demands to quality of transient processes optimal control methods (Pontryagin, *et al.*, 1983) together with modern computers have been becoming powerfull tools while stabilizing dynamical systems. The first results in this directions were obtained in the theory of linear–quadratic problems of optimal control with infinite horizon (Kalman, 1961; Letov, 1960) and later on with a finite horizon (Kwon and Pearson, 1977; Mayne and Michalska, 1990). Both (Kalman, 1961; Letov, 1960) and (Kwon and Pearson, 1977; Mayne and Michalska, 1990) deal with explicit form of positional solutions of linear–quadratic problems of optimal control. It had been a consequence of "no" restrictions on controls because of what problems in questions did not leave the frame of calculus of variations. In the papers (Gabasov, *et al.*, 1991; Gabasov, *et al.*, 1992) methods of optimal control which take into account geometrical restrictions on control functions and allows to consruct bounded stabilizing feedbacks are suggested. As the construction of bounded stabilizing feedback in explicit form represent an especially difficult problem, in (Gabasov, *et al.*, 1991) methods of realization of positional solutions of optimal control problems by modern computer tools are justified. This approach was developed for stabilization of dynamic systems in (Gabasov, *et al.*, 1992). In the paper we at first describe principles of constructing bounded stabilizing feedbacks for dynamic systems. Then we investigate problems of robust stabilization and dynamic systems stabilization with additional properties of transients.

The main features of the approach under considereation consist in the following: 1) structure of feedbacks is not given beforehand; 2) stabilizing controls are restricted; 3) auxiliary constrained optimization problems are introduced to be solved for the initial position of the dynamic system; 4) principle of continuous correcting current solutions is used during a finite period of time.

2. PROBLEM STATEMENT

Consider dynamical system on the interval $t \geq 0$

$$\dot{x} = Ax + bu \ (x \in R^n, \ u \in R) \qquad (1)$$

where A, b a given matrix and a vector.

Let G be a vicinity of $x = 0$ of (1) ($u = 0$), L, $0 \leq L < \infty$ be a given number.

A function

$$u = u(x), \ x \in G \qquad (2)$$

is said to be a bounded stabilizing feedback for (1) if: 1) $u(0) = 0$; 2) $|u(x)| \leq L$, $x \in G$; 3) the equation

$$\dot{x} = Ax + bu(x) \qquad (3)$$

obtained after closing (1) by (2) has a solution $x(t)$, $t \geq 0$, for all $x(0) \in G$; 4) the trivial solution $x(t) = 0$, $t \geq 0$, of (3) is asymptotically stable in G.

The problem consists in constructing such function (2) for which the domain of attraction G of asymptotically stable solution $x(t)$, $t \geq 0$, of equation (3) are close to maximum one. The construction of $u(x)$, $x \in G$ in explicit form is a very difficult problem. By analogy with (Gabasov, *et al.*, 1992) one can inrtoduce a new statement of the problem which is based on the use of modern computer technology.

We assume that a bounded stabilizing feedback (2) has been constructed. Consider the behavior $x^*(t)$, $t \geq 0$, of closed system (3) in some particular process of stabilization having started from $x^*(0) = x_0^*$, $x^* \in G$:

$$\dot{x}^*(t) = Ax^*(t) + bu(x^*(t)), \ x^*(0) = x_0^*. \qquad (4)$$

According to (4) in this particular process the control

$$u^*(t) = u(x^*(t)), \ t \geq 0 \qquad (5)$$

is fed to input of the dynamic system, i.e. in a particular process of stabilization feedback (2) is not used as a whole. It is needed only its meaning along isolated continuous curve $x^*(t)$, $t \geq 0$. This values is supposed to be used not beforhand but at every current moment $\tau \geq 0$.

Function (5) is said to be a realization of the feedback for a particular process of stabilization. A device which is capable to calculate its values in the course of stabilization process is said to be Stabilizer.

Thuth, the problem of stabilization is reduced to the construction for Stabilizer an real–time algorithm.

The basic idea of the stated approach consists in introduction auxiliary (accompanying) problem of optimal control, in construction, following (Gabasov, *et al.*, 1991), an algorithm of work of an optimal regulator and in the proof that the optimal regulator appears to be Stabilizer of dynamic system (1).

3. STABILIZATION OF DYNAMICAL SYSTEM USING A MINIMUM INTENSITY CONTROL PROBLEM

Assume that system (1) is controllable:

$$\text{rank}(b, Ab, \dots, A^{n-1}b) = n.$$

Choose a positive number Θ (parameter of a method) and in the class of piecewise continuous functions $u(t)$, $t \in T = [0, \Theta]$, consider the accompanying optimal control problem

$$\rho(z) = \min_u \max_{t \in T} |u(t)|,$$

$$\dot{x} = Ax + bu, \ x(0) = z, \ (z \in R^n) \qquad (6)$$

$$x(\Theta) = 0, \ |u(t)| \leq \rho, \ t \in T.$$

Designate: $u^0(t|z)$, $t \in T$, is optimal open loop control (6) for an initial condition $z \in R^n$

$$u^0(t|z) = \rho(z)\text{sign}\Delta^0(t), \ z \in R^n,$$

where $\Delta^0(t) = \Delta^0(t|z) = \psi'(t)b = -y'(z)F(\vartheta - t)b$ is the optimal cocontrol, $y = y(z)$ is the optimal n-vector of potentials, $F(t)$ is the fundamental matrix of solutions to system $\dot{x} = Ax$; $\rho = y'F(\Theta)z$ is the optimal intensity of control. $G(\Theta)$ is the set of all condition z, for which problem (6) has a solution.

A function

$$u^0(z) = u^0(0|z), \ z \in G(\Theta), \qquad (7)$$

is said to be an optimal start feedback for (6).

It can be proved that the function $u^0(x) = u^0(0|x)$, $x \in R^n$, is the bounded stabilizing feedback for system (1) (Balashevich, *et al.*, 1994).

Two ways of realization of the bounded stabilizing feedback are possible:

1) "continuous", at which in the optimal open loop control a finite set of defining elements (structure of optimal control $u^0(t|x^*(\tau))$, $t \in T$) is extracted, and for it the system of defining equations is introduced that for problem (6) has the form:

$$\Delta^0(t_i(\tau) + 0) = 0, \ x(\Theta) = 0$$

or

$$\sum_{i=0}^{p} \int_{t_i(\tau)}^{t_{i+1}(\tau)} F(\Theta - t)bdtk_i\rho(\tau) + F(\Theta)x^*(\tau) = 0$$

$$-y_\tau' F(\Theta - t_i(\tau))b = 0, \ i = \overline{1,p} \qquad (8)$$

$$y'_r(\tau) = F(\Theta)x^*(\tau) = \rho(x^*(\tau)) = \rho(\tau))$$
$$t \in [0,\Theta),\ t_0(\tau) = 0,\ t_{p+1}(\tau) = \Theta,$$
$$k_i = \text{sign}\Delta^0(t_i(\tau)+0),$$

where $t_i(\tau), i = 1,\dots,p$ (point of switching of the optimal open loop control), $y(\tau)$ (optimal n-vector of potentials), $\rho(\tau)$ (optimal intensity of the control) are unknowns under rather general conditions. The Jacobi matrix for (8) is nonsingular. An algorithm of solution of (8) (Gabasov, *et al.*, 1992), allowing in the course of process to calculate realizations $u^*(\tau)$, $\tau \geq 0$, of the bounded stabilizing feedback (7) is described.

2) a "discrete" way, at which the accompanying problem is considered in the class piecewise constant controls with the constant period of quantization $h > 0$, $h = \Theta/N$: $N < +\infty$; $u(t) = u_k$, $t \in [kh], k+1)h[$, $k = 0,1,\dots,N-1$. The optimal open loop control $u^0(t|x^*(kh))$, $t = 0,h,\dots,(N-1)h$, in the current moment $\tau = kh$ is under construction by a dual method of linear programming by correction of the control $u^0(t|x^*((k-1)h)0,h,\dots,(N-1)h)$, constructed on the previous step $\tau - h = (k-1)h$. At each of the mentioned ways used methods of correction allow quickly to calculate values $u^*(\tau)$, $\tau \geq 0$.

Using the discrete way of realization of the bounded stabilizing feedback, robust stabilization problem and stabilization of dynamic systems with of additional properties of transients are solved below.

4. ROBUST STABILIZATION OF DYNAMIC SYSTEM WITH THE HELP OF BOUNDED CONTROLS

Assume that the accessible information on parameters A, b of system (1) inexact: $n \times n$-matrix A and n-vector b are those that

$$A = A_0 + \Delta A,\ b = b_0 + \Delta b,$$

where A_0, b_0 are an known $n \times n$-matrix and an n-vector accordingly, ΔA, Δb are unknowns satisfying the inequalities:

$$||\Delta A|| \leq \alpha,\ ||\Delta b|| \leq \beta\ (\alpha,\ \beta > 0).$$

At a given $\varepsilon > 0$ and a fixed numbers $\nu > 0$, $L > 0$, a function

$$u(t,x),\ x \in G,\ t \in [0,\nu[, \qquad (9)$$

is said to be a robust bounded stabilizing open-closed loop control of system (1) in G if 1) $u(t,0) = 0$, $t \in [0,\nu[$; 2) $|u(t,x)| \leq L$, $x \in G$, $t \in [0,\nu[$; 3) the trajectory of the closed system

$$\dot{x} = Ax + bu(t,x),\ x(0) = x_0,\ x_0 \in G, \qquad (10)$$

is a continuous solution of the equation $\dot{x} = Ax + bu(t)$, $x(0) = x_0$, at $u(t) = u(t - k\nu, x(k\nu))$, $t \in [k\nu,(k+1)\nu[$, $k = 0,1,\dots$; 4) system (10) at $A \equiv A_0$, $b \equiv b_0$ is assymototically stable in G; 5)there exists a finite number $t(\varepsilon) > 0$, such that every solution $x(t)$, $t \geq 0$, of system (10) satisfies the condition $||x(t)|| \leq \varepsilon$, $t \geq t(\varepsilon)$.

Choose natural numbers $N, m (N > m > n)$, a real number $h > 0$. Assume $\nu = mh, \Theta = Nh$.

In the class piecewise constant functions $u(t)$, $t \in T = [0,\Theta]$, satisfying the restriction $|u(t)| \leq L$, $t \in T$, we shall consider the accompanying problem of optimal control:

$$\rho(z) = \min \rho,$$
$$\dot{x} = A_0x + b_0u,\ x(0) = z,\ (z \in R^n) \qquad (11)$$
$$x(\Theta) = 0,\ |u(t)| \leq \rho,\ t \in T.$$

Optimal start open–closed loop control is defined by the equality

$$u^0(t,z) = u^0(t|z),\ t \in [0,\nu[,\ z \in R^n.$$

Introduce the set
$G_\Theta = \{z \in R^n : |u^0(t,z)| \leq L,\ t \in [0,\nu]\}$.

For any $\varepsilon > 0$ there exists such $\Theta > 0$ that an ε-vicinity of G_Θ contains all states of (1) which can be transfered to $x = 0$ for a finite time. Using the Lyapunov function method (Barbashin, 1967; Bromberg, 1967) it is possible to show that at given ε, G, α, β and an appropriate choice of parameters $\Theta > 0$, $\nu > 0$, $h > 0$ of problem (11) the feedback $u(t,x) = u^0(t,x)$, $x \in G_\Theta$, $t \in [0,\nu[$, will satisfy all requirements of definition of the robust bounded stabilizing open-closed loop control with $G = G_\Theta$. As the Lyapunov function one can take the optimal values of the criterion of quality $\rho(z), z \in G_\Theta$, of problem (11).

Consider stabilization problem of a mathematical pendulum in the upper unstable state of equilibrium (Malkin, 1966). The mathematical model of such system has the form

$$\dot{x}_1 = x_2,\ \dot{x}_2 = x_1 + x_3,\ \dot{x}_3 = u \qquad (12)$$

where x_1 is the angle of deviation of the pendulum from the vertical, x_2 is the angular speed of the pendulum, x_3 is the moment enclosed to the pendulum.

Let at the initial moment $t = 0$ system (12) is in the condition $x_1(0) = 0.3$, $x_2(0) = 1.0$, $x_3(0) = -1.2$. It is required to stabilize it in the upper vertical state $x_1 = 0$, $x_2 = 0$, $x_3 = 0$.

The accompanying problem is

$$\rho \to \min$$
$$\dot{x}_1 = x_2,\ \dot{x}_2 = x_1 + x_3,\ \dot{x}_3 = u$$

$$x_1(0) = x_1^*(\tau),\ x_2(0) = x_2^*(\tau),\ x_3(0) = x_3^*(\tau)$$
$$x_1(\Theta) = 0,\ x_2(\Theta) = 0,\ x_3(\Theta) = 0$$
$$|u(t)| \le \rho,\ t \in T = [0, \Theta]$$

where $x^*(\tau) = (x_1^*(\tau), x_2^*(\tau), x_3^*(\tau))$ is the condition of system (12) in the current moment τ.

For the solution of the accompanying problem the following parameters were chosen: $\Theta = 1$, $h = 0.025$, $\nu = 5h$.

During work of Stabilizer the coefficient at x_3 is changed (Fig. 1) (curves 1 corresponds to the value $1x_3$, the curves 2, 3 correspond to the values $0.5x_3$ and $1.5x_3$).

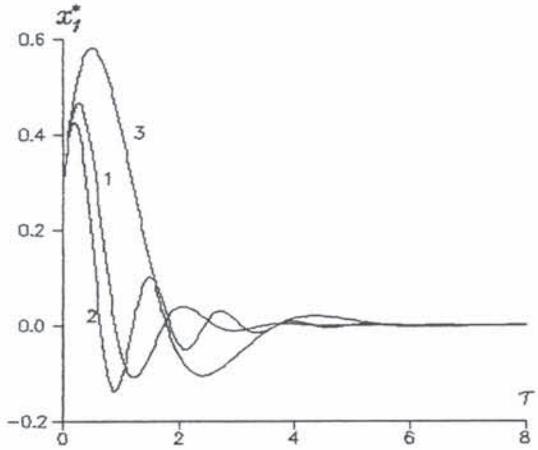

Fig. 1. The trajectory of system (12).

5. STABILIZING FEEDBACKS PROVIDING GIVEN PROPERTIES OF TRANSIENT PROCESSES

Consider system (1). Let $y = Hx$, $H \in R^{m \times n}$, an m-vector of output. Introduce the sets

$$Y(t) = \{y \in R^m : g_*(t) \le y \le g^*(t)\},\ t \ge 0,$$

where $g_*(t)$, $g^*(t)$, $-\infty < g_*(t) \le g^*(t) < \infty$, $t \ge 0$, are given continuous m-vector functions.

Definition 1. At fixed numbers $\nu > 0, h > 0, L > 0$ a function

$$u_s(t, x),\ s \in [0, \nu[,\ t \ge 0,\ x \in G, \qquad (13)$$

is said to be an bounded stabilizing open-closed loop control of (1) in G, if 1) $u_s(t, 0) = 0$, $s \in [0, \nu[$, $t \ge 0$; 2) $|u_s(t, x)| \le L$, $s \in [0, \nu[$, $t \ge 0$, $x \in G$; 3) a trajectory of the closed system

$$\dot{x} = Ax + bu_s(t, x),\ x(0) = x_0,\ x_0 \in G, \qquad (14)$$

is a continuous solution of the equation

$$\dot{x} = Ax + bu(t),\ x(0) = x_0,$$

at $u(t) = u_s(k\nu, x(k\nu))$, $s \in [0, \nu[$, $t \in [k\nu, (k+1)\nu[$, $k = 0, 1, \ldots$; 4) system (14) is assymototically stable in G.

Definition 2. A bounded stabilizing open-closed control (13) is said to be a stabilizing open-closed loop control with the property A if the output signal $y(t)$, $t \ge 0$, of system (14) is contained in set $Y(t)$, $t \ge 0$, where $g_*(t) = -a\exp(-\alpha t)$, $g^*(t) = a\exp(-\alpha t)$, $\alpha > 0$. The number $\alpha > 0$ is called degree of stability of a transient.

For construction of the bounded stabilizing feedback with the property A two accompanying problems were considered.

The first problem is the optimal control problem

$$B(\tau, z) = \min \int_0^\Theta |u(t)| dt,$$
$$\dot{x} = Ax + bu,\ x(0) = z,\ x(\Theta) = 0, \qquad (15)$$
$$g_*(t+\tau) \le Hx(t) \le g^*(t+\tau),$$
$$|u(t)| \le 1,\ t \in T = [0, \Theta].$$

Let $G(\Theta, \tau)$ is the set of all $z \in R^n$ for which problem (15) has a solution.

An optimal start open-slosed loop control is defined by the equality

$$u_s^0(\tau, z) = u^0(s|\tau, z), s \in T_h, z \in G(\Theta, \tau), \tau \in R_\nu.$$

By the Lyapunov function method it is possible to show that at given $g_*(t)$, $g^*(t)$, $t \ge 0$, and appropriate choice of parameters $\Theta > 0$, $\nu > 0$, $h > 0$ of problem (15) the feedback $u_s(t, x) = u_s^0(t, x)$, $s \in T_h$, $x \in G(\Theta, t)$, $t \in R_\nu$, will satisfy all requirements of definition of bounded stabilizing open-closed loop controls with the property A. As the Lyapunov function we shall take optimal values of the criterion of quality of problem (15).

Illustrate the obtained results on an example of stabilization of oscillatory system (Sussmann, *et al.*, 1994)

$$\dot{x}_1 = x_2,\ \dot{x}_2 = -x_1 + x_3, \qquad (16)$$
$$\dot{x}_3 = x_4,\ \dot{x}_4 = -x_3 + u,$$

where (x_1, x_2, x_3, x_4) is a state of system (16), u is a control.

In the work Sussmann'a, *et al.*, 1994 the bounded stabilization feedback:

$$u = -\varepsilon \mathrm{sat}\left(\frac{x_4}{\varepsilon} + \frac{1}{29}\mathrm{sat}\left(\frac{29}{\varepsilon}(-x_1 + x_3 + x_4)\right)\right),$$
$$|u(t)| \le \varepsilon,\ \mathrm{sat}(s) = \mathrm{sign}(s)\min\{|s|, 1\} \qquad (17)$$

is constructed.

For comparison of results we at first shall construct the bounded stabilizing feedback by the above method for the case when the restriction are not imposed on the output $y(t)$, $t \ge 0$. The accompanying problem of optimal control in this case is taken as

$$\int_0^\Theta |u(t)|dt \to \min,$$

$$\dot{x}_1 = x_2,\ \dot{x}_2 = -x_1 + x_3,\ \dot{x}_3 = x_4,\ \dot{x}_4 = -x_3 + u,$$
$$x_1(0) = x_1^*(\tau),\ x_2(0) = x_2^*(\tau), \qquad (18)$$
$$x_3(0) = x_3^*(\tau),\ x_4(0) = x_4^*(\tau),$$
$$x_1(\Theta) = 0,\ x_2(\Theta) = 0,\ x_3(\Theta) = 0,\ x_4(\Theta) = 0,$$
$$|u(t)| \le 1,\ t \in T = [0, \Theta].$$

It was solved at the following values of parameters: $\Theta = 8$, $N = 25$, $\nu = 0.32$. As an initial condition the vector $x_0^* = (0.1, 0.1, 0.1, 0.1)$ was taken.

In Fig. 2 the transients of system (16) are given with feedback (17) at $\varepsilon = 1$ (curve 1) and with the feedback constructed on accompanying problem (18) (curve 2).

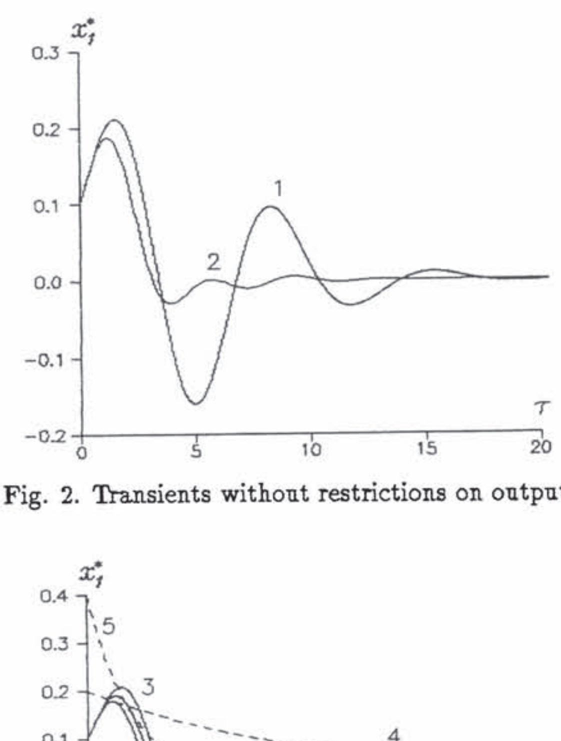

Fig. 2. Transients without restrictions on output.

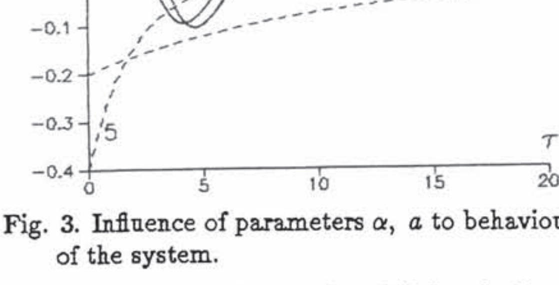

Fig. 3. Influence of parameters α, a to behaviour of the system.

Now construct the bounded stabilizing feedback for system (16) in view of restrictions on output. As the output we shall take $y(t) = x_1(t), t \ge 0$. Assume $g_*(t) = -a\exp(-\alpha t), g^*(t) = a\exp(-\alpha t)$, $t \ge 0$.

The accompanying problem of optimal control:

$$\int_0^\Theta |u(t)|dt \to \min,$$

$$\dot{x}_1 = x_2,\ \dot{x}_2 = -x_1 + x_3,\ \dot{x}_3 = x_4,\ \dot{x}_4 = -x_3 + u,$$
$$x_1(0) = x_1^*(\tau),\ x_2(0) = x_2^*(\tau), \qquad (19)$$
$$x_3(0) = x_3^*(\tau),\ x_4(0) = x_4^*(\tau),$$
$$x_1(\Theta) = 0,\ x_2(\Theta) = 0,\ x_3(\Theta) = 0,\ x_4(\Theta) = 0,$$
$$-a\exp(-\alpha t) \le x_1(t) \le a\exp(-\alpha t),$$
$$|u(t)| \le 1,\ t \in T = [0, \Theta].$$

At the solution of problem (19) the following paramaters were chosen: $\Theta = 8, h = 0.4, \nu = 5h, x_0^* = (0.1, 0.1, 0.1, 0.1)$. In Fig. 3 the behaviour of the output $y(\tau) = x_1^*(\tau)$, $\tau \ge 0$, for various values α, a have been shown. The curve 1 corresponds to $\alpha = 0.1, a = 0.2$, the curve 2 stands for $\alpha = 0.5, a = 0.4$. For comparison the transient process is shown when on the output $y(t) = x_1(t)$, $t \ge 0$, the restrictions (curve 3) are not imposed. The dotted lines stand for the case of restrictions on values of output. The curves 4, 5 correspond to $g_*(t) = -0.2\exp(-0.1t)$, $g^*(t) = 0.2\exp(-0.1t)$; $g_*(t) = -0.4\exp(-0.5t)$, $g^*(t) = 0.4\exp(-0.5t)$ respectively.

The second problem in question has been the accompanying problem of minimization of intensity of control:

$$\rho \to \min,$$
$$\dot{x}_1 = x_2,\ \dot{x}_2 = -x_1 + x_3,\ \dot{x}_3 = x_4,\ \dot{x}_4 = -x_3 + u,$$
$$x_1(0) = x_1^*(\tau),\ x_2(0) = x_2^*(\tau),$$
$$x_3(0) = x_3^*(\tau),\ x_4(0) = x_4^*(\tau), \qquad (20)$$
$$x_1(\Theta) = 0,\ x_2(\Theta) = 0,\ x_3(\Theta) = 0,\ x_4(\Theta) = 0,$$
$$-a\exp(-\alpha t) \le x_1(t) \le a\exp(-\alpha t),$$
$$|u(t)| \le \rho,\ t \in T = [0, \Theta].$$

At the solution of problem (20) the following parameters were chosen: $\Theta = 2$, $h = 0.08$, $\nu = 5h$, $\alpha = 0.1$, $a = 0.11$.

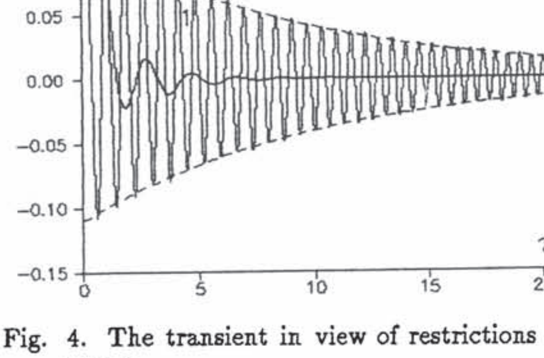

Fig. 4. The transient in view of restrictions on output.

In Fig. 4. the results of calculations are presented. The curve 1 corresponds to the solution of (20). The dotted lines show restrictions on a output. For comparison, the transitive process is shown when on output of the system the restrictions (curve 2) are not imposed.

Definition 3. A bounded stabilizing open–closed control (13) is said to be a stabilizing open–closed loop control with the property B if derivatives of the output $y(t)$, $t \geq 0$, corresponding to the solution $x(t)$, $t \geq 0$, of systems (14), change values in given moments of time $t = k/q$, $k = 1, 2, \ldots$. The number q is degree of oscillation.

For system (16) one can construct the stabilizing open–closed control with the property B ($y = x_1$, $\dot{y} = x_2$).

Let l be an integer of periods of constant signs of length $1/q$ on the interval $[0, \Theta]$, $N_1 = \{1, 3, 5, 7, \ldots\}$, $N_2 = \{2, 4, 6, 8, \ldots\}$.

The accompanying problem of optimal control has the form

$$\rho \to \min,$$
$$\dot{x}_1 = x_2,\ \dot{x}_2 = -x_1 + x_3,\ \dot{x}_3 = x_4,\ \dot{x}_4 = -x_3 + u,$$
$$x_1(0) = x_1^*(\tau),\ x_2(0) = x_2^*(\tau),$$
$$x_3(0) = x_3^*(\tau),\ x_4(0) = x_4^*(\tau), \qquad (21)$$
$$x_1(\Theta) = 0,\ x_2(\Theta) = 0,\ x_3(\Theta) = 0,\ x_4(\Theta) = 0,$$
$$|u(t)| \leq \rho,\ t \in T.$$

Let $\Theta < 1/q$.

If $(k-1)/q \leq \tau \leq k/q - \Theta$, $k \in N_1$, then $x_2(t) \geq 0$, $t \in T = [0, \Theta]$;

if $(k-1)/q \leq \tau \leq k/q - \Theta$, $k \in N_2$, then $x_2(t) \leq 0$, $t \in T$;

if $k/q - \Theta < \tau < k/q + \Theta$, $k \in N_1$, then $x_2(t) \geq 0$, $t \in [0, k/q - \tau]$, $x_2(t) \leq 0$, $t \in [k/q - \tau, \Theta]$;

if $k/q - \Theta < \tau < k/q + \Theta$, $k \in N_2$, then $x_2(t) \leq 0$, $t \in [0, k/q - \tau]$, $x_2(t) \geq 0$, $t \in [k/q - \tau, \Theta]$.

Let $\Theta \geq 1/q$.

If $\tau = (k-1)/q$, $k \in N_1$, then $x_2(t) \geq 0$,

$$t \in T_1 = \bigcup_{s=0,2,4,\ldots,l-1} [s/q, (s+1)/q] \bigcup_{l-even} [l/q, \Theta],$$

$x_2(t) \leq 0$,

$$t \in T_2 = \bigcup_{s=1,3,5,\ldots,l-1} [s/q, (s+1)/q] \bigcup_{l-odd} [l/q, \Theta];$$

if $\tau = (k-1)/q$, $k \in N_2$, then $x_2(t) \leq 0$, $t \in T_1$, $x_2(t) \leq 0$, $t \in T_2$;

if $(k-1)/q < \tau < k/q$, $k \in N_1$, then $x_2(t) \geq 0$,

$$t \in T_3 = [0, k/q - \tau] \bigcup_{s=1,3,5,\ldots,l-1} [k/q - \tau + s/q,$$
$$k/q - \tau + (s+1)/q] \bigcup_{l-even} [(l-1)/q + k/q - \tau, \Theta],$$

$x_2(t) \leq 0$,

$$t \in T_4 = \bigcup_{s=0,2,4,\ldots,l-1} [k/q-\tau+s/q, k/q-\tau+(s+1)/q]$$
$$\bigcup_{l-odd} [(l-1)/q + k/q - \tau, \Theta];$$

if $(k-1)/q < \tau < k/q$, $k \in N_2$, then $x_2(t) \leq 0$, $t \in T_3$, $x_2(t) \geq 0$, $t \in T_4$.

At the solution of (21) the following values of parameters were chosen: $\Theta = 2$, $h = 0.1$, $\nu = 5h$. As a initial condition the vector $x_0^* = (-0.1, 0.1, 0.1, 0.1)$ was taken.

In Fig. 5 the trajectories for three processes of stabilization are presented corresponding to various degrees of oscillation: 1) on the interval $[0, \Theta]$ there are two periods of constant signs of speed x_2 (curves 1); 2) on an interval $[0, \Theta]$ there are four periods of constants signs of speed x_2 (curves 2); 3) the period of constant signs of speed x_2 is equal to Θ (curves 3).

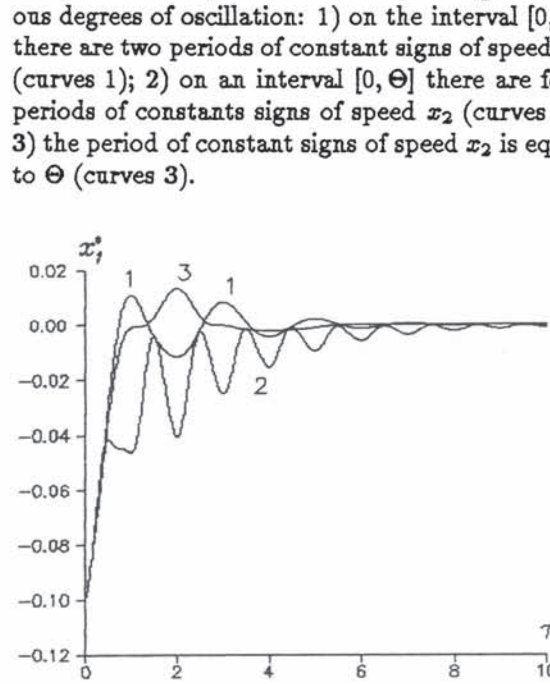

Fig. 5. The transients at various degrees of oscillation.

Definition 4. A bounded stabilizing open–closed control (13) is said to be a stabilizing open–closed loop control with the property C if the output $y(t)$, $t \geq 0$, of systems (14) satisfies conditions: $y_i(t) \geq p_i$, $t \geq 0$, at $y_i(0) > 0$, $p_i \leq 0$; $y_i(t) \leq p_i$, $t \geq 0$, at $y_i(0) < 0$, $p_i \geq 0$, $i = \overline{1, m}$. The numbers p_i, $i = \overline{1, m}$, refer to as degrees of overcontrol. If $p_i = 0$, $i = \overline{1, m}$, then in the closed system is no any overcontrol.

Construct the stabilizing open–close control without overcontrol ($p_1 = 0$) for system (16) with the output $y(t) = x_1(t)$, $t \geq 0$.

The accompanying problem has the form

$$\rho \to \min,$$
$$\dot{x}_1 = x_2,\ \dot{x}_2 = -x_1 + x_3,\ \dot{x}_3 = x_4,\ \dot{x}_4 = -x_3 + u,$$

$$x_1(0) = x_1^*(\tau),\ x_2(0) = x_2^*(\tau),$$
$$x_3(0) = x_3^*(\tau),\ x_4(0) = x_4^*(\tau), \quad (22)$$
$$x_1(\Theta) = 0,\ x_2(\Theta) = 0,\ x_3(\Theta) = 0,\ x_4(\Theta) = 0,$$
$$x_1(t) \ge 0,\ |u(t)| \le \rho,\ t \in T = [0, \Theta].$$

For the solution of (22) the following values of parameters were chosen: $\Theta = 2$, $h = 0.08$, $\nu = 5h$. As an initial condition the vector $x_0^* = (0.1, 0.1, 0.1, 0.1)$ was taken.

In Fig. 6. the output is given for the process of stabilization (curve 1).

Definition 5. A bounded stabilizing open–closed control (13) is said to be a stabilizing open–closed loop control with the property D if the derivative of the output $y(t)$, $t \ge 0$, of systems (14) keep constant signs on an interval $t \ge 0$.

Let construct for system (16) a stabilizing open–closed loop control with property D (the output $y = x_1$). The accompanying problem

$$\rho \to \min,$$
$$\dot{x}_1 = x_2,\ \dot{x}_2 = -x_1 + x_3,\ \dot{x}_3 = x_4,\ \dot{x}_4 = -x_3 + u,$$
$$x_1(0) = x_1^*(\tau),\ x_2(0) = x_2^*(\tau),$$
$$x_3(0) = x_3^*(\tau),\ x_4(0) = x_4^*(\tau), \quad (23)$$
$$x_1(\Theta) = 0,\ x_2(\Theta) = 0,\ x_3(\Theta) = 0,\ x_4(\Theta) = 0,$$
$$x_2(t) \le 0,\ |u(t)| \le \rho,\ t \in T = [0, \Theta].$$

As an initial condition the vector $x_0^* = (0.1, -0.1, 0.1, 0.1)$ was taken. The following values of parameters of problem (23) were taken: $\Theta = 2$, $h = 0.08$, $\nu = 5h$. In Fig. 6 the output given (curve 2).

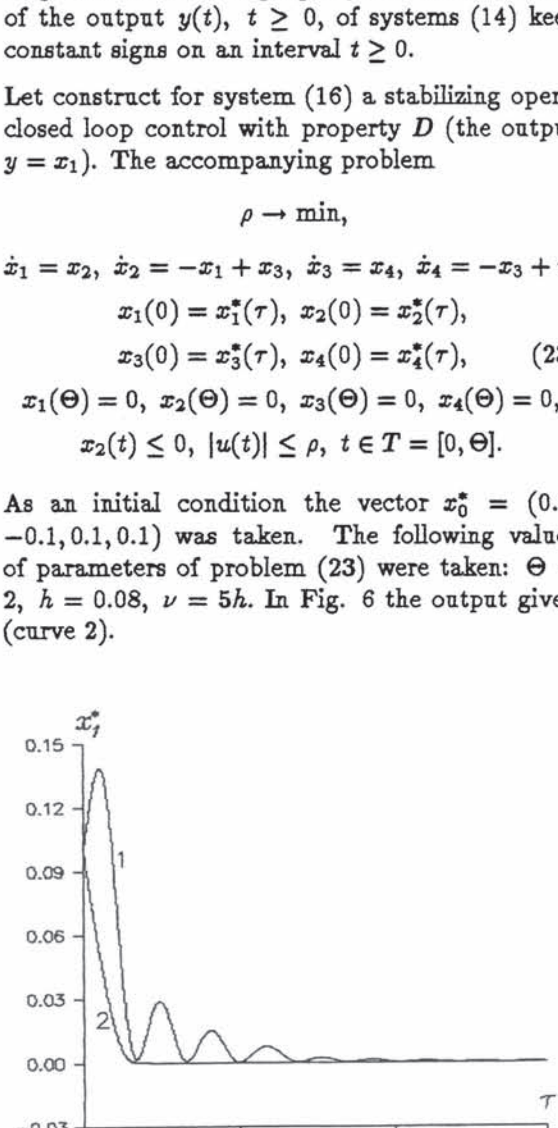

Fig. 6. Transient of "no" overcontrol; monotone transient.

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