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ON P-NONINVARIANT WAVE EQUATION FOR SPIN 1/2 PARTICLE WITH ANOMALOUS MAGNETIC MOMENT, INTERACTION WITH EXTERNAL FIELDS

Within the theory of relativistic wave equations with extended sets of Lorentz group representations, a new P-noninvariant 20-component wave equation for spin 1/2 particle is proposed. The presence of external electromagnetic field and Riemannian space-time background are taken into account. Due to internal structure of the particle, additional interaction terms appear, they correspond to additional characteristic-anomalous magnetic moment.

1. Gel’fand-Yaglom Basis

The goal of the paper is to construct a new *P*-noninvariant wave equation for a massive spin 1/2 particle. We apply general theory of relativistic wave equations with extended sets of representation of the Lorentz group [1-4]. We start with the following set of irreducible representations (this is 20-component theory)

$$T = (0,1/2) \oplus (1/2,0) \oplus (0,1/2)' \oplus (1/2,0)' \oplus (1,1/2) \oplus (1/2,1),$$

where the "prime" serves to distinguish repeated representations of the Lorentz group. The matrix Γ_4 of corresponding wave equation has the following structure $\Gamma_4 = (C^{(1/2)} \otimes I_2) \oplus (C^{(3/2)} \otimes I_4)$, where $C^{(1/2)}$, $C^{(3/2)}$ represent spin-blocks related to spins 1/2 and 3/2. With the use of the numeration of irreducible components

$$(0,1/2) \,\square\, 1, \quad (0,1/2)' \,\square\, 2, \quad (1,1/2) \,\square\, 3,$$
$$(1/2,0) \,\square\, 4, \quad (1/2,0)' \,\square\, 5, \quad (1/2,1) \,\square\, 6,$$

the blocks $C^{(1/2)}$ and $C^{(3/2)}$ are given by the formulas

$$C^{(\frac{1}{2})} = \begin{vmatrix} 0 & 0 & 0 & c_{14}^{(\frac{1}{2})} & c_{15}^{(\frac{1}{2})} & c_{16}^{(\frac{1}{2})} \\ 0 & 0 & 0 & c_{24}^{(\frac{1}{2})} & c_{25}^{(\frac{1}{2})} & c_{26}^{(\frac{1}{2})} \\ 0 & 0 & 0 & c_{34}^{(\frac{1}{2})} & c_{35}^{(\frac{1}{2})} & c_{36}^{(\frac{1}{2})} \\ c_{41}^{(\frac{1}{2})} & c_{42}^{(\frac{1}{2})} & c_{43}^{(\frac{1}{2})} & 0 & 0 & 0 \\ c_{51}^{(\frac{1}{2})} & c_{52}^{(\frac{1}{2})} & c_{53}^{(\frac{1}{2})} & 0 & 0 & 0 \\ c_{61}^{(\frac{1}{2})} & c_{62}^{(\frac{1}{2})} & c_{63}^{(\frac{1}{2})} & 0 & 0 & 0 \end{vmatrix}, \quad C^{(\frac{3}{2})} = \begin{vmatrix} 0 & c_{36}^{(\frac{3}{2})} \\ c_{63}^{(\frac{3}{2})} & 0 \end{vmatrix}. \qquad (1)$$

From invariance of the equation under proper Lorentz group follow the constraints

$$c_{36}^{(\frac{3}{2})} = 2c_{36}^{(\frac{1}{2})}, \quad c_{63}^{(\frac{3}{2})} = 2c_{63}^{(\frac{1}{2})}.$$

Besides, without loss of generality, the links between repeated components may be broken:

$$c_{15}^{(\frac{1}{2})} = c_{51}^{(\frac{1}{2})} = c_{24}^{(\frac{1}{2})} = c_{42}^{(\frac{1}{2})} = 0.$$

Because, we wish to construct the model of a particle with single spin $1/2$, we require that eigenvalues of the block $C^{(3/2)}$ be equal to zero. Therefore, we set $c_{36}^{(\frac{3}{2})} = c_{63}^{(\frac{3}{2})} = 0$, whence it follows $c_{36}^{(\frac{1}{2})} = c_{63}^{(\frac{1}{2})} = 0$.

2. Modified Gel'fand-Yaglom basis

Let us find the form of the matrix Γ_4 in so-called modified Gel'fand-Yaglom basis. Here the listing of basis elements of the complete wave function ψ is slightly different. We find the following decomposition for spin block $C^{(1/2)}$:

$$C^{(1/2)} = \frac{1}{2} \begin{vmatrix} (c_{14}^{(\frac{1}{2})} + c_{41}^{(\frac{1}{2})}) & 0 & (c_{16}^{(\frac{1}{2})} + c_{43}^{(\frac{1}{2})}) \\ 0 & (c_{25}^{(\frac{1}{2})} + c_{52}^{(\frac{1}{2})}) & (c_{26}^{(\frac{1}{2})} + c_{53}^{(\frac{1}{2})}) \\ (c_{34}^{(\frac{1}{2})} + c_{61}^{(\frac{1}{2})}) & (c_{35}^{(\frac{1}{2})} + c_{62}^{(\frac{1}{2})}) & 0 \end{vmatrix} \otimes \gamma_4 +$$

$$+ \frac{1}{2} \begin{vmatrix} (c_{14}^{(\frac{1}{2})} - c_{41}^{(\frac{1}{2})}) & 0 & (c_{16}^{(\frac{1}{2})} - c_{43}^{(\frac{1}{2})}) \\ 0 & (c_{25}^{(\frac{1}{2})} - c_{52}^{(\frac{1}{2})}) & (c_{26}^{(\frac{1}{2})} - c_{53}^{(\frac{1}{2})}) \\ (c_{34}^{(\frac{1}{2})} - c_{61}^{(\frac{1}{2})}) & (c_{35}^{(\frac{1}{2})} - c_{62}^{(\frac{1}{2})}) & 0 \end{vmatrix} \otimes \gamma_5 \gamma_4; \qquad (2)$$

here the first term corresponds to purely P-invariant model, the second term relates to purely P-noninvariant model. We will we restrict ourselves to the second variant of the theory.

It is convenient to employ shortening notations, then the spin block $C^{(1/2)}$ reads

$$C^{(1/2)} = \begin{vmatrix} a_1 & 0 & a_2 \\ 0 & a_3 & a_4 \\ a_5 & a_6 & 0 \end{vmatrix} \otimes \gamma_4 + \begin{vmatrix} ib_1 & 0 & ib_2 \\ 0 & ib_3 & ib_4 \\ ib_5 & ib_5 & 0 \end{vmatrix} \otimes \gamma_5 \gamma_4 . \quad (3)$$

For purely P-noninvariant model it becomes simpler

$$C^{(1/2)} = i \begin{vmatrix} b_1 & 0 & b_2 \\ 0 & b_3 & b_4 \\ b_5 & b_5 & 0 \end{vmatrix} \otimes \gamma_5 \gamma_4 . \quad (4)$$

We create the model for particle with one mass, so the matrix

$$\begin{vmatrix} b_1 & 0 & b_2 \\ 0 & b_3 & b_4 \\ b_5 & b_5 & 0 \end{vmatrix}$$

must have only one non-vanishing eigenvalue. In accordance with this, parameters b_i obey restrictions

$$b_1 + b_3 = 1, \quad b_1 b_3 - b_2 b_5 - b_4 b_6 = 0, \quad b_2 b_3 b_5 + b_1 b_4 b_6 = 0. \quad (5)$$

3. Spinor form of the wave equation

After some technical manipulations we derive the set of spinor equations (where $\partial_{a\dot{b}} = \frac{1}{i} \partial_\mu \sigma^\mu_{a\dot{b}}$)

$$i \left\{ b_1 \partial^{\dot{a}b} \Psi_b + \sqrt{\frac{2}{3}} b_2 \partial^b_{\dot{c}} \Psi^{(\dot{a}\dot{c})}_b \right\} + M \Psi^{\dot{a}} = 0,$$

$$-i \left\{ b_1 \partial_{a\dot{b}} \Psi^{\dot{b}} + \sqrt{\frac{2}{3}} b_2 \partial^c_{\dot{b}} \Psi^{\dot{b}}_{(ac)} \right\} + M \Psi_a = 0,$$

$$i \left\{ b_3 \partial^{\dot{a}b} \Psi'_b + \sqrt{\frac{2}{3}} b_4 \partial^b_{\dot{c}} \Psi^{(\dot{a}\dot{c})}_b \right\} + M \Psi'^{\dot{a}} = 0,$$

$$-i \left\{ b_3 \partial_{a\dot{b}} \Psi'^{\dot{b}} + \sqrt{\frac{2}{3}} b_4 \partial^c_{\dot{b}} \Psi^{\dot{b}}_{(ac)} \right\} + M \Psi'_a = 0,$$

$$-\frac{i}{\sqrt{6}} b_5 (\partial^{\dot{c}}_a \Psi_b + \partial^{\dot{c}}_b \Psi_a) - \frac{i}{\sqrt{6}} b_6 (\partial^{\dot{c}}_a \Psi'_b + \partial^{\dot{c}}_b \Psi'_a) + M \Psi^{\dot{c}}_{(ab)} = 0,$$

$$\frac{i}{\sqrt{6}} b_5 (\partial^{\dot{a}}_c \Psi^{\dot{b}} + \partial^{\dot{b}}_c \Psi^{\dot{a}}) + \frac{i}{\sqrt{6}} b_6 (\partial^{\dot{a}}_c \Psi'^{\dot{b}} + \partial^{\dot{b}}_c \Psi'^{\dot{a}}) + M \Psi^{(\dot{a}\dot{b})}_c = 0. \quad (6)$$

4. Equations in spin-tensor form

After additional work we arrive at the spin-tensor system (where $\hat{\partial} = \partial_\mu \gamma_\mu$)

$$i\gamma_5 \left\{ b_1 \hat{\partial}(\gamma_\mu \Psi_\mu) - \frac{4b_2}{\sqrt{6}} \left[-\frac{1}{4} \hat{\partial}(\gamma_\mu \Psi_\mu) + (\partial_\mu \Psi_\mu) \right] \right\} + M(\gamma_\mu \Psi_\mu) = 0,$$

$$i\gamma_5 \left\{ b_3 \hat{\partial} \Psi_0 - i\frac{4b_4}{\sqrt{6}} \left[(\partial_\mu \Psi_\mu) - \frac{1}{4} \hat{\partial}(\gamma_\mu \Psi_\mu) \right] \right\} + M\Psi_0 = 0,$$

$$\frac{2i}{\sqrt{6}} \gamma_5 \left\{ b_5 \left[\partial_\lambda (\gamma_\mu \Psi_\mu) - \frac{1}{4} \gamma_\lambda \hat{\partial}(\gamma_\mu \Psi_\mu) \right] - \right.$$

$$\left. -ib_6 \left[\partial_\lambda \Psi_0 - \frac{1}{4} \gamma_\lambda \hat{\partial} \Psi_0 \right] + M\{\Psi_\lambda - \frac{1}{4} \gamma_\lambda (\gamma_\mu \Psi_\mu) \right\} = 0. \quad (7)$$

5. Reducing the system to minimal equation

In absence of external fields, the main component of the wave function is

$$\Phi(x) = b_5 \gamma_\mu \Psi_\mu(x) - ib_6 \Psi_0(x). \quad (8)$$

The main bispinor $\Phi(x)$ satisfies modified Dirac–like P-noninvariant equation

$$\left\{ i\gamma_5 (\gamma_\mu \partial_\nu) + M \right\} \Phi(x) = 0. \quad (9)$$

Concomitant bispinors may be constructed by the rules

$$\gamma_\mu \Psi_\mu(x) = \frac{b_1^2}{b_5 (b_1^2 - b_3^2)} \Phi(x), \quad \Psi_0(x) = \frac{-ib_3^2}{b_6 (b_1^2 - b_3^2)} \Phi(x). \quad (10)$$

6. The minimal equation in presence of electromagnetic field

In presence of electromagnetic fields, the main component is the same

$$\Phi(x) = b_5 \gamma_\mu \Psi_\mu(x) - ib_6 \Psi_0(x).$$

This component obeys the following P-noninvariant equation for a particle with anomalous magnetic moment

$$\left\{ i\gamma^5 \gamma_\mu (\partial_\mu + ieA_\nu) - \frac{4b_1 b_3}{M} ieF_{\mu\nu} \frac{\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu}{4} + M \right\} \Psi = 0. \quad (11)$$

Expressions for $\Psi = 0$ and $(\gamma_\mu \Psi_\mu)$ are

$$(\gamma_\mu \Psi_\mu) = \frac{b_1^2}{b_5 (b_1^2 - b_3^2)} \left\{ 1 + \frac{4}{3} \left(\frac{b_1 b_3}{M} \right)^2 ieF_{\mu\nu} \frac{\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu}{4} \right\} \Phi,$$

$$\Phi_0 = -i\frac{b_3^2}{b_6(b_1^2 - b_3^2)}\left\{1 + \frac{4}{3}\left(\frac{b_1 b_3}{M}\right)^2 ieF_{\mu\nu}\frac{\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu}{4}\right\}\Phi. \quad (12)$$

7. Extension of the model to General relativity

In order to follow extension of the model from flat Minkowski space to any Riemannian space-time we should turn back and make several simple modifications.

1. In Riemannian space we use the metric $g_{\alpha\beta}(x)$, related to signature $(+,-,-,-)$, we must make the change:

$$M \dashrightarrow iM. \quad (13)$$

2. Now Dirac matrices in spinor basis are

$$\gamma^0 = \begin{vmatrix} 0 & I \\ I & 0 \end{vmatrix}, \quad \gamma^i = \begin{vmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{vmatrix}. \quad (14)$$

3. Derivatives are modified according to the rules

$$D_\alpha(x) = \nabla_\alpha + \Gamma_\alpha(x) + ieA_\alpha(x), \quad \hat{D} = \gamma^\alpha(x) D_\alpha(x),$$

where $\Gamma_\alpha(x)$ is bispinor connection, and $\gamma^\alpha(x) = \gamma^a e^\alpha_{(a)}(x)$.

4. Note important commutation rules

$$\hat{D}(x) = \gamma^\rho(x) D_\beta = D_\beta \gamma^\rho(x), \quad D_\sigma(x) g_{\alpha\beta}(x) = g_{\alpha\beta}(x) D_\sigma(x),$$

$$\hat{D}\hat{D} = D^\alpha D_\alpha - \Sigma(x), \quad \Sigma(x) = -ieF_{\alpha\beta}\sigma^{\alpha\beta}(x) + \frac{R}{4},$$

where $R(x)$ is the Ricci scalar.

5. Note the notations

$$\gamma^5(x) = \frac{i}{4!}\varepsilon_{\alpha\beta\rho\sigma}(x)\gamma^\alpha(x)\gamma^\beta(x)\gamma^\rho(x)\gamma^\sigma(x),$$

$$\varepsilon^{\alpha\beta\rho\sigma}(x) = \varepsilon^{abcd}\; e^\alpha_{(a)}(x)\, e^\beta_{(b)}(x)\, e^\rho_{(c)}(x)\, e^\sigma_{(d)}(x), \quad \varepsilon_{0123} = -1.$$

Levi-Civita object $\varepsilon^{\alpha\beta\rho\sigma}(x)$ changes under tetrad transformations as follows

$$\varepsilon'^{\alpha\beta\rho\sigma}(x) = \det[L_a^{\;b}(x)]\, \varepsilon^{\alpha\beta\rho\sigma}(x).$$

In particular, at the tetrad P–reflection, it transforms as a tetrad pseudoscalar

$$\varepsilon^{(p)\alpha\beta\rho\sigma}(x) = (-1)\, \varepsilon^{\alpha\beta\rho\sigma}(x).$$

The above analysis for the generally covariant system remains in fact the same. We can write down final result without repeating the calculation.

$$\Phi = b_5(\gamma_\mu \Psi_\mu) - ib_6\Psi_0,$$

$$\left\{ i\gamma^5(x)\hat{D}(x) - \frac{4b_1b_3}{M}\left[-ieF_{\mu\nu}\sigma^{\mu\nu}(x) + \frac{R(x)}{4}\right] + iM \right\}\Phi = 0. \quad (15)$$

Expressions for concomitant components are given the formulas

$$\gamma^{\mu}(x)\Psi_{\mu}(x) = \frac{b_1^2}{b_5(b_1^2 - b_3^2)}\left\{1 - \frac{4}{3}\left(\frac{b_1b_3}{iM}\right)^2\left(-ieF_{\mu\nu}\sigma^{\mu\nu} + \frac{R(x)}{4}\right)\right\}\Phi,$$

$$\Psi_0(x) = -i\frac{b_3^2}{b_6(b_1^2 - b_3^2)}\left\{1 - \frac{4}{3}\left(\frac{b_1b_3}{iM}\right)^2\left(-ieF_{\mu\nu}\sigma^{\mu\nu} + \frac{R(x)}{4}\right)\right\}\Phi. \quad (16)$$

Conclusions

This theory gives a *P*-noninvariant model for spin 1/2 particle with anomalous magnetic moment.

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THE ENERGY-MOMENTUM TENSOR FOR A SPIN 1/2 PARTICLE TAKING INTO ACCOUNT POLARIZABILITIES

Introduction

Interaction of the electromagnetic field with a structural particles in the electrodynamic of hadrons is based on main principles of the relativistic quantum field theory. In the model conceptions where basically the dia-