$$\left\{ i\gamma^5(x)\hat{D}(x) - \frac{4b_1b_3}{M}\left[-ieF_{\mu\nu}\sigma^{\mu\nu}(x) + \frac{R(x)}{4} \right] + iM \right\}\Phi = 0. \quad (15)$$

Expressions for concomitant components are given the formulas

$$\gamma^{\mu}(x)\Psi_{\mu}(x) = \frac{b_1^2}{b_5(b_1^2 - b_3^2)}\left\{ 1 - \frac{4}{3}\left(\frac{b_1b_3}{iM}\right)^2 \left(-ieF_{\mu\nu}\sigma^{\mu\nu} + \frac{R(x)}{4} \right) \right\}\Phi,$$

$$\Psi_0(x) = -i\frac{b_3^2}{b_6(b_1^2 - b_3^2)}\left\{ 1 - \frac{4}{3}\left(\frac{b_1b_3}{iM}\right)^2 \left(-ieF_{\mu\nu}\sigma^{\mu\nu} + \frac{R(x)}{4} \right) \right\}\Phi. \quad (16)$$

Conclusions

This theory gives a P-noninvariant model for spin 1/2 particle with anomalous magnetic moment.

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THE ENERGY-MOMENTUM TENSOR FOR A SPIN 1/2 PARTICLE TAKING INTO ACCOUNT POLARIZABILITIES

Introduction

Interaction of the electromagnetic field with a structural particles in the electrodynamic of hadrons is based on main principles of the relativistic quantum field theory. In the model conceptions where basically the diagram technique is used a number of features for interaction of photons with hadrons have been determined [1, 2]. However, the diagram technique is mainly employed for a description of electromagnetic processes on a simplest quark systems. In the case of interaction for the electromagnetic field with complex quark-gluon systems in the low-energy region perturbative methods of QCD are nonapplicable. That is why the low-energy theorems and sum rules are widely used lately [3-6].

In the present time the low-energy electromagnetic characteristics which connect with hadron structure, such as formfactor and polarizabilities, it is possible to obtain from nonrelativistic theory [5]. Passing on from the nonrelativistic electrodynamics to the relativistic field theory one can make use the correspondence principle. But it is necessary step by step to investigate a transition from the covariant Lagrangian formalism to the Hamiltonian one [7-9].

This work is a continuation of the researches which have been presented in the our previous articles [6-8]. Using the covariant Lagrangian of interaction of the electromagnetic field with a structural polarizable particle, the equations of motion, canonical and metric energy-momentum tensors have been obtained.

1. Total Lagrangian

The total interaction Lagrangian of the spin-1/2 particles with the electromagnetic field will be consists from the Lagrangian for free electromagnetic field L_{e-m}, the spinor or Dirac's field L_D, the interaction Lagrangian of the free electromagnetic field with the Dirac's field L_{int-D} and the Lagrangian which considers electric and magnetic polarizabilities of particles $L_{\alpha_0\beta_0-D}$:

$$L_{total-D} = -\frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} + \overline{\psi}(\frac{1}{2}i\gamma_{\alpha}\overleftrightarrow{\partial}^{\alpha} + m)\psi - e(\overline{\psi}\gamma_{\alpha}\psi)A^{\alpha} + K_{\sigma\nu}\Theta^{\sigma\nu}, \qquad (1)$$

where $K_{\sigma\nu} = \frac{2\pi}{m}\left(\alpha_0 F_{\sigma\mu}F^{\mu}_{\nu} + \beta_0 \tilde{F}_{\sigma\mu}\tilde{F}^{\mu}_{\nu}\right)$, $\overleftrightarrow{\partial}_{\nu} = \overleftarrow{\partial}_{\nu} - \overrightarrow{\partial}_{\nu}$, $\Theta^{\sigma\nu} = \frac{i}{2}\left(\overline{\psi}\gamma^{\sigma}\overleftrightarrow{\partial}^{\nu}\psi\right)$, ψ is the wave function of spin-1/2 particles. In this expression $\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, where $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ are the tensors of the electromagnetic field, α_0 and β_0 are electric and magnetic polarizabilities , $\varepsilon_{\mu\nu\rho\sigma}$ – Levi-Chevita antisymmetric tensor ($\varepsilon^{0123} = 1$). The part of the Lagrangian with polarizabilities it is possible to rewrite as:

$$L^{(\alpha\beta)} = -\frac{1}{4}F_{\mu\nu}G^{\mu\nu} = K_{\sigma\nu}\Theta^{\sigma\nu}, \qquad (2)$$

where $G^{\mu\nu}$ is antisymmetric tensor $G^{\mu\nu} = -G^{\nu\mu}$ and equal

$$G^{\mu\nu} = -\frac{\partial L^{(\alpha\beta)}}{\partial(\partial_\mu A_\nu)} = \frac{4\pi}{m}((\alpha_0 + \beta_0)(F^\mu_\rho \Theta^{\rho\nu} - F^\nu_\rho \Theta^{\rho\mu}) - \beta_0 \Theta^\rho_\rho F^{\mu\nu}). \qquad (3)$$

2. The equations of motion

For interaction of the spinor and electromagnetic fields the next system of equations are used:

$$-\frac{\partial L}{\partial A_\mu} + \partial_\gamma \frac{\partial L}{\partial(\partial_\gamma A_\mu)} = 0, \qquad (4)$$

$$-\frac{\partial L}{\partial \overline{\psi}} + \partial_\gamma \frac{\partial L}{\partial(\partial_\gamma \overline{\psi})} = 0, \qquad (5)$$

$$-\frac{\partial L}{\partial \psi} + \partial_\gamma \frac{\partial L}{\partial(\partial_\gamma \psi)} = 0, \qquad (6)$$

where A_μ is the vector-potential of the electromagnetic field.

From the Lagrangian (1) and expressions (4-6) we will be have the equations of motion for charged particle spin-1/2 α_0-electric and β_0-magnetic polarizabilities taking into account:

$$\partial_\mu F^{\mu\nu} = e\overline{\psi}\gamma^\nu \psi - \partial_\mu G^{\mu\nu}, \qquad (7)$$

$$(i\gamma^\nu \overrightarrow{\partial}_\nu - m)\psi = eA_\nu \gamma^\nu \psi - \frac{i}{2}(\partial^\nu K_{\sigma\nu}\gamma^\sigma)\psi - iK_{\sigma\nu}\gamma^\sigma \partial^\nu \psi, \qquad (8)$$

$$\overline{\psi}(i\overleftarrow{\partial}_\nu \gamma^\nu + m) = -e\overline{\psi} A_\nu \gamma^\nu - \frac{i}{2}\overline{\psi}(\partial^\nu K_{\sigma\nu}\gamma^\sigma) - i(\partial^\nu \overline{\psi})\gamma^\sigma K_{\sigma\nu}. \qquad (9)$$

Using the equations (7-8) the Lagrangian (1) can be presented as the next formula:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\overline{\psi}(i\overrightarrow{D} - m)\psi - \frac{1}{2}\overline{\psi}(i\overleftarrow{D} + m)\psi, \qquad (10)$$

where

$$\overrightarrow{D} = \overrightarrow{\partial}_\mu \gamma^\mu + ieA_\mu \gamma^\mu + K_{\sigma\nu}\gamma^\sigma \overrightarrow{\partial}^\nu,$$

$$\overleftarrow{D} = \gamma^\mu \overleftarrow{\partial}_\mu - ieA_\mu \gamma^\mu + \overleftarrow{\partial}^\nu K_{\sigma\nu}\gamma^\sigma.$$

With the help of the Lagrangian (10) and equations (7-8) the canonical energy-momentum tensor looks like

$$T^{\mu\nu}_{can} = \frac{\partial L}{\partial(\partial_\mu A_\rho)}(\partial^\nu A_\rho) + \partial^\nu \overline{\psi}\frac{\partial L}{\partial(\partial_\mu \overline{\psi})} + \frac{\partial L}{\partial(\partial_\mu \psi)}\partial^\nu \psi - g^{\mu\nu}L.$$

As a result will get

$$T_{can}^{\mu\nu} = -(F^{\mu\rho} + G^{\mu\rho})\partial^{\nu} A_{\rho} + \frac{1}{4} g^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} + \frac{1}{4} g^{\mu\nu} G^{\rho\sigma} F_{\rho\sigma}. \quad (11)$$

Using an unambiguous of the energy-momentum tensor definition we will construct the metric energy-momentum tensor:

$$T_{metr}^{\mu\nu} = T_{can}^{\mu\nu} + \partial_{\rho}[(F^{\mu\rho} + G^{\mu\rho})A^{\nu}]. \quad (12)$$

Thus $T_{metr}^{\mu\nu}$ reads as:

$$T_{metr}^{\mu\nu} = F^{\mu\rho} F_{\rho}^{\nu} + \frac{1}{4} g^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} + G^{\mu\rho} F_{\rho}^{\nu} - j^{\mu} A^{\nu} + \frac{1}{4} g^{\mu\nu} G^{\rho\sigma} F_{\rho\sigma}, \quad (13)$$

where $j^{\mu} = \partial_{\nu}(F^{\nu\mu} + G^{\nu\mu})$.

Extracting a part of the metric tensor responsible for the polarizabilities in the (12) we find that

$$T_{metr}^{(\alpha\beta)\mu\nu} = G^{\mu\rho} F_{\rho}^{\nu} - \partial_{\rho} G^{\mu\rho} A^{\nu} + \frac{1}{4} g^{\mu\nu} G^{\rho\sigma} F_{\rho\sigma}. \quad (14)$$

Integrating by parts and using the definition $\vec{E} = -\vec{\nabla}\varphi$ according to [10] in the rest frame of the particle we obtain the energy density of interaction for the particle with polarizabilities and electromagnetic field:

$$\mathrm{E} = -\frac{2\pi}{m} \Theta^{00} (\alpha_0 \vec{E}^2 + \beta_0 \vec{H}^2),$$

where Θ^{00} is the energy density of the particle spin-1/2.

Conclusion

To assume the covariant Lagrangian of interaction of the electromagnetic field with a polarizable particle spin-1/2 as a basis in the Lagrangian covariant formalism the equations of motion have been found. The correlations between the covariant Lagrangian, canonical and metric energy-momentum tensors have been obtained. In the rest frame of the particle the energy density of interaction for the particle with polarizabilities and electromagnetic field have been found.

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P-NONINVARIANT EQUATION FOR SPIN 1/2 PARTICLE, TAKING INTO ACCOUNT OF THE EXTERNAL COULOMB FIELD

Within the theory of relativistic wave equations with extended sets of Lorentz group representations, a new P-noninvariant 20-component wave equation for spin 1/2 particle was proposed. The quantum mechanical Dirac-like P-noninvariant equation is solved in presence of external Cou-