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THE RAYLEIGH-GANS-DEBYE MODEL OF SHG FROM THE FINITE CYLINDER IN CASE OF NORMAL INCIDENCE OF A PLANE WAVE

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The exact solution of the problem of second-harmonic generation (SHG) by surfaces of particles requires a lot of computations. So the generalized nonlinear Rayleigh-Gans-Debye approximation [1] is used to find the solution. It is applicable for parameters of the media $\{|\xi-1|,|\eta-1|\} \ll 1; \xi = n^{2\omega}/n^{\omega}; \eta = n_p/n_m$, where $n^{\omega,2\omega}_{p,m}$ are the refractive indices of the material of the particle (p) and surrounding media (m) for the frequencies of electromagnetic waves ω and 2ω.

A cylindrical particle (height and radius of the base are h and a accordingly) coated with nonlinear material (of the thickness d_0) is placed at the origin of the Cartesian coordinate system in such a way that the center of the particle coincides with the origin and the axis of the cylindrical particle coincides with the Oz-axis. Equations for an incoming plane wave $\vec{E}^{in}$ propagating along Ox-axis and the nonlinear polarization $\vec{P}^{2\omega}$ are

$$\vec{E}^{in}(\vec{r}) = \vec{e}^{in} E_0 \exp\left(ik^{\omega}x - i\omega t\right);\ P_i^{2\omega}(\vec{x}) = \chi_{ijk}(\vec{x}) E_j^{in}(\vec{x}) E_k^{in}(\vec{x}), \tag{1}$$

where $\vec{e}^{in}$ is the unit vector of polarization of the wave, E_0 is the magnitude of the wave,

$$\chi_{ijk} = \chi_1 n_i n_j n_k + \chi_2 n_i \delta_{jk} + \chi_3\left(n_j \delta_{ki} + n_k \delta_{ij}\right) \tag{2}$$

is the nonlinear susceptibility tensor of the second order depending on the components of the normal vector of the surface and anisotropy coefficients χ_1, χ_2, χ_3. The scheme of the problem is presented in fig. 1.

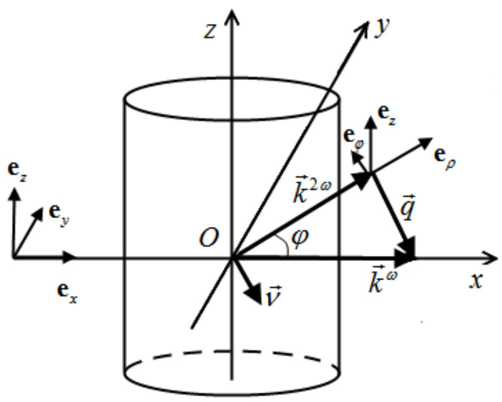

Fig. 1 The scheme of the problem

Then the electric intensity vector of the second harmonic generated by the lateral surface of the cylinder in the far zone is

$$\begin{gathered}\vec{E}_l(\vec{r}) = -2\pi i \mu^{2\omega}\left[(2\omega)^2/c^2\right]\left[\exp\left(ik^{2\omega}r\right)/r\right] d_0 a h E_0^2 \times \\ \times\left(1-\mathbf{e}_r \circ \mathbf{e}_r\right)\left[\sin\left(q_{\|}h/2\right)/\left(q_{\|}h/2\right)\right]\vec{f}_l,\end{gathered} \tag{3}$$

where $\mu^{2\omega}$ is the magnetic permeability of the surrounding media and

$$\vec{f}_l = \vec{v}\left[\left(\vec{e}_\perp^{in}\vec{v}\right)^2 \Gamma_1^{(J)}\left(q_\perp a\right) - \left(\vec{e}^{in}\vec{e}^{in}\right)\Gamma_2^{(J)}\left(q_\perp a\right) - \left(\vec{e}_\perp^{in}\vec{e}_\perp^{in}\right)\Gamma_4^{(J)}\left(q_\perp a\right)\right] - \\ -2\vec{e}^{in}\left(\vec{e}_\perp^{in}\vec{v}\right)\Gamma_3^{(J)}\left(q_\perp a\right) - 2\vec{e}_\perp^{in}\left(\vec{e}_\perp^{in}\vec{v}\right)\Gamma_4^{(J)}\left(q_\perp a\right) \tag{4}$$

is the vector that characterizes spatial distribution of the second harmonic wave. It depends on the auxiliary functions based on cylindrical Bessel functions $J_i\left(z\right)$

$$\Gamma_1^{(J)}\left(z\right) = \chi_1 J_3\left(z\right);\ \Gamma_2^{(J)}\left(z\right) = \chi_2 J_1\left(z\right);\ \Gamma_3^{(J)}\left(z\right) = \chi_3 J_1\left(z\right); \\ \Gamma_4^{(J)}\left(z\right) = \chi_1\left(J_1\left(z\right) + J_3\left(z\right)\right)/4 \tag{5}$$

The unit vector $\vec{v}$ is directed along the scattering vector $\vec{q}$:

$$\vec{q} = 2\vec{k}^{\omega} - \vec{k}^{2\omega}; q_\parallel = \left(\vec{q}\mathbf{e}_z\right);\quad q_\perp = \left|\vec{q} - \left(\vec{q}\mathbf{e}_z\right)\mathbf{e}_z\right|;\quad \vec{v} = \frac{\vec{q} - \left(\vec{q}\mathbf{e}_z\right)\mathbf{e}_z}{\left|\vec{q} - \left(\vec{q}\mathbf{e}_z\right)\mathbf{e}_z\right|}. \tag{6}$$

The electric intensity vector due to the butt-end of the cylinder is

$$\vec{E}_b\left(\vec{r}\right) = 2\pi i \mu^{2\omega}\frac{\left(2\omega\right)^2}{c^2}\frac{\exp\left(ik^{2\omega}r\right)}{r} d_0 a^2 E_0^2 \times \\ \times \sin\left(q_\parallel h/2\right)\left(J_0\left(q_\perp a\right) + J_2\left(q_\perp a\right)\right)\left(1 - \mathbf{e}_r \circ \mathbf{e}_r\right)\vec{f}_b, \tag{7}$$

$$\vec{f}_b = \chi_1\left(\vec{e}^{in}\mathbf{e}_z\right)\left(\vec{e}^{in}\mathbf{e}_z\right)\mathbf{e}_z + \chi_2\left(\vec{e}^{in}\vec{e}^{in}\right)\mathbf{e}_z + 2\chi_3\left(\vec{e}^{in}\mathbf{e}_z\right)\vec{e}^{in}. \tag{8}$$

The radiation patterns for SHG are presented in fig. 2. The parameters are

$$\vec{e}^{in} = -\mathbf{e}_y;\ \xi = 1.015;\ \chi_1 = 0;\ \chi_2 = 1;\ \chi_3 = 0;\ h = 0.075\pi;\ k^{\omega}a = 1.0. \tag{9}$$

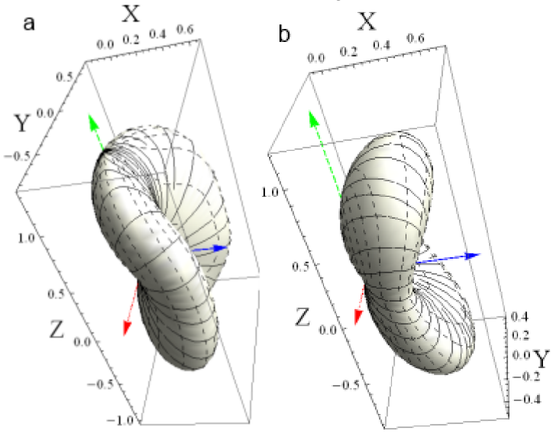

Fig. 2 The radiation patterns for SHG by the lateral surface of the cylindrical particle (a) and the butt-ends of the particle (b)

The solid arrow (along Ox) represents the direction of propagation of the incident wave, the dashed arrow (along Oz) coincides with the axis of the particle, the dotted arrow (opposite Oy) represents the direction of the polarization vector. The polarization of the generated wave is linear. The solid lines on the surface of the radiation patterns represent the projection of the polarization vector of the second harmonic wave on the surface of the corresponding pattern.

[1] Viarbitskaya, S. Size dependence of second-harmonic generation at the surface of microspheres / S. Viarbitskaya, V. Kapshai, P. van der Meulen, T. Hannson // Phys. Rev. A – 2010. – № 81. – P. 053850-1–053850-12.